Overview of Some Results in Energy Market Modelling and Clean Energy Vision *

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Outline of Presentation

- *Introduction*

- *Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market*

- *Variance and Volatility Swaps in Energy Markets*

- *Weather Derivatives in Energy Markets*

- *Pricing Crude Oil Options using Lévy Processes*

- *Energy Market Contracts with Delayed and Jumped Volatilities*

- *A Vision to Transition to 100% Wind, Water & Solar Energy in Canada*

Introduction: Abstract

The talk overviews my recent results in energy market modelling, including:
- option pricing formula for a mean-reversion asset,
- variance and volatility swaps in energy markets,
-applications of weather derivatives in energy markets,
- pricing crude oil options using Levy processes,
-energy contracts modelling with delayed and jumped volatilities.

I will also talk about
-the clean renewable energy prospective.

A Brief Overview

A Brief Overview

•**Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market**
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233*)
•**Variance and Volatility Swaps in Energy Markets**
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)
•**Weather Derivatives in Energy Markets**
(*The J. Energy Markets, V.8, N.1, March 2015, 59-76)*
•**Pricing Crude Oil Options using Lévy Processes**
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64*)
•**Energy Market Contracts with Delayed and Jumped Volatilities**
(*Handbook of Energy Finance: Theories, Practices and Simulations, World Scientific, 2019*)
•**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada**

A Brief Overview

• **Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market**
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

Some commodity prices, like oil and gas, exhibit the mean reversion, unlike stock price. It means that they tend over time to return to some long-term mean.

Mean-Reversion

We present explicit option pricing formula for a mean-reverting asset in energy market.

A Brief Overview

•**Variance and Volatility Swaps in Energy Markets**
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

We calculate variance and volatility swaps in energy market

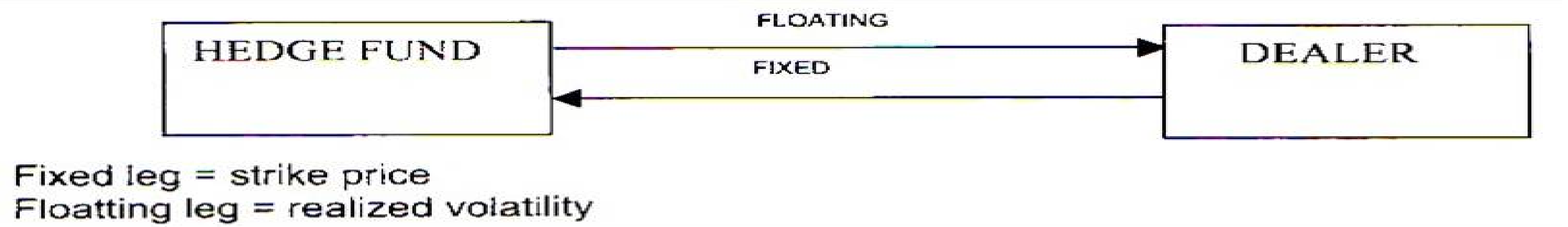


Fig. 1.Hedge Fund+Dealer

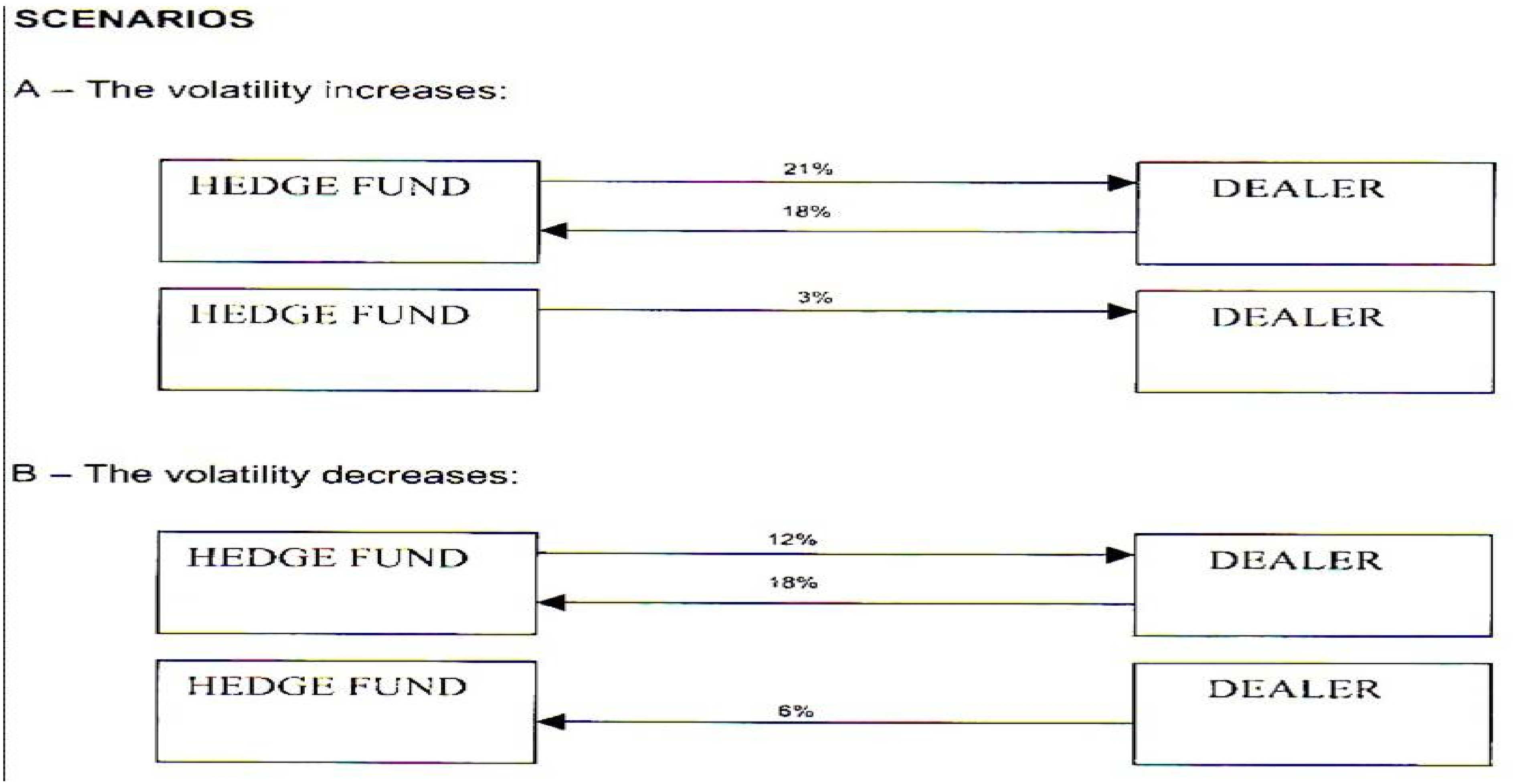
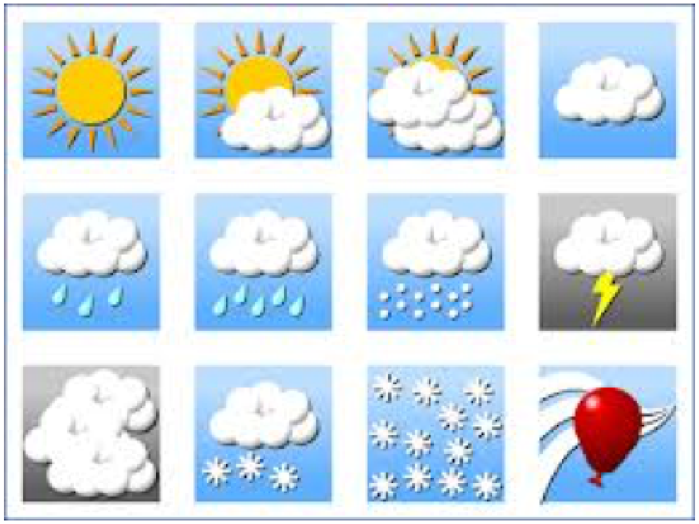


Fig. 2. Scenarios: A-volatility increases and B-volatility decreases

A Brief Overview

•**Weather Derivatives in Energy Markets**
(*The J. Energy Markets, V.8, N.1, March 2015, 59-76)*



A Brief Overview

•**Weather Derivatives in Energy Markets**
(*The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

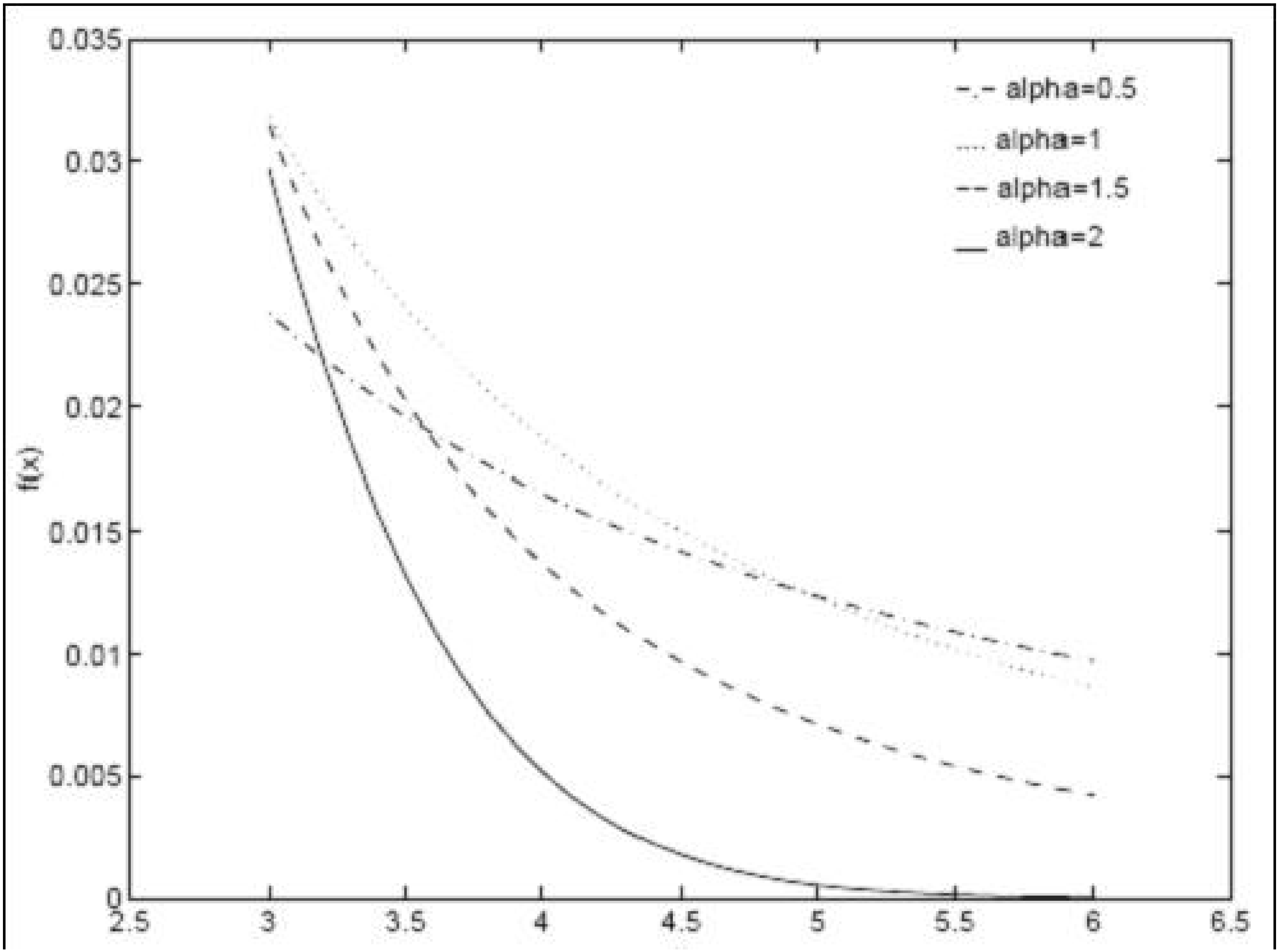
We use future contracts written on temperature to demonstrate the hedging strategies for commodities as an application of weather derivatives.

Our focus will be on the dynamic hedging strategy of energy futures using temperature futures and constructing the hedge ratio.

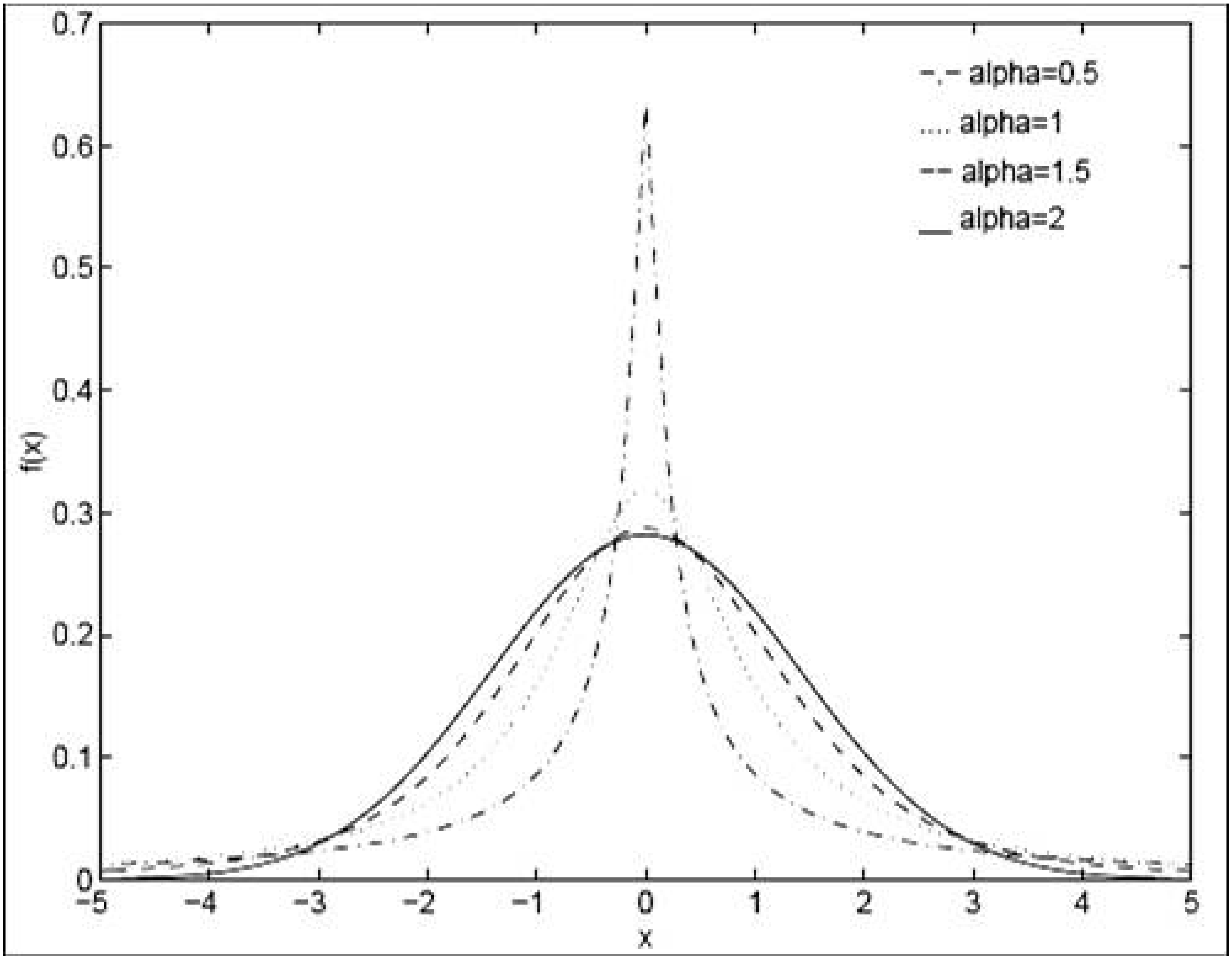
A Brief Overview

•**Pricing Crude Oil Options using Lévy Processes**
(*The J. Energy Markets, V.9, N 1, March 216, 47-64***)**

Crude oil prices exhibit significant volatility over time and the distribution of returns on crude oil prices show fat tails and skewness, and they barely follow normal distribution.



