PIMS Mini-workshop on calculus of variations and PDEs around the work of Alessio Figalli

February 8-9, 2019

University of British Columbia

Program Schedule

Getting Started

Get connected: Select the "ubcvisitor" wireless network on your wireless device. Open up a web browser, and you will be directed to the login page.

Locations

Friday February 8: ESB 2012

Saturday February 9: MATH 102

Schedule

Friday February 8:

3:00pm- 3:30pm — Light Reception at PIMS Lounge, ESB 4133

3:30pm- 4:30pm — PIMS -UBC Math Distinguished Colloquium: **Alessio Figalli, ETH Zurich**:

Regularity of interfaces in phase transitions via obstacle problems

4:45pm- 5:45pm — **Robert McCann, University of Toronto:**

Displacement convexity of Boltzmann's entropy characterizes positive energy in general relativity

Saturday February 9: Location, MATH 100 (Please note room change)

9:00am - 10:00am — **Ovidiu Savin, Columbia University**

Sharp $W^{2,p}$ regularity results in the optimal transport problem between convex domains

10:00am - 10:20 am — **Coffee break**

10:20am - 10:50am — **Aaron Zeff Palmer, University of British Columbia**

A solution to the Monge transport problem for Brownian martingales

11:00am -11:30pm — **Arunima Bhattacharya, University of Oregon**

Regularity Bootstrapping for 4th-order Nonlinear Elliptic Equations

11:40pm- 12:10pm — **Seunghyeok Kim, Hanyang University**

A compactness theorem of the fractional Yamabe problem

12:10pm - 1:30pm — **Lunch**

1:30pm - 2:00pm — **Yong Liu, University of Science and Technology of China**

Travelling wave solutions of the GP equation

2:10pm - 2:40pm — **Hyunju Kwon, University of British Columbia**

Strong ill-posedness of the logarithmically regularized 2D Euler equations in the borderline spaces

2:40pm - 3:00pm — **Break**

3:00pm- 4:00pm — **Francesco Maggi, University of Texas**

Soap films, soap bubbles, and almost critical points in geometric variational problems

4:00pm — End of workshop

Titles and Abstracts

Arunima Bhattacharya, University of Oregon

Regularity Bootstrapping for 4th-order Nonlinear Elliptic Equations

We consider nonlinear 4th-order elliptic equations of double divergence type. We show that for a certain class of equations where the nonlinearity is in the Hessian, solutions that are $C^{2,\alpha}$ enjoy interior estimates on all derivatives.

Alessio Figalli, ETH Zurich

Regularity of interfaces in phase transitions via obstacle problems

The so-called Stefan problem describes the temperature distribution in a homogeneous medium undergoing a phase change, for example ice melting to water. An important goal is to describe the structure of the interface separating the two phases. In its stationary version, the Stefan problem can be reduced to the classical obstacle problem, which consists in finding the equilibrium position of an elastic membrane whose boundary is held fixed and which is constrained to lie above a given obstacle. The aim of this talk is to give a general overview of the classical theory of the obstacle problem, and then discuss recent developments on the structure of interfaces, both in the static and the parabolic settings.

Seunghyeok Kim, Hanyang University, Korea.

A compactness theorem of the fractional Yamabe problem

Since Schoen raised the question of compactness of the full set of solutions of the Yamabe problem in the C^0 topology (in 1988), it had been generally expected that the solution set must be C^0-compact unless the underlying manifold is conformally equivalent to the standard sphere.
In 2008-09, Khuri, Marques, Schoen himself and Brendle gave the surprising answer that the expectation holds whenever the dimension of the manifold is less than 25 (under the validity of the positive mass theorem whose proof is recently announced by Schoen and Yau) but does not if the dimension is 25 or greater. On the other hand, concerning the fractional Yamabe problem on a conformal infinity of an asymptotically hyperbolic manifold, Kim, Musso, and Wei considered an analogous question and constructed manifolds of high dimensions whose solution sets are C^0-noncompact (in 2017). In this talk, we show that the solution set is C^0-compact if the conformal infinity is non-umbilic and its dimension is 7 or greater. Our proof provides a general scheme toward other possible compactness theorems for the fractional Yamabe problem. This is joint work with Monica Musso (University of Bath, United Kingdom) and Juncheng Wei (University of British Columbia, Canada).