Normal and α-stable Tails



Normal and α-stable Densities

A Brief Overview

•**Pricing Crude Oil Options using Lévy Processes**
(*The J. Energy Markets, V.9, N 1, March 216, 47-64***)**

This is the reason we use Normal Gaussian Process (NIG), Jump Diffusion Process (JD), and Variance-Gamma Process (VG) as three Lévy processes that do not have these drawbacks and their tails carry heavier mass than normal distribution. Our results indicate that all these three Levy processes have very good out of sample results for near at the money options than others.

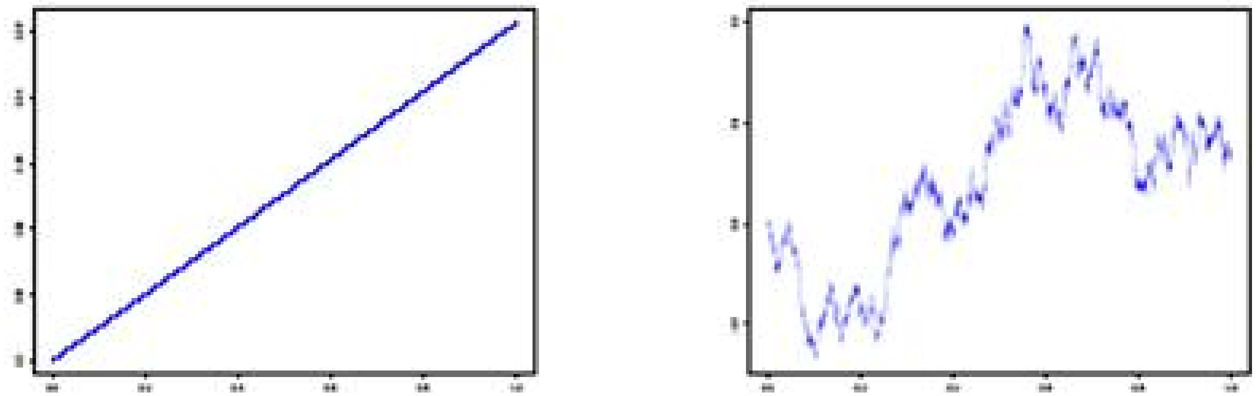


FIGURE 2.4. Examples of Lévy processes: linear drift (left) and Brownian motion

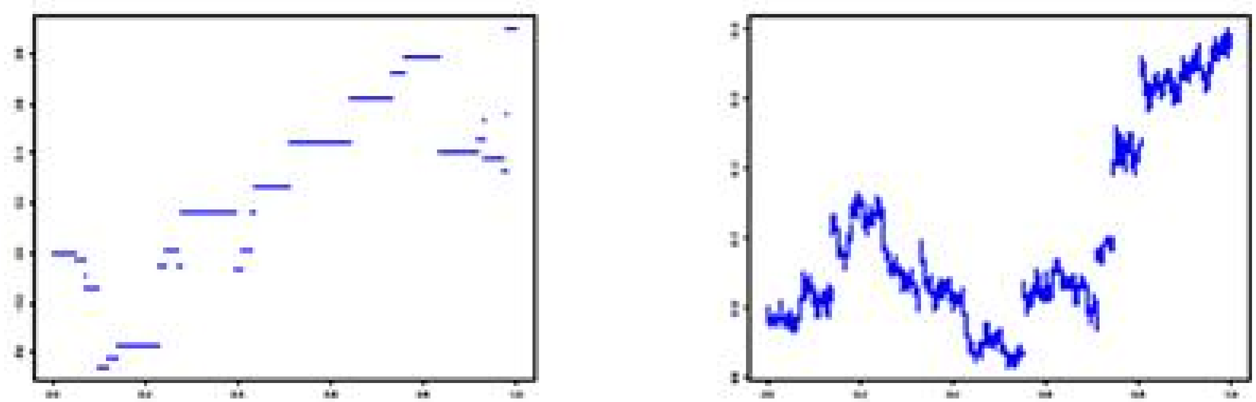


FIGURE 2.5. Examples of Lévy processes: compound Poisson process (left) and Lévy jump-diffusion

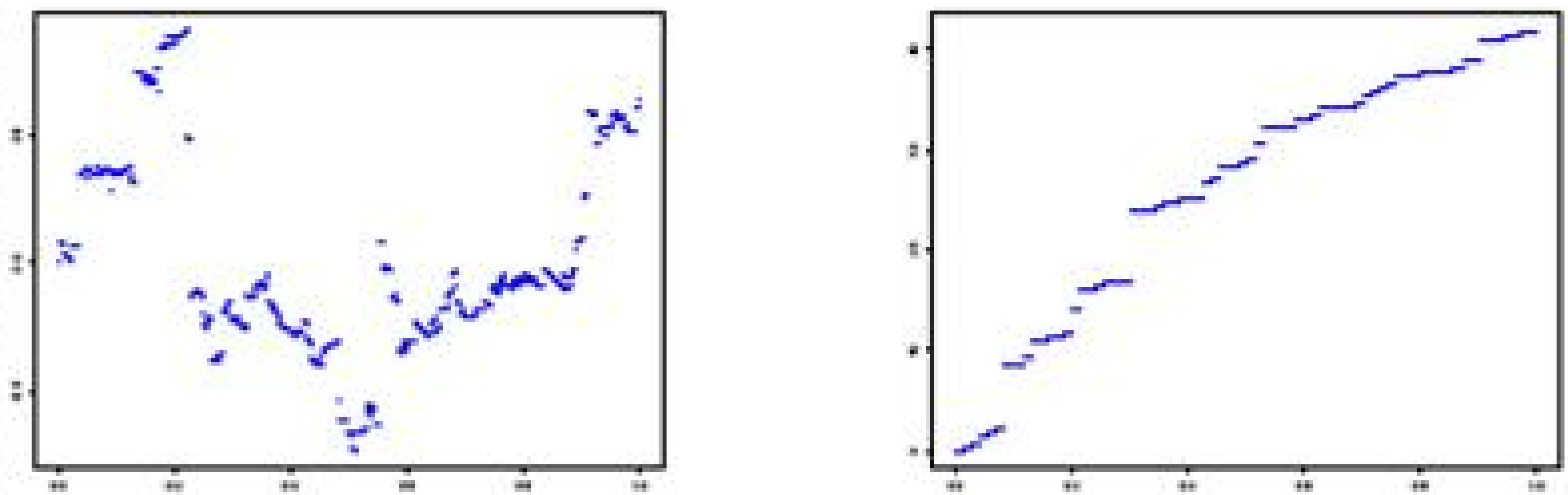
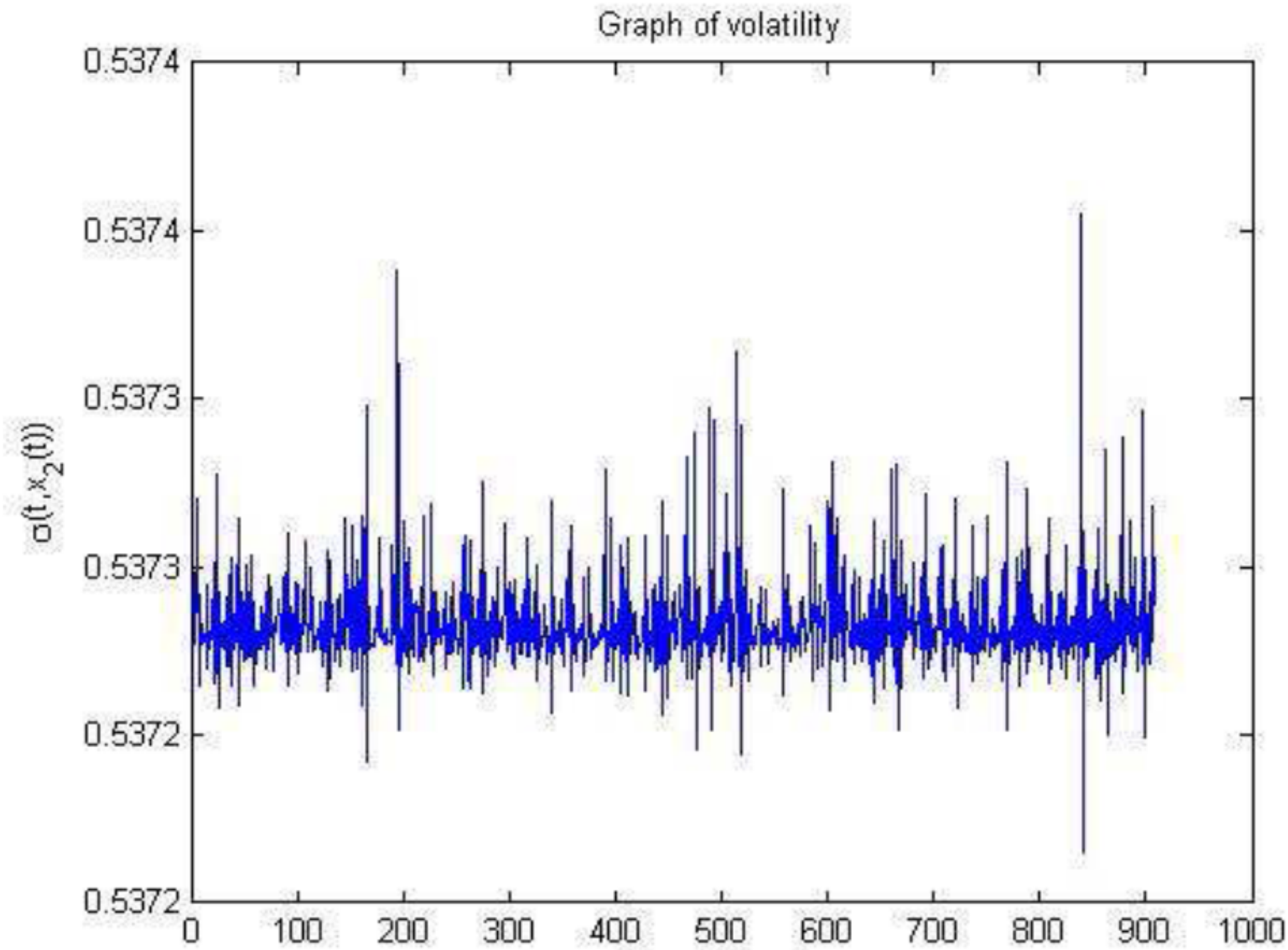


FIGURE 7.10. Simulated path of a normal inverse Gaussian (left) and an inverse Gaussian process

A Brief Overview

•**Energy Market Contracts with Delayed and Jumped Volatilities**
(*Handbook of Energy Finance: Theories, Practices and Simulations, World Scientific, 2019***)**
In this Part we concentrate on stochastic modelling and pricing of energy markets' contracts for stochastic volatilities with delay and jumps. Our model of stochastic volatility exhibits jumps and also past-dependence: the behaviour of a stock price right after a given time t not only depends on the situation at t, but also on the whole past (history) of the process $S(t)$ up to time t.


Graph of volatility
σ(t,x_2(t))
0.5374
0.5374
0.5373
0.5373
0.5372
0.5372
0
100
200
300
400
500
600
700
800
900
1000

A Brief Overview

• **Energy Market Contracts with Delayed and Jumped Volatilities**
(*Handbook of Energy Finance: Theories, Practices and Simulations, World Scientific, 2019***)**

The basic products in these markets are spot, futures and forward contracts and options written on these. We study forwards and swaps. A numerical examples is presented for stochastic volatility with delay using the Henry Hub daily natural gas data (1997-20011).

A Brief Overview

•A Vision to Transition to 100% Wind, Water & Solar Energy in Canada

A group of U.S. civil engineering has calculated that Canada could be completely powered by renewable energy, if we just decide to do it.

They say that would save \$110.1 billion on health care costs every year and prevent $9,884$ annual air pollution deaths.

Their research is available at thesolutionsproject.org.

PART I

Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market

(*J. Numer. Appl. Math., V.1(96), 2008, 216-233*)

Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

Some commodity prices, like oil and gas, exhibit the mean reversion, unlike stock price. It means that they tend over time to return to some long-term mean. In this talk we consider a risky asset S_t following the mean-reverting stochastic process given by the following stochastic differential equation

$$dS_t = a(L - S_t)dt + \sigma S_t dW_t,$$

where W is a standard Wiener process, $\sigma > 0$ is the volatility, the constant L is called the 'long-term mean' of the process, to which it reverts over time, and $a > 0$ measures the 'strength' of mean reversion.

Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233*)

This mean-reverting model is a one-factor version of the two-factor model made popular in the context of energy modelling by Pilipovic (1997). We call it *continuous-time GARCH* or *inhomogeneous geometric Brownian motion* model.

Using a change of time method we find an explicit solution of this equation and using this solution we are able to find the variance and volatility swaps pricing formula under the physical measure. Then, using the same argument, we find the option pricing formula under risk-neutral measure.

Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

Black's model (1976) and Schwartz's model (1997) have become a standard approach to the problem of pricing options on commodities. These models have the advantage of mathematical convenience, in that they give rise to closed-form solutions for some types of options (See Wilmott (2000)).

Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

$$\begin{aligned} C_T &= e^{-(r+a)T}S(0)N(y_+) - e^{-rT}KN(y_-) \\ &+ Le^{-(r+a)T}[(e^{aT}-1) - \textstyle\int_0^{y_0} zF_T(dz)], \end{aligned}$$

where

$$y_+ := \sigma\sqrt{T} - y_0 \quad and \quad y_- := -y_0,$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}dy.$$

Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

$$y_0 = \frac{\ln(\frac{K}{S(0)}) + (\frac{\sigma^2}{2} + a)T}{\sigma\sqrt{T}} - \frac{\ln(1 + \frac{aL}{S(0)} \int_0^T e^{as} e^{-\sigma y_0 \sqrt{s} + \frac{\sigma^2 s}{2}} ds)}{\sigma\sqrt{T}}$$

and $F_T(dz)$ is the distribution function of

$$\eta(T) = \frac{4ae^{-aT}}{\sigma^2} e^{-2B_{\frac{\sigma^2 T}{4}}} \int_0^{\sigma^2 T/4} e^{2((2a/\sigma^2+1)+W_s)} ds$$

(Can be estimated and calculated by M. Yor's (1992) result: On some exponential functions of Brownian motion, *Advances in Applied Probability*, Vol. 24, No. 3, 509-531).

Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market

(*J. Numer. Appl. Math., V.1(96), 2008, 216-233*)

$$\begin{aligned} C_T^* &= e^{-(r+a^*)T} S(0) N(y_+) - e^{-rT} K N(y_-) \\ &+ L^* e^{-(r+a^*)T} [(e^{a^*T} - 1) - \int_0^{y_0} z F_T^*(dz)], \end{aligned}$$

where

$$y_+ := \sigma\sqrt{T} - y_0 \quad and \quad y_- := -y_0,$$

$$a^* := a + \lambda\sigma, \quad L^* := \frac{aL}{a + \lambda\sigma},$$

Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

y_0 is the solution of the following equation

$$y_0 = \frac{\ln(\frac{K}{S(0)}) + (\frac{\sigma^2}{2} + a^*)T}{\sigma\sqrt{T}} - \frac{\ln(1 + \frac{a^* L^*}{S(0)} \int_0^T e^{a^* s} e^{-\sigma y_0 \sqrt{s} + \frac{\sigma^2 s}{2}} ds)}{\sigma\sqrt{T}},$$

and $F_T^*(dz)$ is the probability distribution $F_T(dz)$ as above, but instead of a we have to take $a^* = a + \lambda\sigma$, λ is a market price of risk.

Remark: When $L^* = 0$ and $a^* = -r$, then the explicit option pricing formula is the well-known Black-Scholes formula!

Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market
(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

Numerical Example: AECO Natural GAS Index (1 May 1998-30 April 1999)
We shall calculate the value of a European call option on the price of a daily natural gas contract. To apply our formula for calculating this value we need to calibrate the parameters $a, \quad L, \quad \sigma$ and λ. These parameters may be obtained from futures prices for the AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999 (see Bos, Ware and Pavlov (2002), p.340). The parameters pertaining to the option are the following:

Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market

(*J. Numer. Appl. Math., V.1(96), 2008, 216-233***)**

Price and Option Process Parameters						
T	a	σ	L	λ	r	K
6 months	4.6488	1.5116	2.7264	0.1885	0.05	3

From this table we can calculate the values for a^* and L^* :

$$a^* = a + \lambda\sigma = 4.9337,$$

and

$$L^* = \frac{aL}{a + \lambda\sigma} = 2.5690.$$

For the value of S_0 we can take $S_0 \in [1, 6]$.

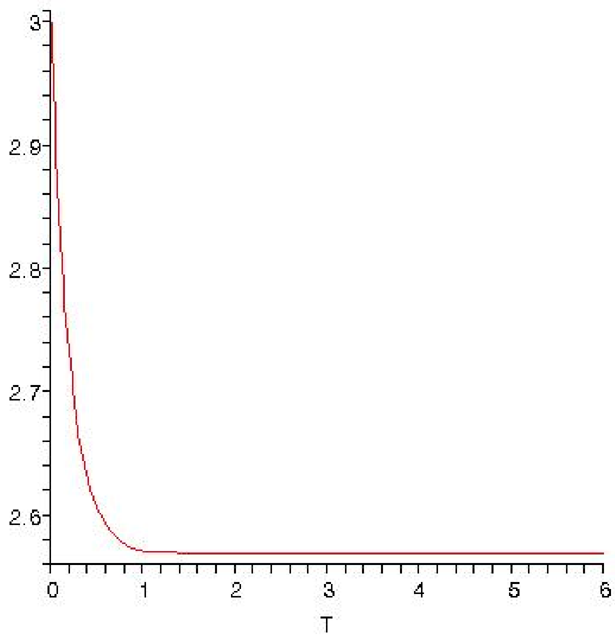


Fig. 1. Dependence of ES_t on T (AECO Natural Gas Index (1 May 1998-30 April 1999))

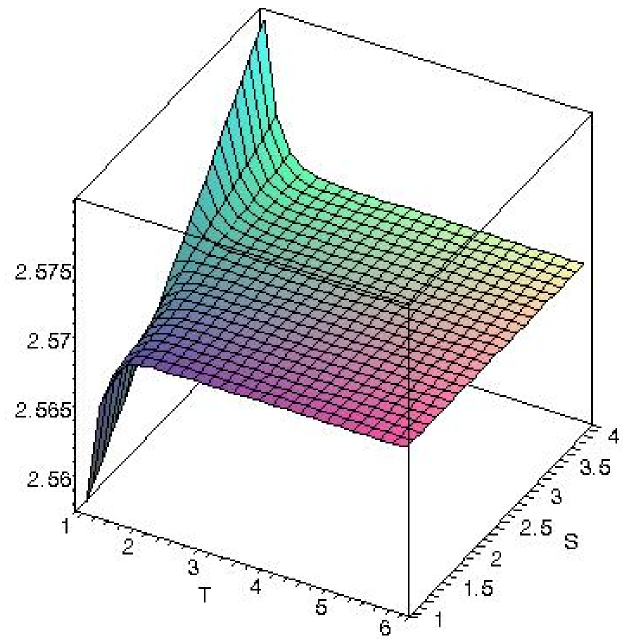


Fig. 2. Dependence of ES_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

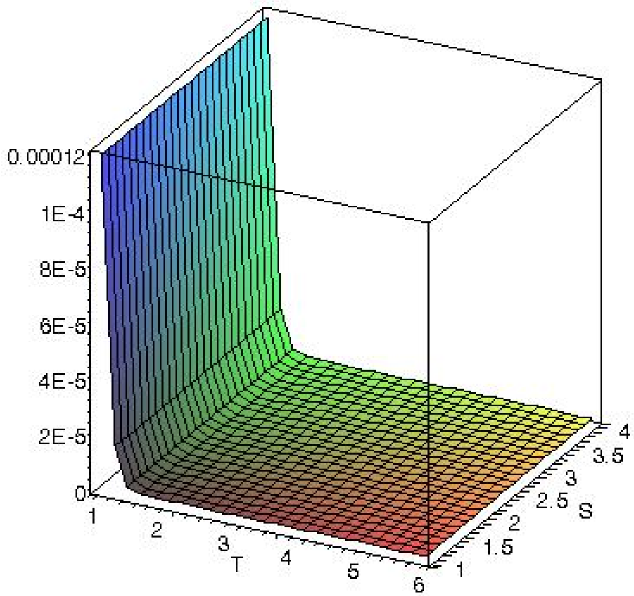


Fig. 3. Dependence of variance of S_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

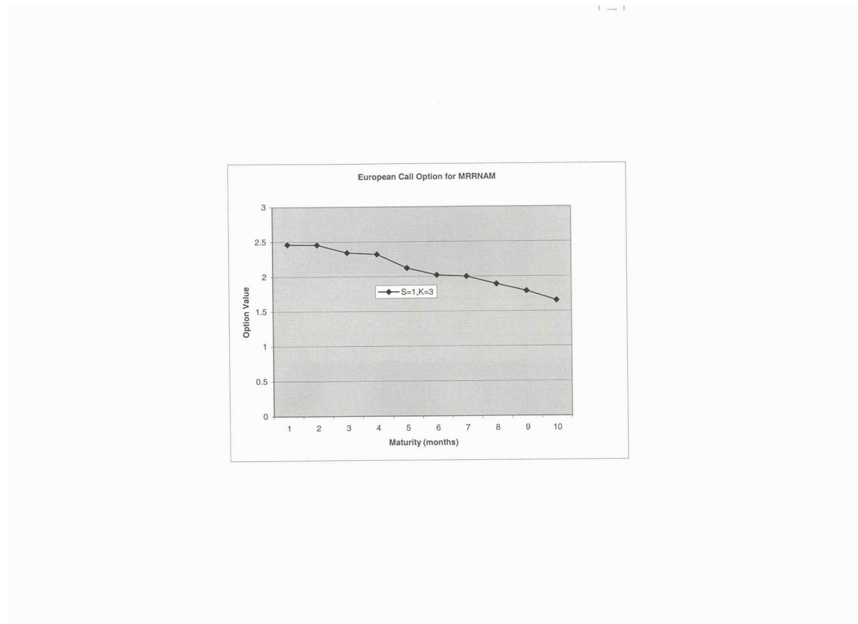


Fig. 4. Dependence of European Call Option Price on Maturity (months) ($S(0) = 1$ and $K = 3$) (AECO Natural Gas Index (1 May 1998-30 April 1999))

PART II
Variance and Volatility Swaps in Energy Markets

(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

Variance swaps are quite common in commodity, e.g., in energy market, and they are commonly traded. We consider Ornstein-Uhlenbeck process for commodity asset with stochastic volatility following continuous-time GARCH model or Pilipovic (1998) one-factor model. The classical stochastic process for the spot dynamics of commodity prices is given by the Schwartz' model (1997). It is defined as the exponential of an Ornstein-Uhlenbeck (OU) process, and has become the standard model for energy prices possessing mean-reverting features.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

Our focus on energy commodities derives from two reasons:

1) energy is the most important commodity sector, and crude oil and natural gas constitute the largest components of the two most widely tracked commodity indices: the Standard & Poors Goldman Sachs Commodity Index (S & P GSCI) and the Dow Jones-AIG Commodity Index (DJ-AIGCI);

2) existence of a liquid options market: crude oil and natural gas indeed have the deepest and most liquid options marketss among all commodities.

The idea is to use variance (or volatility) swaps on futures contracts.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

At maturity, a variance swap pays off the difference between the realized variance of the futures contract over the life of the swap and the fixed variance swap rate. And since a variance swap has zero net market value at initiation, absence of arbitrage implies that the fixed variance swap rate equals to conditional risk-neutral expectation of the realized variance over the life of swap. Therefore, e.g., the time-series average of the payoff and/or excess return on a variance swap is a measure of the variance risk premium. Variance risk premia in energy commodities, crude oil and natural gas, has been considered by A. Trolle and E. Schwartz (2009). The same methodology as in Trolle & Schwartz (2009) was used by Carr & Wu (2009) in their study of equity variance risk premia.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

The S & P GSCI is comprised of 24 commoditieswith the weight of each commodity determined by their relative levels of world production over the past five years. The DJ-AIGCI is comprised of 19 commodities with the weight of each component determined by liquidity and world production values, with liquidity being the dominant factor. Crude oil and natural gas are the largest components in both indices. In 2007, their weight were 51.30% and 6.71%, respectively, in the S & P GSCI and 13.88% and 11.03%, respectively, in the DJ-AIGCI.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

The Chicago Board Options Exchange (CBOE) recently introduced a Crude Oil Volatility Index (ticker symbol OVX). This index also measures the conditional risk-neutral expectation of crude oil variance, but is computed from a cross-section of listed options on the United States Oil Fund (USO), which tracks the price of WTI as closely as possible.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

The CBOE Crude Oil ETF Volatility Index (Oil VIX, Ticker - OVX) measures the market's expectation of 30-day volatility of crude oil prices by applying the VIX methodology to United States Oil Fund, LP (Ticker - USO) options spanning a wide range of strike prices (see Figures below. Courtesy-CBOE: http://www.cboe.com/micro/oilvix/introduction.aspx).
We have to notice that crude oil and natural gas trade in units of 1,000 barrels and 10,000 British thermal units (mmBtu), respectively. Usually, prices are quoted as US dollars and cents per barrel or mmBtu.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

In this talk, we consider a risky asset in energy market with stochastic variance following a mean-reverting stochastic process satisfying the following SDE (continuous-time GARCH(1,1) model):

$$d\sigma^2(t) = a(L - \sigma^2(t))dt + \gamma\sigma^2(t)dW_t,$$

where a is a speed of mean reversion, L is the mean reverting level (or equilibrium level), γ is the volatility of volatility $\sigma(t)$, W_t is a standard Wiener process.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

Using a change of time method we find an explicit solution of this equation, and using this solution we are able to find the variance and volatility swaps pricing formula under the physical measure. Then, using the same argument, we find the option pricing formula under risk-neutral measure. We applied Brockhaus-Long (2000) approximation to find the value of volatility swap. A numerical example for the AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999 is presented.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**
How Does the Volatility/Variance Swap Work

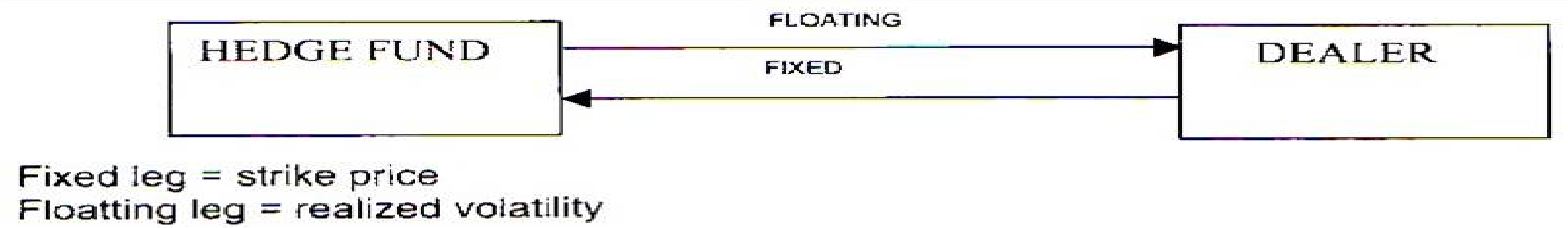


Fig. 1.Hedge Fund+Dealer

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**
How Does the Volatility/Variance Swap Work
Possible Scenarios