Hyunju Kwon, University of British Columbia

Strong ill-posedness of the logarithmically regularized 2D Euler equations in the borderline spaces

The question of whether the incompressible Euler equations are well-posed in the borderline spaces has attracted a lot of attention in the recent decades. In order to understand how the solution behaves in the borderline spaces, the logarithmically regularized 2D Euler equations were introduced. The velocity u in the original vorticity equation is replaced by $T_\gamma u$ where $T_\gamma = \ln^{-\gamma}(e-\Delta)$, $\gamma>0$. In the borderline Sobolev space, the local-in-time well-posedness has been proved for $\gamma>\frac 12$ by Chae and Wu, while the strong ill-posedness of 2D Euler vorticity equation ($\gamma=0$) was established recently by Bourgain and Li.
In this talk, I will outline how these equations are strongly ill-posed also in the intermediate regime $0<\gamma \le \frac 12$. This work completes our understanding regarding the local well-posedness of the logarithmically regularized 2D Euler equations in the borderline Sobolev space.

Yong Liu, University of Science and Technology of China

Travelling wave solutions of the GP equation

We consider the existence of travelling wave solutions of the Gross-Petaevskii equation in the plane. When the travelling speed is close zero, we use Adler-Moser polynomials to construct mutli-vortex solutions. When the travelling speed is in the subsonic region, we use rational solutions of KP-I equation to construct solution which are close to 1. This provides another point of view of the relation between the KdV equation and the KP-I equation, which are both classical integrable systems.

Francesco Maggi, University of Texas

Soap films, soap bubbles, and almost critical points in geometric variational problems

We explain how the study of almost critical points of surface energies arise in the study of the equilibrium and evolution of soap films, soap bubbles, and crystal grains. We introduce some theorems that describe (qualitatively and, sometimes, quantitatively) the possible bubbling configurations, and that provide useful criteria to exclude bubbling. This talk is based on several papers written in the past few years in collaboration with G. Ciraolo (U Palermo), M. Delgadino (Imperial College London), D. King (UT Austin), C. Mihaila (U Chicago), R. Neumayer (Nothwestern U), A. Scardicchio (ICTP Trieste), and S. Stuvard (UT Austin).

Robert McCann, University of Toronto

Displacement convexity of Boltzmann's entropy characterizes positive energy in general relativity

Einstein's theory of gravity is based on assuming that the fluxes of a energy and momentum in a physical system are proportional to a certain variant of the Ricci curvature tensor on a smooth 3+1 dimensional spacetime. The fact that gravity is attractive rather than repulsive is encoded in the positivity properties which this tensor is assumed to satisfy. Hawking and Penrose (1971) used this positivity of energy to give conditions under which smooth spacetimes must develop singularities. By lifting fractional powers of the Lorentz distance between points on a globally hyperbolic spacetime to probability measures on spacetime events, we show that the strong energy condition of Hawking and Penrose is equivalent to convexity of the Boltzmann-Shannon entropy along the resulting geodesics of probability measures. This new characterization of of the strong energy condition on globally hyperbolic manifolds also makes sense in (non-smooth) metric measure settings, where it has the potential to provide a framework for developing a theory of gravity which admits certain singularities and can be continued beyond them. It provides a Lorentzian analog of Lott, Villani and Sturm's metric-measure theory of lower Ricci bounds, and hints at new connections linking gravity to the second law of thermodynamics. Preprint available at http://www.math.toronto.edu/mccann/papers/GRO.pdf

Aaron Zeff Palmer, University of British Columbia

A solution to the Monge transport problem for Brownian martingales

The Monge transport problem for Brownian martingales is an optimal transport problem where the transport plans are given by stopped Brownian motion. We solve this problem for transport costs that satisfy a stochastic version of the twist condition, by finding a stopping time (analogous to a transport map) that minimizes the expected cost. This in particular includes the distance cost case $c=|x-y|$. We prove existence and uniqueness of the solution, and characterize it as the first hitting time of a barrier that is given by the coincidence set for the dual obstacle problem. This is joint work with N Ghoussoub, YH Kim, and TS Lim.

Ovidiu Savin, Columbia University

Sharp $W^{2,p}$ regularity results in the optimal transport problem between convex domains

Given two domains with the same volume, the optimal transport, in its most basic form, consists in mapping one domain into the other by a measure preserving transformation which minimizes a total transport cost. For the quadratic cost, the regularity theory of the map was developed by L. Caffarelli in the early 90s, by making use of its connection with the Monge-Ampere equation. In my talk I will review these results and discuss some recent work in collaboration with Hui Yu concerning the global $W^{2,p}$ estimates for the convex potential.

Yannick Sire, Johns Hopkins University

Geometric bifurcations for constant Q-curvature metrics

I will describe some recent results dealing with non-uniqueness of prescribing constant Q-curvature metrics on closed manifolds of dimension 5 and higher. We will provide explicit examples and design a general method to obtain such non-uniquess results for in the regular and singular cases of the problem.