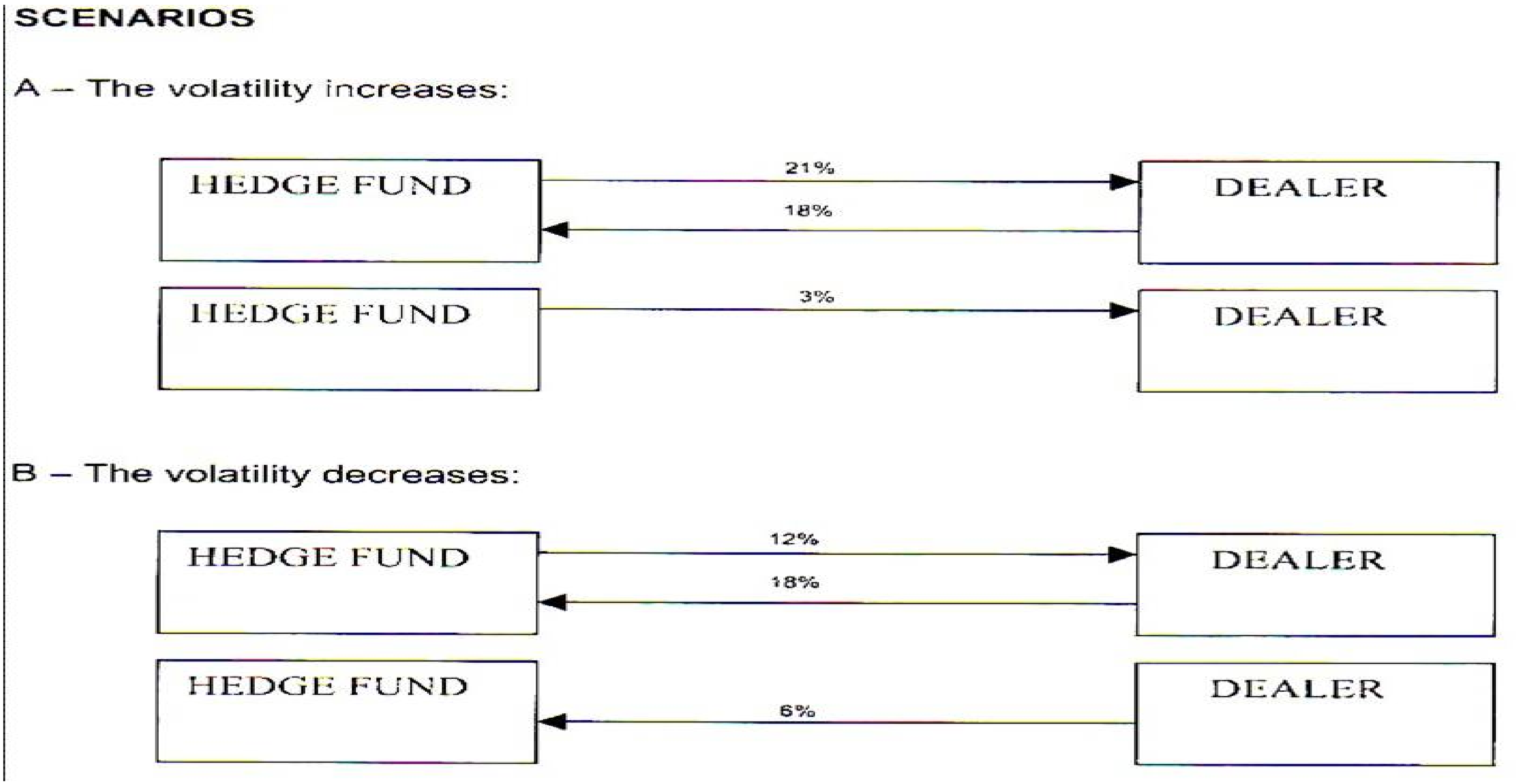


Fig. 2. Scenarios: A-volatility increases and B-volatility decreases

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**
Risk-neutral Stochastic Volatility Model (SVM)

$$d\sigma^2(t) = a^*(L^* - \sigma^2(t))dt + \gamma\sigma^2(t)dW_t^*,$$

where

$$a^* := a + \lambda\gamma, \quad L^* := \frac{aL}{a + \lambda\gamma},$$

$W_t^* := W_t + \lambda t$, and λ is the market price of risk.

Variance and Volatility Swaps in Energy Markets
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**
Variance and Volatility Swaps for Risk-Neutral SVM

For the variance swap we have:

$$E^*\sigma_R^2 := EV := \frac{1}{T}\int_0^T E\sigma^2(t)dt = \frac{(\sigma^2(0) - L^*)}{a^*T}(1 - e^{-a^*T}) + L^*.$$

For the volatility swap we obtain:

$$E^*\sqrt{V} \;\approx\; \sqrt{E^*V} - \frac{Var^*(V)}{8(E^*V)^{3/2}}.$$

Variance and Volatility Swaps in Energy Markets: Numerical Example

(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

Parameters			
a	γ	L	λ
4.6488	1.5116	2.7264	0.18

Variance and Volatility Swaps in Energy Markets: Numerical Example

(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

From this table we can calculate the values for risk adjusted parameters a^* and L^* :

$$a^* = a + \lambda\gamma = 4.9337,$$

and

$$L^* = \frac{aL}{a + \lambda\gamma} = 2.5690.$$

For the value of $\sigma^2(0)$ we can take $\sigma^2(0) = 2.25$.

For variance swap and for volatility swap with risk adjusted parameters we use formula obtained above.

Variance and Volatility Swaps in Energy Markets: Figures
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

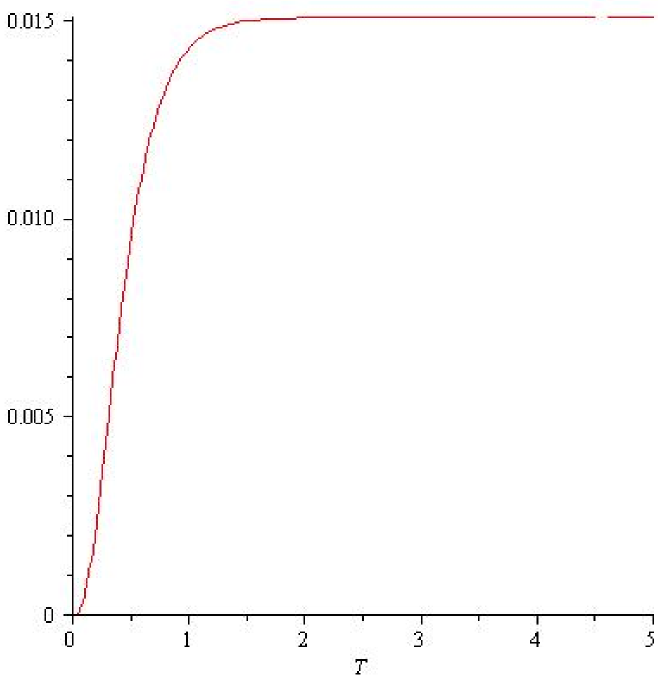


Fig. 1: Variance Swap

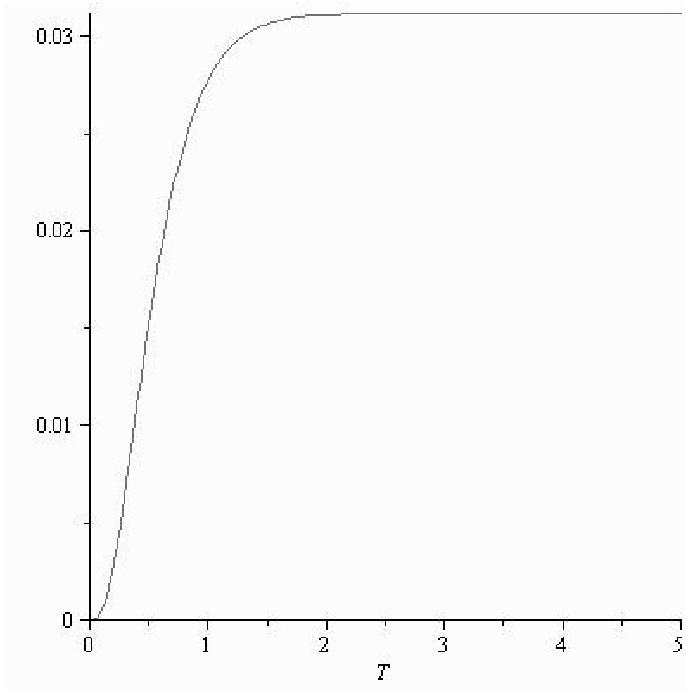


Fig. 2: Volatility Swap

Variance and Volatility Swaps in Energy Markets: Figures
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

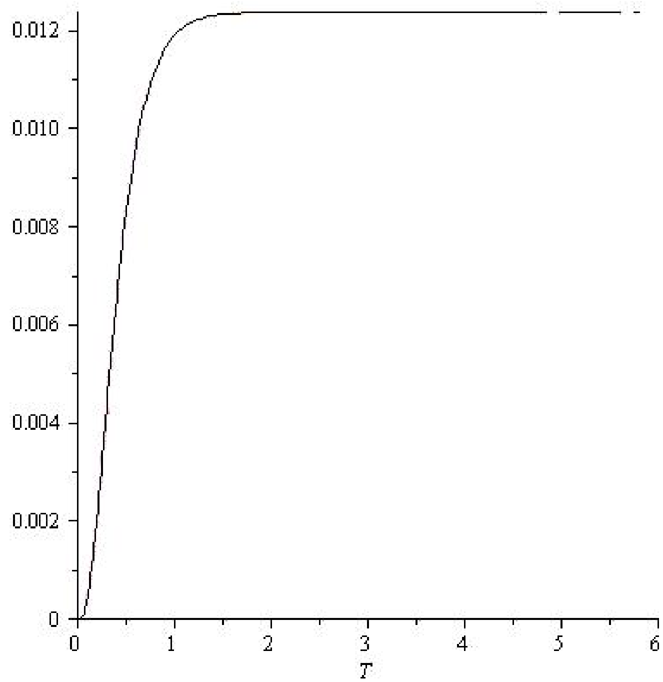


Fig. 3: Variance Swap (Risk Adjusted Parameters)

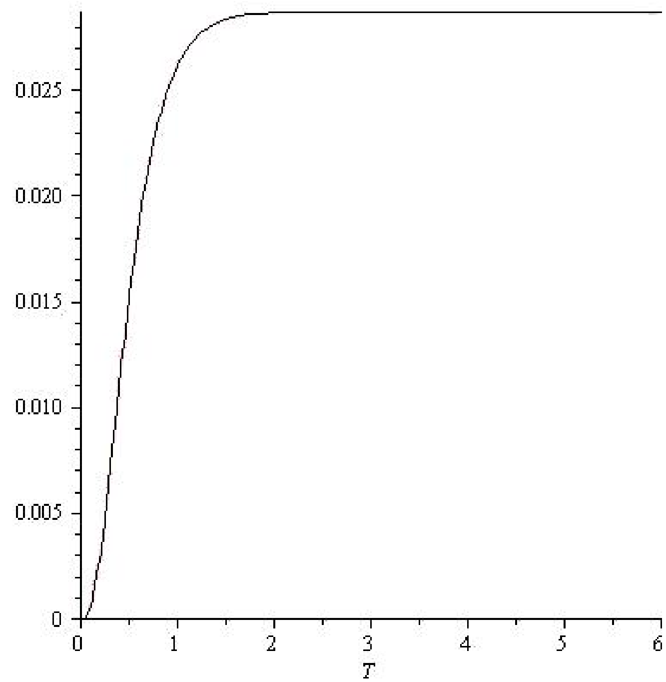


Fig. 4: Volatility Swap (Risk Adjusted Parameters)

Variance and Volatility Swaps in Energy Markets: Figures
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50***)**

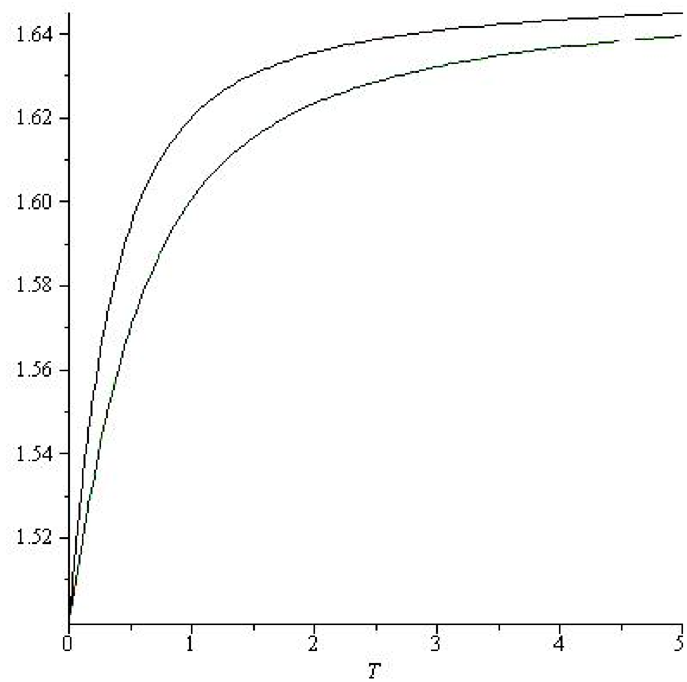


Fig. 5: Comparison: Adjusted and Non-Adjusted Price

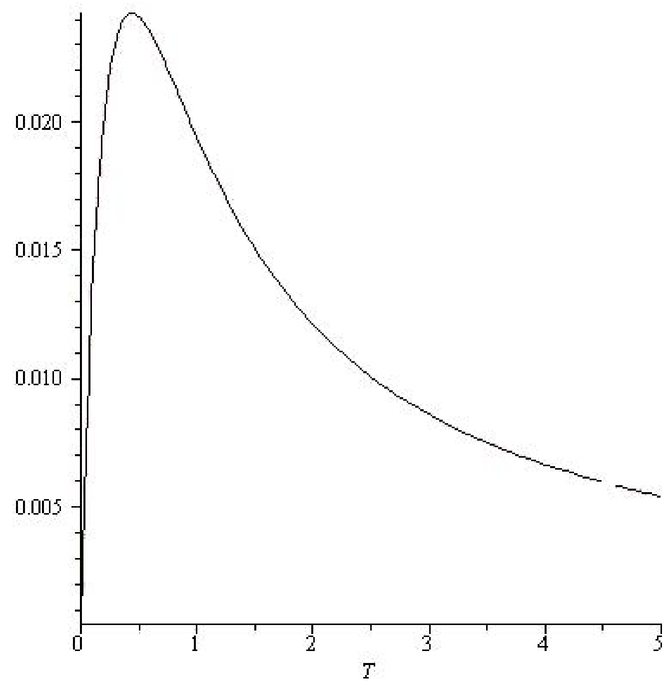


Fig. 6: Convexity Adjustment

PART III
Weather Derivatives in Energy Markets

(*The J. Energy Markets, V.8, N.1, March 2015, 59-76*)

Weather Derivatives in Energy Markets
(*The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

The weather derivatives market, in which contracts written on weather indices was firstly appeared over-the-counter (OTC) in July 1996 between Aquila Energy and Consolidated Edison Co. from United States. After that, companies accustomed to trading weather contracts based on electricity and gas prices in order to hedge their price risks realized by weather during the end of 1990s and the beginning of 2000s. Consequently, the market grew rapidly and expanded to other industries and to Europe and Japan.

Weather Derivatives in Energy Markets
(*The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

Reported from Weather Risk Management Association (WRMA), an industry body that represents the weather market, recently, the total notional value of the global weather risk market has reached $11.8 billion in last year. With geographic expansion, the OTC market boosted nearly 30% in last year. In this article, we will concentrate on the market of temperature derivatives found at the Chicago Mercantile Exchange (CME), which is one of the largest weather derivatives trading platforms. Up to now, the CME has weather futures and options traded based on a range of weather indices for 47 cities from United States, Canada, Europe, Australia and Asia.

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As a common sense, weather affects different entities in different ways. In order to hedge these different types of risks, weather derivatives are written on different types of weather variables or weather indices. The most commonly used weather variable is the temperature. Widely used temperature indices include cumulative average temperature (CAT), heating degree days (HDD) and cooling degree days (CDD). They are originated from the energy industry, and designed to correlate well with the local demands for heating or cooling.

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CAT is defined as the sum of the daily average temperature over the period $[\tau_1, \tau_2]$ of the contract, the index CAT$:=\sum_{t=\tau_1}^{\tau_2} T(t) = \int_{\tau_1}^{\tau_2} T(t)dt,$ where $T(t)$ is the daily average temperature. It is mainly used in Europe and Canada. In winter, HDD are used to measure the demand for heating, i.e. they are a measure of how cold the weather is and usually used in United States, Europe, Canada and Australia. In contrast, CDD are used in summer to measure the demand of energy used for cooling and a measure of how hot the weather is. They are usually used in United States, Canada and Australia.

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The definitions for HDD and CDD are given by HDD:=max$(T(t)-c,0)$ and CDD:=max$(c-T(t),0)$, where the constant c denotes the threshold, say $65^\circ F(18^\circ C)$. Since most air conditioners are switched on when temperatures are above or below c.

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With respect to our model, consider the weather index $T(t)$, which is the daily average temperature (DAT). We suppose the DAT has a generalization of the Ornstein-Uhlenbeck dynamics

$$dT(t) = ds(t) + k(T(t) - s(t))dt + \sigma(t)dL(t),$$

where $L(t)$ is a Lévy process (jump-diffusion), $s(t)$ is the seasonal mean level and k is the speed in which the temperature reverts to $s(t)$. $\sigma(t)$ is assumed to be a measurable and bounded function represents the seasonal volatility of temperature.

In the simplest case, $L(t) = W(t)$-a standard Wiener process.

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This model was firstly introduced by Dornier and Queruel (2000) with Brownian motion as the random noise. Benth and Saltyte-Benth (2005) has successfully applied this model with generalized hyperbolic Lévy process to the Norwegian temperature data. We applied this model to our Canadian temperature data (Swishchuk & Cui (2013)).

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We define the temperature futures prices written on CAT, CDD and HDD, which constitute the three main classes of futures products at CME market. Consider the price dynamic of future written on CAT over specific time period $[\tau_1, \tau_2]$, with $\tau_1 < \tau_2$. Firstly, assume the daily average temperature follows stochastic differential equation with $L(t)$ being Lévy process and a constant continuously compounding interest rate r.

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The future price $F_{CAT}(t,\tau_1,\tau_2)$ at time $0 \leq t \leq \tau_1$ based on CAT under risk-neutral probability measure Q is:

$$F_{CAT}(t,\tau_1,\tau_2) = E^Q[\int_{\tau_1}^{\tau_2} T(s)ds|\mathcal{F}_t],$$

where Q is the risk-neutral measure (specified through Esscher transform) and $\mathcal{F}_t$ is σ-algebra generated by $L(t)$.

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Similarly, the risk-neutral CDD and HDD future prices are defined as:

$$F_{CDD}(t,\tau_1,\tau_2) = E^Q[\int_{\tau_1}^{\tau_2} \max(T(s)-c,0)ds|\mathcal{F}_t],$$

and

$$F_{HDD}(t,\tau_1,\tau_2) = E^Q[\int_{\tau_1}^{\tau_2} \max(c-T(t),0)ds|\mathcal{F}_t],$$

The relationship between futures prices of CAT, CDD and HDD is defined as

$$F_{CAT}(t,\tau_1,\tau_2) + F_{HDD}(t,\tau_1,\tau_2) = c(\tau_2-\tau_1) - F_{CDD}(t,\tau_1,\tau_2).$$

newpage **Weather Derivatives in Energy Markets**
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We use future contracts written on temperature to demonstrate the hedging strategies for commodities as an application of weather derivative.

Within several forms of weather derivatives, the future contract does not require cost to enter a position, since when entering a future contract, the probability of weather event being lower or higher than the threshold is the same to both side, either side has the same chance of receiving payoff from the counter party.

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There are two types of hedging strategies using temperature futures in the following contents. The first strategy is a static hedging mainly focusing on mitigating the volume risk of commodity sales using temperature futures.

The other strategy consider the dynamic hedging strategy of commodity future using temperature futures. Without loss of generality, we choose the energy market as the one to hedge using temperature futures.

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a) In a static hedge, the number of hedging contracts is not changed over the course of the hedge in response to any movement in the values of the hedging instrument or the hedged asset.

b) In a dynamic hedge, on the other hand, more hedging contracts are bought or sold to bring back the hedge ratio to the target hedge ratio.

A hedge ratio is the ratio of exposure to a hedging instrument to the value of the hedged asset. A ratio of 1 or 100% means that the position is fully hedged and a ratio of 0 means it is not hedged at all.

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Our focus will be on the dynamic hedging strategy of energy futures using temperature futures. In the spirit of Broadie and Jain (2008), consider a portfolio at time t containing one unit of energy (e.g. heating oil) future F_E and β_t (β_t is the hedge ratio for energy future F_E) units of weather futures F_W, both with maturity (delivery) at time T. Assume the portfolio has value $\Pi(t)$ at time t, a constant risk-free interest rate r, then

$$\Pi(t) = e^{-r(T-t)}[F_E(t) + \beta_t F_W(t)]. \tag{1}$$

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The portfolio is self-financing, so the change in this portfolio in a small amount of time dt is given by

$$d\Pi(t) = r\Pi(t)dt + e^{-r(T-t)}[dF_E(t) + \beta_t dF_W(t)]. \tag{2}$$

Hence, in order to dynamically hedge the energy future F_E with maturity T, the stochastic component of portfolio vanishes, the hedge ratio β_t could be defined as

$$\beta_t = -\frac{dF_E(t)}{dF_W(t)}, \tag{3}$$

with an assumption that $dF_W(t) \neq 0$. Therefore, from the last equation, to hedge an energy futures, we are required to hold β_t units of temperature future at time t.

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Therefore, we need to specify two models for energy and temperature futures so that we could get the explicit dynamics of energy and temperature futures, and hence get a closed form of the hedge ratio β_t. For futures pricing purpose, these models will be built on the underlings of futures, namely the energy spot price and the daily average temperature.

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Rermark: Note that another this type of hedging ratio is called the optimal hedge ratio (see for example Hatemi-J and Roca (2006) and Yeh, Huang and Hsu (2008)), which takes the form:

$$\beta_t = -\frac{\mathrm{Cov}(dF_E(t), dF_W(t))}{\mathrm{Var}(dF_W(t))}, \tag{4}$$

where Cov is the covariance and Var is the variance. If we are clear about the dynamics of the energy and temperature futures, it is also possible to apply this dynamic hedging strategy.

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Our energy and temperature models under risk-neutral measure Q are:

$$dX(t) = \left(\theta\sigma_E + \kappa_E\left(\mu - \frac{\sigma_E^2}{2\kappa_E} - X(t)\right)\right)dt + \sigma_E dW_E^\theta(t),$$

and

$$dT(t) = ds(t) + (\theta\sigma_W(t) + \kappa_W(T(t) - s(t)))dt + \sigma_W(t)dW_T^\theta(t),$$

where θ is the market price of risk, $X(t) = \ln S(t)$, $W_E^\theta(t)$ and $W_T^\theta(t)$ are Brownian motions (with correlation ρ) w.r.t. Q.

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Combined Q dynamics system for energy futures F_E and CAT futures F_W is

$$\begin{cases} dF_E(t,T) & = & \sigma_E e^{-\kappa_E(T-t)} \exp\left(\mu_X(T) + \frac{1}{2}\sigma_X^2(T)\right) dW_E^\theta(t); \\ dF_W(t,\tau_1,\tau_2) & = & \kappa_W^{-1}(e^{\kappa_W(\tau_2-t)} - e^{\kappa_W(\tau_1-t)})\sigma_W(t) dW_W^\theta(t); \\ dW_E^\theta(t) dW_W^\theta(t) & = & \rho dt, \end{cases}$$

where $\mu_X(T)$ and $\sigma_X^2(T)$ are expectation and variance of the log-spot price $X(t)$.

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$$\beta_t = -\frac{c_1(t)}{c_2(t)}\rho,$$

where $c_1(t)$ and $c_2(t)$ are time-dependent constant defined as:

$$\begin{cases} c_1(t) & := & \sigma_E e^{-\kappa_E(T-t)} \exp\left(\mu_X(T) + \frac{1}{2}\sigma_X^2(T)\right); \\ c_2(t) & := & \kappa_W^{-1}(e^{\kappa_W(\tau_2-t)} - e^{\kappa_W(\tau_1-t)})\sigma_W(t). \end{cases}$$

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We choose the crude oil (crude oil is the world's most actively traded commodity, and the NYMEX (CME) division light, sweet crude oil futures contract is the world's most liquid forum for crude oil trading) futures as the the one that we want to hedge using CAT futures. Followed by the calibration method described in Schwarz (1997), the log-future prices $\ln F_E(t,T)$ need to be rewritten as the standard state-space form and then applied to Kalman filter to get the parameter set $\Theta_E = \{\kappa_E, \mu_E, \sigma_E, \theta_E\}$ and spot price series $S(t)$.

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The data used to calibrate the energy future consist of daily generic observations of WTI light, sweet crude oil futures prices (these data are obtained from Bloomberg financial service) with delivery periods in the first two front months. The WTI crude oil futures data used in calibration cover the CME exchange daily settlement prices ranging from January 2nd, 2001 to December 31st, 2010, resulting in 2508 record for each future contracts set (this choice of data set is consistent with that in Swishchuk and Cui (2013), which is 10 years of temperature data from January 1st, 2001 to December 31st, 2010 in Calgary, AB, Canada).

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Since there is no exact delivery date for each contract, instead, the CME contract specification defines a delivery period ranging from the first calendar day to the last calendar day of the delivery month, we simply assume that the delivery date for each contract is the first calendar day in the delivery month to calculate the time to maturity value $T_i - t$.

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Table below presents the estimation results for the energy model applied to the WTI crude oil future price data. The last two parameters ξ_1 and ξ_2 are the diagonal entries of matrix $H := Var(\epsilon_t)$ with random noise ϵ_t.

Parameter	μ	σ_E	κ_E	θ	ξ_1	ξ_2
Estimation	3.9187	0.0215	0.0025	0.2009	0.0003	0.0123

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For the temperature market, we follow the calibration procedure described in Swishchuk and Cui (2013) to get the parameter set $\Theta_W = \{\kappa_W, \sigma_W\}$. For illustration purpose, we choose the estimated parameters in Calgary as the ones under the temperature market to calculate the hedge ratio. Recall the calibration results for Calgary in Swishchuk and Cui (2013), we could get the parameter set $\Theta_W = \{\kappa_W, \sigma_W\}$ in Calgary as follows:
$\kappa_W = -0.2411$
and annual seasonal volatility
$\sigma_W = 4.424 + 1.633\cos(0.0167t) + 0.1912\sin(0.0167t)$.

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To calculate the correlation parameter ρ, we use the correlation between the filtered log-spot price and daily average temperature as a natural approximation to ρ. By taking all the daily average temperature on the dates with future prices available, and calculating the correlation coefficient between log-spot prices and average temperature of these days over 10 years (from January 2nd, 2001 to December 31st, 2010), we have the correlation $\rho = 0.1058$. This correlation indicates a positive correlation between the log-spot price of crude oil and daily average temperature.

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With the calibrated parameters in energy model and temperature model, we could then calculate the dynamic hedge ratio β_t explicitly. In the Figure below, we plot the initial hedge ratio β_0 along the crude oil future delivery time (in days) and initial log-spot price dimensions.

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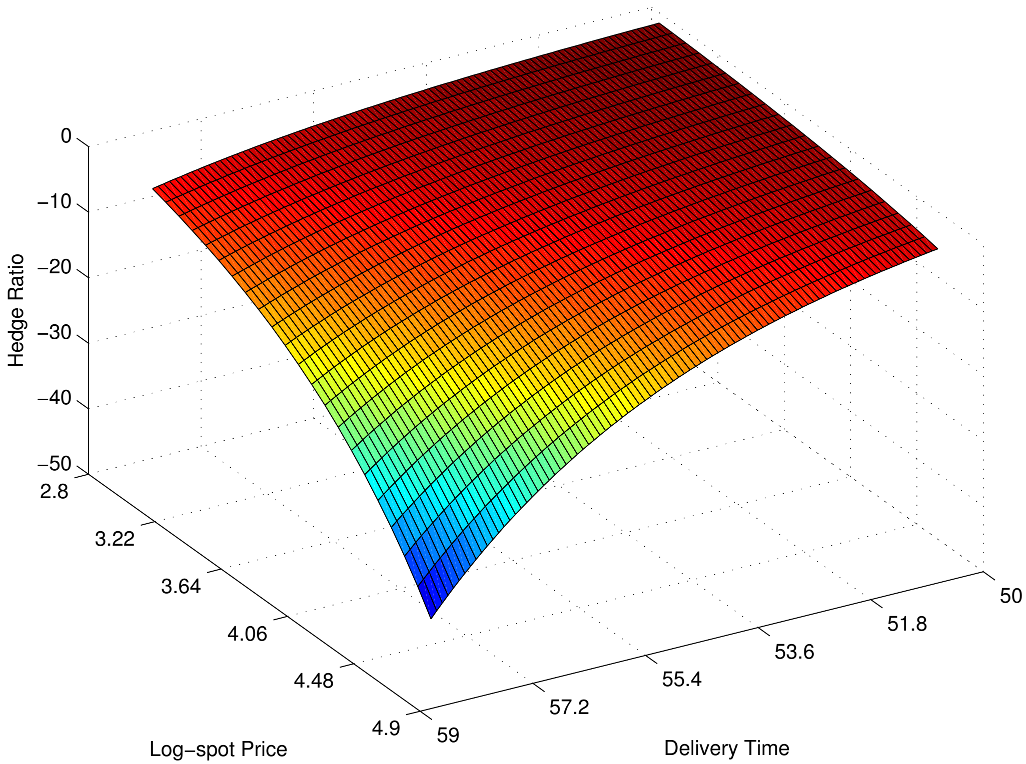

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From this Figure, we could find that if one hold a crude oil futures, initially he need to short some CAT futures in the portfolio depending on the spot price of the crude oil and the time to delivery (trade termination) length. Basically the number of temperature futures one need to hold will be more with longer time to delivery and higher spot price of the crude oil. Moreover, we could conclude that the same effect holds for other energy commodities, such as heating oil, gas and so on, since they are usually positively correlated to the crude oil market movement.

PART IV

Pricing Crude Oil Options using Lévy Processes

(*The J. Energy Markets, V.9, N 1, March 2016, 47-64*)

Pricing Crude Oil Options using Lévy Processes
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64***)**

Crude oil prices exhibit significant volatility over time and the distribution of returns on crude oil prices show fat tails and skewness, and they barely follow normal distribution. This is the reason we use Normal Gaussian Process (NIG), Jump Diffusion Process (JD), and Variance-Gamma Process (VG) as three Levy processes that do not have these drawbacks and their tails carry heavier mass than normal distribution. We use fractional fast Fourier transform to calibrate parameters in an optimization setup, using data on European-style options on crude oil futures in NYMEX for the settlement date of April 24th, 2015. Our results indicate that all these three Levy processes have very good out of sample results for near at the money options than others.

Pricing Crude Oil Options using Lévy Processes
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Merton's (1976) Jump Diffusion Model:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + (e^{\alpha+\beta\epsilon} - 1)dN_t,$$

where Brownian motion W_t and Poisson process N_t are independent, $\epsilon \approx N(0,1)$.

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Normal Inverse Gaussian (NIG) Model:

$$S_t = S_0 \exp\{\mu^Q t + X_t\},$$

where μ^Q is the drift under Q measure,

$$X_t = \beta\delta^2 I_t + \delta W_{I_t}$$

is a NIG process, I_t is the inverse Gaussian process. NIG process has three parameters, tail-heaviness α, skewness β and scale δ.

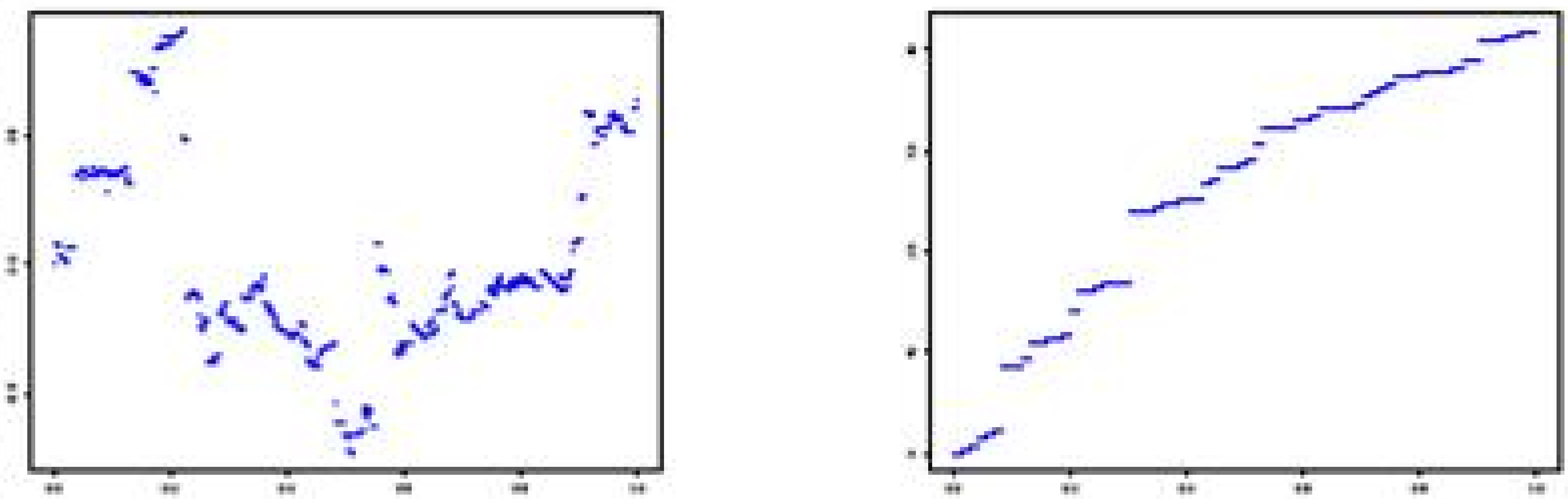

FIGURE 7.10. Simulated path of a normal inverse Gaussian (left) and an inverse Gaussian process

Pricing Crude Oil Options using Lévy Processes
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Variance Gamma (VG) Model:

$$S_t = S_0 \exp\{\mu^Q t + X_t\},$$

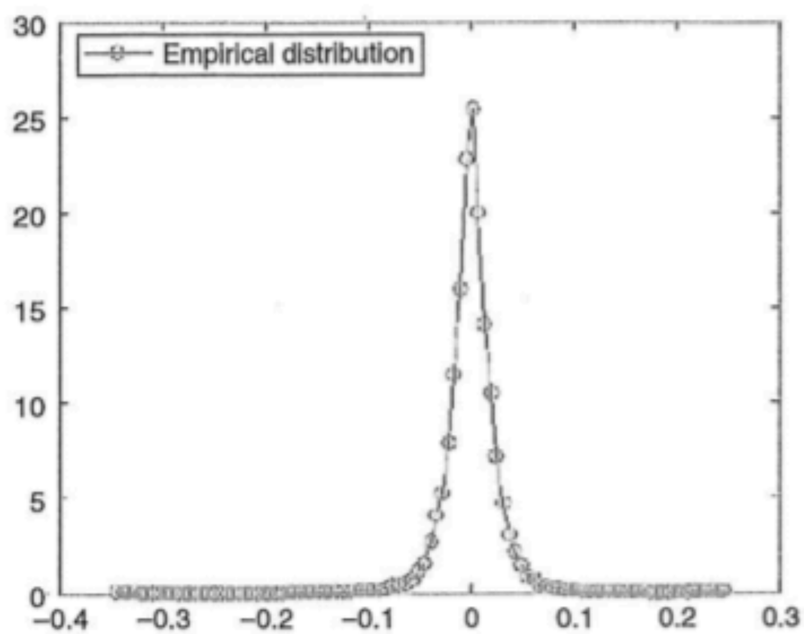
where X_t is a VG process such that

$$X_t = \theta I_t + \sigma W_{I_t},$$

and I_t is a gamma process with parameter v.

Pricing Crude Oil Options using Lévy Processes
Figures, Tables, Estimations
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64***)**

FIGURE 1 Empirical distribution of returns on WTI spot crude oil prices from January 4, 1983 to April 21, 2015.



Source: Bloomberg.

TABLE 1 Descriptive statistics of returns on WTI spot crude oil from 1990 Q1 through 2015 Q4.

Period	Observation	Mean	SD	Skewness	Kurtosis
January 1, 1990–December 31, 1992	750	0.0003	0.031	−1.6856	28.3106
January 1, 1993–December 31, 1996	753	0.0001	0.016	−0.4139	5.9106
January 1, 1997–December 31, 1999	503	−0.0012	0.0262	0.9451	11.6558
January 1, 2000–December 31, 2002	747	0.0006	0.0265	−0.4389	5.5271
January 1, 2003–December 31, 2005	750	0.0012	0.0241	−0.294	4.3453
January 1, 2006–December 31, 2008	749	0	0.0279	1.0834	14.507
January 1, 2009–December 31, 2012	753	0.0014	0.0258	0.1359	6.7841
January 1, 2013–December 31, 2015	826	−0.0006	0.0167	0.0406	5.7803

Source: Bloomberg. SD denotes standard deviation.

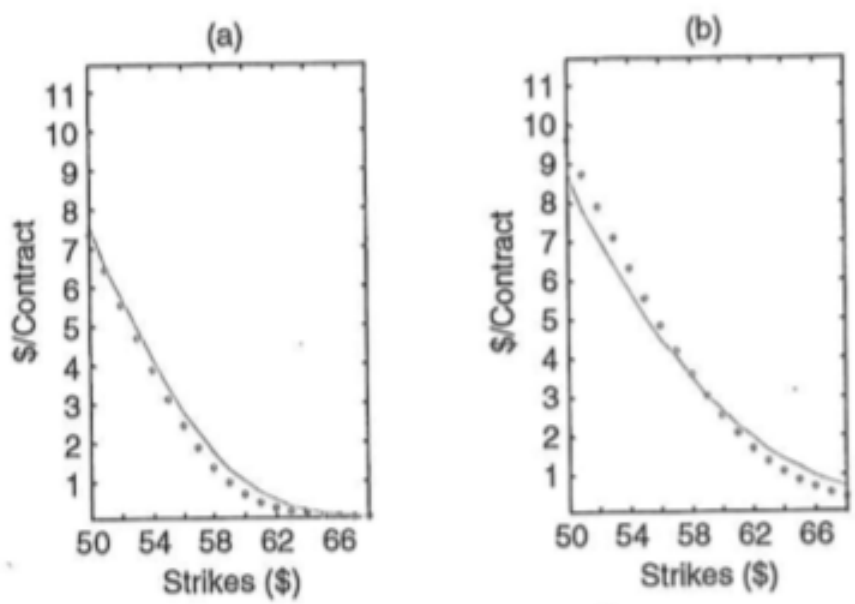
TABLE 3 WTI crude futures and options prices with strikes.

	Days to expiry after market close on April 24, 2015						
	18	52	113	205	297	387	417
	Settlement of each contract						
Strike	**June 2015**	**July 2015**	**Sept 2015**	**Dec 2015**	**March 2016**	**June 2016**	**July 2016**
WTI crude futures options							
50.00	7.36	9.60	11.78	13.86	14.94	15.61	15.61
51.00	6.43	8.73	10.96	13.06	14.31	15.04	15.04
52.00	5.53	7.89	10.15	12.28	13.38	14.29	14.29
53.00	4.68	7.08	9.37	11.52	12.80	13.55	13.55
54.00	3.87	6.30	8.62	10.78	11.88	12.83	12.83
55.00	3.11	5.54	7.89	10.07	11.16	11.88	11.88
56.00	2.44	4.84	7.19	9.37	10.65	11.19	11.19
57.00	1.86	4.19	6.52	8.70	9.77	10.52	10.52
58.00	1.35	3.58	5.88	8.05	9.11	9.87	9.87
59.00	0.96	3.01	5.27	7.42	8.48	9.24	9.24
60.00	0.67	2.51	4.69	6.81	7.87	8.63	8.63
61.00	0.45	2.07	4.15	6.23	7.29	8.05	8.05
62.00	0.31	1.67	3.66	5.67	6.73	7.48	7.48
63.00	0.21	1.35	3.21	5.16	6.19	6.94	6.94
64.00	0.15	1.08	2.79	4.67	6.00	6.43	6.43
65.00	0.11	0.85	2.40	4.22	5.24	5.95	5.95
66.00	0.09	0.68	2.07	3.79	5.10	5.80	5.80
67.00	0.07	0.53	1.78	3.40	4.39	5.05	5.05
68.00	0.06	0.42	1.53	3.05	4.00	4.64	4.64
Futures							
	57.15	58.90	60.50	62.03	62.98	63.57	63.68

TABLE 5 Calibrated parameters of JDM, VG and NIG processes.

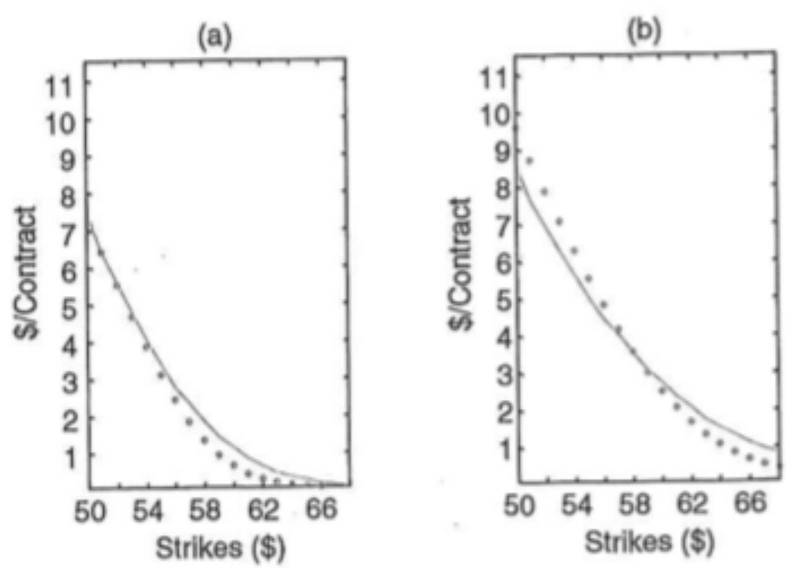
Parameters	JDM	VG	NIG
θ	N/A	0.1820	N/A
δ	0.4628	N/A	2.5327
α	0.1247	N/A	−18.5203
λ	0.3318	N/A	N/A
σ	0.4272	0.4104	N/A
ν	N/A	0.0823	N/A
β	N/A	N/A	−7.4572

FIGURE 3 WTI June 2015 and July 2015 market versus NIG-based option prices.



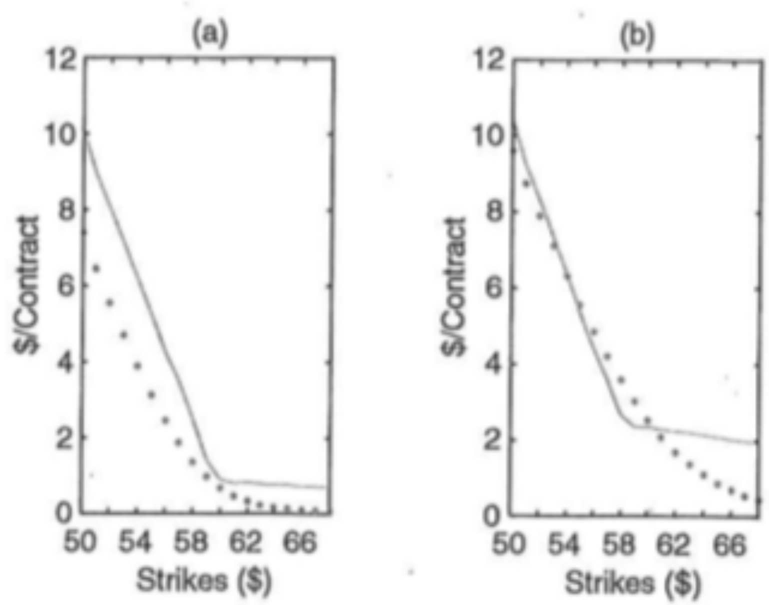
(a) June contract. (b) July contract. Market prices (circles) versus NIG output (line).

FIGURE 4 WTI June 2015 and July 2015 market versus Black–Scholes-based option prices.



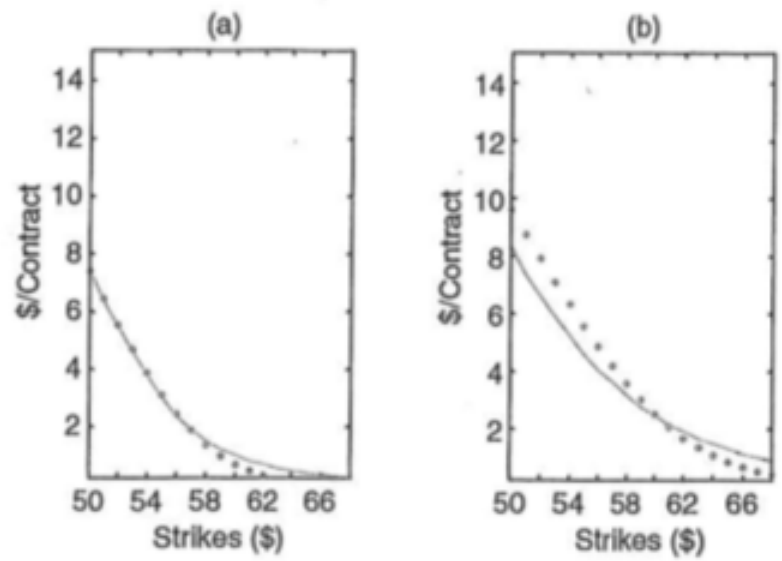
(a) July contract. (b) June contract. Market prices (circles) versus Black–Scholes output (line).

FIGURE 5 WTI June and July market versus JDM-based option prices.



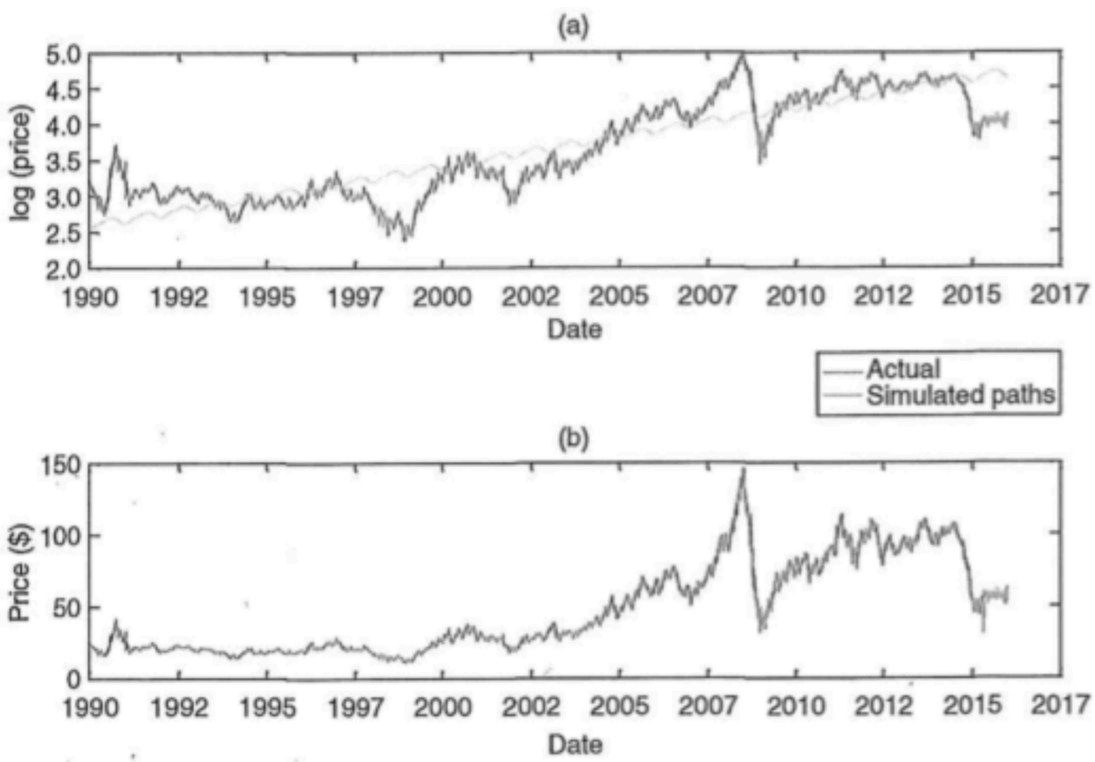
(a) June contract. (b) July contract. Market prices (circles) versus JDM output (line).

FIGURE 6 WTI June 2015 and July 2015 market versus VG-based option prices.



(a) June contract. (b) July contract. Market prices (circles) versus VG output (line).

FIGURE 7 Log and level of WTI crude spot prices with simulated paths using option-based calibrated the MJDM.



(a) Actual log of WTI crude spot and the simulated path as of April 27, 2015. (b) Actual WTI crude spot and the simulated price path as of April 27, 2015.

Pricing Crude Oil Options using Lévy Processes
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64***)**

The volatility of crude oil prices is very important for policy makers, crude oil producers and refineries. We used most recent data through April 2016 from crude oil futures and options markets to model dynamics of crude oil prices. Our results indicate that crude oil prices show significant jumps that are very frequent. Crude oil price returns show skew as well. These findings are consistent across all three models we used in this research.

Pricing Crude Oil Options using Lévy Processes
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64***)**

In the case of JDM, the volatility of size of the jumps is bigger than volatility of the diffusion part. The VG process results in slightly smaller volatility than JDM. The mean of the jump component size implied by JDM, and skew parameter of VG process both indicate existence of right-skew in crude oil price returns, but the NIG process implies that the density of returns are skewed to the left.

PART V
Energy Market Contracts with Delayed and Jumped Volatilities

(*Handbook of Energy Finance: Theories, Practices and Simulations, World Scientific, 2019*)

Energy Market Contracts with Delayed and Jumped Volatilities
(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019***)**

In this Part we concentrate on stochastic modelling and pricing of energy markets' contracts for stochastic volatilities with delay and jumps. Our model of stochastic volatility exhibits jumps and also past-dependence: the behaviour of a stock price right after a given time t not only depends on the situation at t, but also on the whole past (history) of the process $S(t)$ up to time t. The spot price process $S(t)$ is modelled by the OU process driven by independent increments process. The basic products in these markets are spot, futures and forward contracts and options written on these. We study forwards and swaps. A numerical examples is presented for stochastic volatility with delay using the Henry Hub daily natural gas data (1997-20011).

Energy Market Contracts with Delayed and Jumped Volatilities
(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019***)**

Definition of IIP (see [Skorokhod, 1991], [Benth *et al.***, 2008]):** An adapted RCLL stochastic process $I(t)$ starting at zero is an *IIP* (*Independent Increment process*) if it satisfies the following two conditions:

1) The increments $I(t_0), I(t_1) - I(t_0), ..., I(t_n) - I(t_{n-1})$ are independent r.v. for any partition $0 \leq t_0 < t_1 < ... < t_n$, and $n \geq 1$.

2) It is continuous in probability, that is, for every $t \geq 0$ and $\epsilon > 0$, it

$$\lim_{s \to t} P(|I(s) - I(t)| > \epsilon) = 0.$$

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(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019***)**

If we add the condition that increments are stationary, then $I(t)$ is called a *Lévy process.* (See [Sato, 1999], [Schoutens, 2003]). If the increments of Lévy process are normally distributed then we have a *Brownian motion.* Lévy processes which are increasing, that is, having only positive jumps, are often called *subordinators*.

Sometimes the IIP are called *additive processes*. (See [Sato, 1999]).

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Let the stochastic process $S(t)$ be denoted as (er call it Geometric Models with Stochastic Delayed and Jumped Volatility):

$$\ln S(t) = \ln \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t),$$

where for $i = 1, ..., m$

$$dX_i(t) = (\mu_i(t) - \alpha_i(t) X_i(t))dt + \sigma_i(t, X_i(t+\theta))dB(t),$$

and for $j = 1, ..., n$

$$dY_j(t) = (\delta_j(t) - \beta_j(t) Y_j(t))dt + \eta_j(t, Y_j(t+\theta))dI_j(t).$$

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Here, $\theta \in [-\tau, 0], \tau > 0$, is the delay, and on the interval $[-\tau, 0]$, $X_i(t) = \phi_i(t)$ and $Y_j(t) = \psi_j(t)$, where $\phi_i(t)$ and $\psi_j(t)$ are deterministic functions, $i = 1, ..., m$ and $j = 1, ..., n$.

We remark that two factors $X_i(t), i = 1, ..., m,$ and $Y_j(t), j = 1, ..., n,$ represent the long- and short-term fluctuations of the spot dynamics which may be correlated. We suppose that jumps components I_j are independent, which is an obvious restriction of generality.

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The deterministic seasonal price level is modelled by the function $\Lambda(t)$, (seasonal function) which is assumed to be continuously differentiable.

The coefficients $\mu_i, \alpha_i, \delta_j \beta_j$ are all continuous functions. We suppose that volatilities $\sigma_{ik}(t)$ and $\eta_j(t)$ are stochastic volatilities with delay and jumps.

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We consider two cases in this situation:

$$\begin{aligned}\frac{d\sigma_i^2(t,X_i(t+\theta))}{dt} &= \gamma_i^1 V_i + \frac{\alpha}{\tau}[\int_{t-\tau}^t \sigma_i(u, X_i(u+\theta)dB(u)\\ &+ \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta)d\tilde{N}_1(t)]^2\\ &- (a_i+b_i)\sigma_i^2(t, X_i(t+\theta))\end{aligned}$$

and

$$\begin{aligned}\frac{d\eta_j^2(t,Y_j(t+\theta))}{dt} &= \gamma_j^2 W_i + \frac{\alpha}{\tau}[\int_{t-\tau}^t \eta_j(u, X_j(u+\theta)dB_1(u)\\ &+ \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta)d\tilde{N}_2(t)]^2\\ &- (c_j+d_j)\eta_j^2(t, X_i(t+\theta)\end{aligned}$$

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Here, $B(t)$ and $B_1(t)$ are two independent Brownian motions and $\tilde{N}_1(t)$ and $\tilde{N}_2(t)$ are two independent compensated Poisson processes with intensities λ_1 and λ_2, independent of $B(t)$ and $B_1(t)$.

We note, that in [Benth *et al.*, (2008)] it was considered only deterministic $\sigma_i(t)$ and $\eta_j(t)$.

Energy Market Contracts with Delayed and Jumped Volatilities
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Let the stochastic process $S(t)$ be defined as (we call it Arithmetic Models with Stochastic Delayed and Jumped Volatility)

$$S(t) = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t),$$

where $X_i(t), i = 1, ..., m$, and $Y_j(t), j = 1, ..., n$, are defined for the geometric models above and the seasonality function $\Lambda(t)$ is the same.

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We suppose that for this model the volatilities $\sigma_i^2(t, X_i(t+\theta))$ and $\eta_j^2(t, Y_j(t+\theta))$ satisfied the same equations as for the case of Geometric Models.

We study the pricing of forwards and swaps for the above-mentioned model with delayed and jumped volatilities.

Energy Market Contracts with Delayed and Jumped Volatilities

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When entering the forward contract, one agrees on a future delivery time and the price to be paid for receiving the underlying. Suppose that the delivery time is T, with $0 \leq t \leq T < +\infty$, and that the agreed price to pay upon delivery is $f(t,T)$:

$$f(t,T) = E_Q[S(T)|\mathcal{F}_t] -$$

fundamental pricing relation between the spot and forward price. Since the energy markets are incomplete, the choice of martingale measure Q is open.

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Let us consider swaps, using the electricity market as the typical example. The buyer of an electricity futures receives power during a settlement period (physically or financially), against paying a fixed price per MWh. Let $F(t, \tau_1, \tau_2)$ be the electricity futures price at time t for the delivery period $[\tau_1, \tau_2]$ with $\tau_1 \leq \tau_2$.

In general, we can write the link between a swap contract and the underlying spot as

$$F(t, \tau_1, \tau_2) = E_Q[\int_{\tau_1}^{\tau_2} w(u, \tau_1, \tau_2) S(u) du | \mathcal{F}_t],$$

where w is a weight function.

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The dynamics of forward price, $t \to f(t,T)$, wrt Q^θ in the Geometric Model case is

$$\begin{aligned}\frac{df(t,T)}{f(t,T)} &= \{\sum_{i=1}^m \sigma_i(t, X_i(t+\theta)) \exp(-\int_t^T \alpha_i(u)du)\} dB^\theta(t) \\ &+ \sum_{j=1}^n \{\int_R \exp(z\eta_j(t, Y_j(t+\theta)) e^{-\int_t^T \beta_j(u)du}) - 1\} \tilde{N}_j^\theta(dt, dz).\end{aligned}$$

Energy Market Contracts with Delayed and Jumped Volatilities
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The risk-neutral dynamics of the swap price $F(t, \tau_1, \tau_2)$ in the Geometric Models case is given by

$$\begin{aligned} \frac{dF(t,\tau_1,\tau_2)}{F(t-,\tau_1,\tau_2)} &= \sum_{i=1}^{m} \sigma_i(t, X_i(t+\theta)) dB^\theta(t) \\ &+ \sum_{j=1}^{n} \int_R (e^{\eta_j(t,Y_j(t+\theta))z} - 1) \tilde{N}_j^\theta(dz, dt). \end{aligned}$$

Energy Market Contracts with Delayed and Jumped Volatilities
(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019***)**

The risk-neutral dynamics of the swap price $F(t,\tau_1,\tau_2)$ in the Arithmetic Models case is given by

$$\begin{aligned} dF(t,\tau_1,\tau_2) &= \textstyle\sum_{i=1}^{m} \sigma_i(t, X_i(t+\theta)) \int_{\tau_1)}^{\tau_2} w(u,\tau_1,\tau_2) e^{-\int_v^u \alpha_i(s)ds} du dB^{\theta}(t) \\ &+ \textstyle\sum_{j=1}^{n} \int_R z\eta_j(t, Y_j(t+\theta)) \\ &\times \textstyle\int_{\tau_1}^{\tau_2} w(u,\tau_1,\tau_2) e^{-\int_v^u \beta_j(s)ds} du \tilde{N}_j^{\theta}(dt,dz). \end{aligned}$$

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Numerical Example: Henry Hub Natural Gas Daily Spot Prices (1997-2011)

This numerical example and figures are borrowed from [Otunuga and Ladde, 2014]. In this paper, the authors used the model for spot price with delayed stochastic volatility from the paper [Kazmerchuk, Swishchuk and Wu, 2005], and applied it to the Henry Hub daily natural gas data set for the period 02/01/2001-09/30/2004. The data was collected from the United State Energy Information Administration website (www.eia.gov).

Energy Market Contracts with Delayed and Jumped Volatilities
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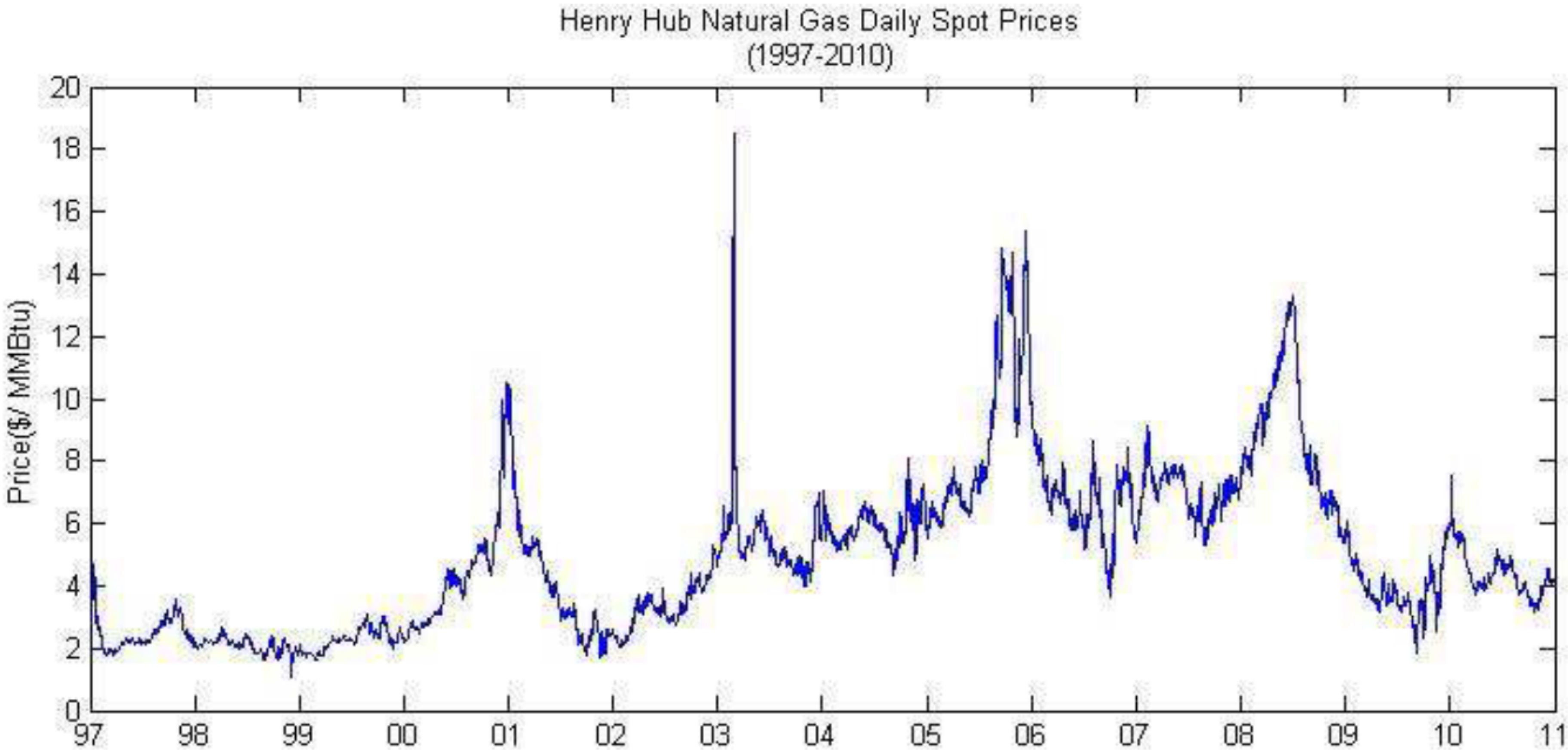
From Figure 1 below we can see the properties of the gas daily spot prices: randomly driven, non-negative, mean reversion, jumps (spikes), unpredictability of spot price volatility:


Henry Hub Natural Gas Daily Spot Prices
(1997-2010)
Price($/ MMBtu)
0
2
4
6
8
10
12
14
16
18
20
97
98
99
00
01
02
03
04
05
06
07
08
09
10
11

Energy Market Contracts with Delayed and Jumped Volatilities
(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019***)**

Table 1 below gives descriptive statistics of Henry Hub Daily Natural Gas spot prices (1997-2011):

	Mean	Variance	Skewness	Kurtosis	Minimum	Maximum
S_t	4.9519	2.4966	1.0391	4.3491	1.05	18.48
$S_{t+1} - S_t$	-0.0001142	0.3189	-0.7735	191.8911	-8.01	6.50
$\ln(S_t)$	1.4754	0.5048	-0.0465	2.1540	0.0488	2.9167
$\ln[S_{t+1}/ S_t]$	2.8485e-5	0.0473	0.4814	22.0473	-0.56	0.5657

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As we can see from the Table 1 above, the logarithmic price is better than the raw price data because the variance for log is the smallest.

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A simple model for the spot price is considered:

$$\ln S(t) = X(t),$$

where

$$dX(t) = \gamma(k - X(t))dt + \sigma(t, X(t))dB(t),$$

and

$$\frac{\sigma^2(t, X(t))}{dt} = [\alpha + \beta \int_{t-\tau}^{t} \sigma(s, X(s))dB(s)]^2 + c\sigma^2(t, X(t)).$$

Here, τ is the delay parameter.

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The model for $\sigma^2(t, X(t))$ above is the same as the model for stochastic volatility with delay that we considered in [Kazmerchuk, Swishchuk and Wu, 2005].

Discrete scheme is implemented: $l = 2 = [\frac{\tau}{\Delta}]$, where Δ is the size of the mesh of the discrete-time grid, $[,]$ is the floor function.

Estimated Parameters are (Courtesy- [Otunuga and Ladde, 2014]):

γ	k	τ	α	β	c
1.8943	1.5627	0.008	0.433	-0.07	-1.5

Energy Market Contracts with Delayed and Jumped Volatilities
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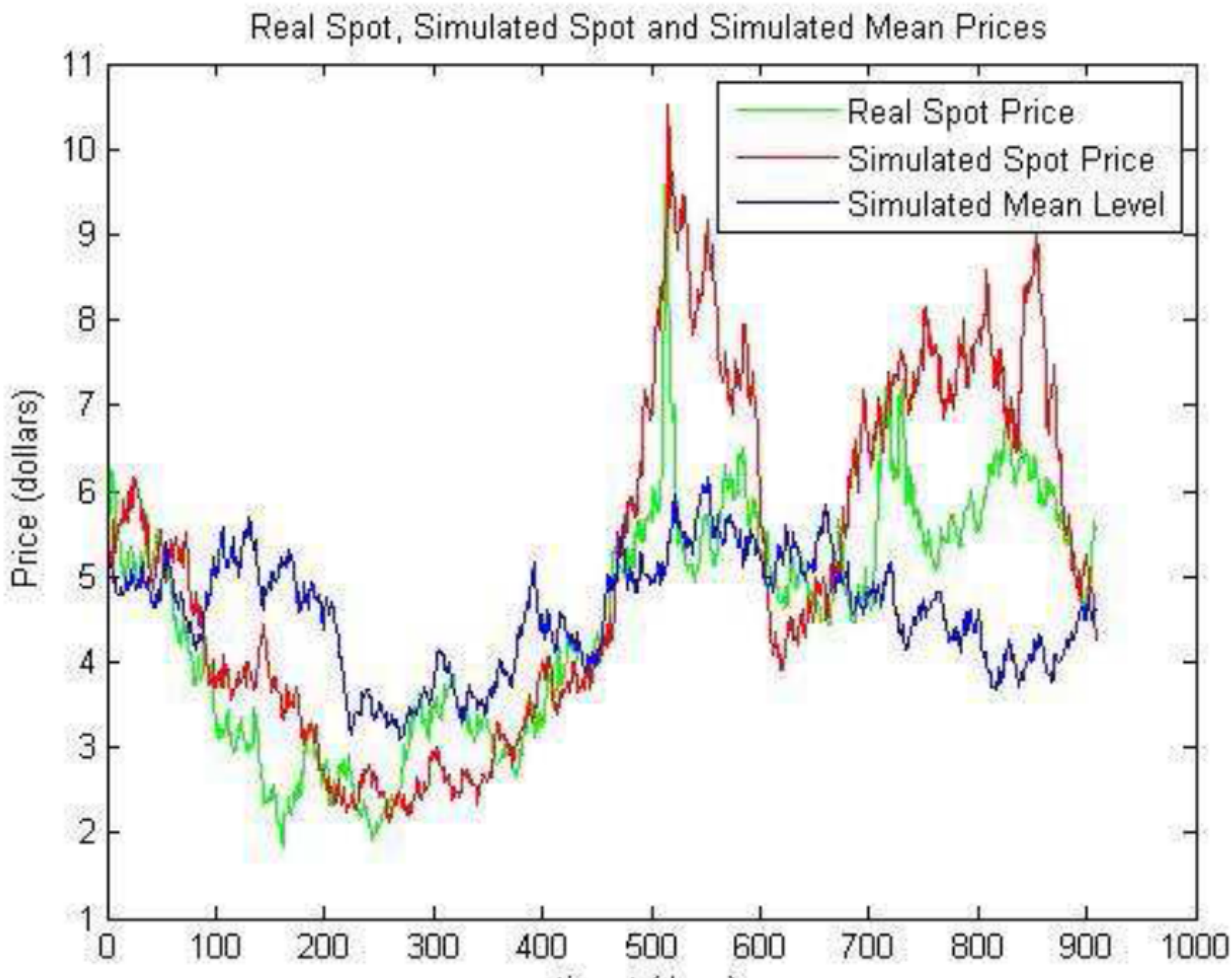
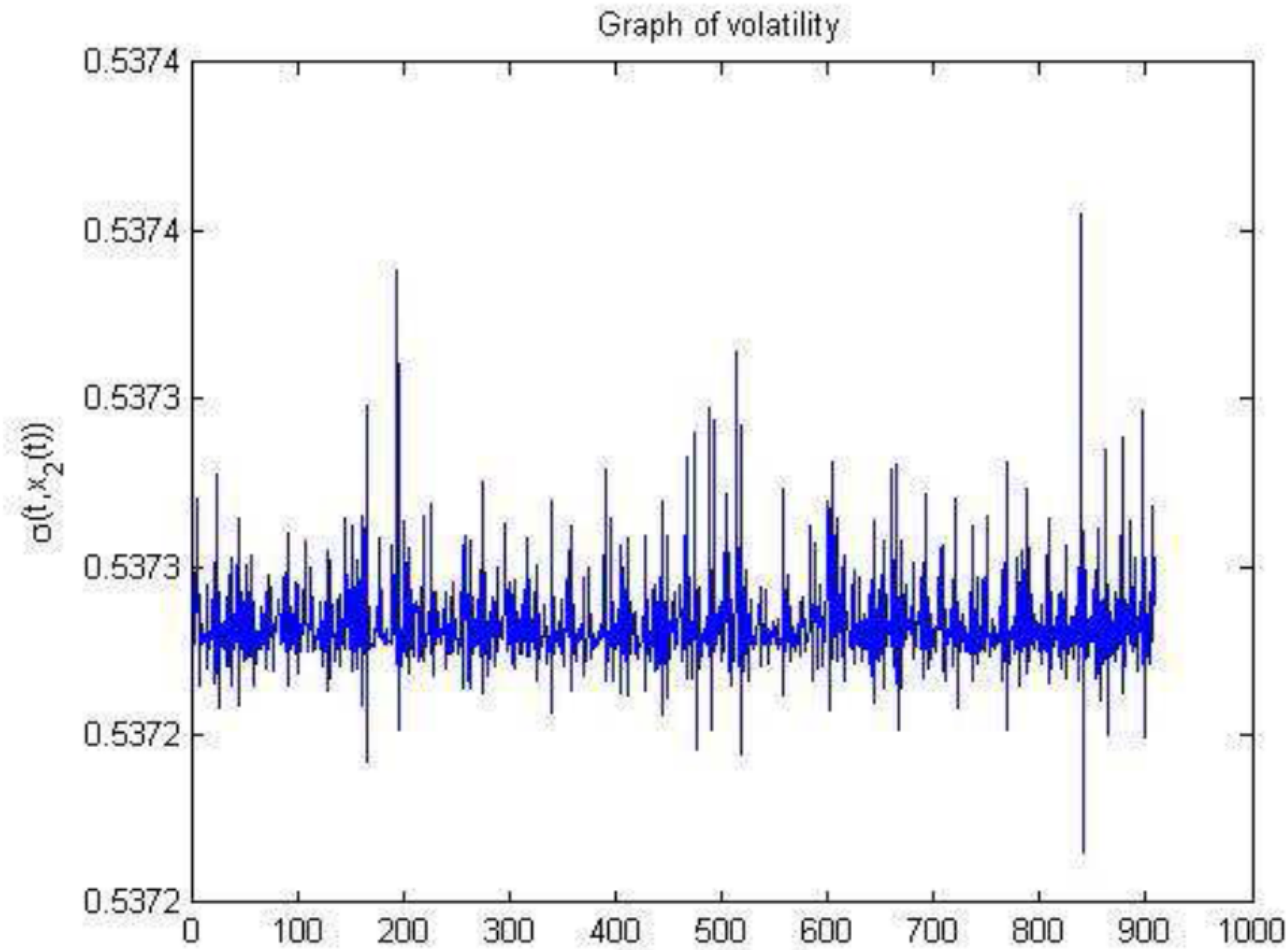
Graphs below, Figure 2, includes Real, Simulated Spot Prices and Simulated Expected Spot Price (Henry Hub Daily Natural Gas Data Set (02/01/2001-09/30/2004)):


Real Spot, Simulated Spot and Simulated Mean Prices
Real Spot Price
Simulated Spot Price
Simulated Mean Level
Price (dollars)
1
2
3
4
5
6
7
8
9
10
11
0
100
200
300
400
500
600
700
800
900
1000

Energy Market Contracts with Delayed and Jumped Volatilities
(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019*)

Graph below, Figure 3, shows simulated $\sigma(t, X(t))$ from Henry Hub Daily Natural Gas Data Set (02/01/2001-09/30/2004)).



Graph of volatility
0.5374
0.5374
0.5373
0.5373
0.5372
0.5372
σ(t,x_2(t))
0
100
200
300
400
500
600
700
800
900
1000

PART VI
A Vision to Transition to 100% Wind, Water & Solar Energy in Canada

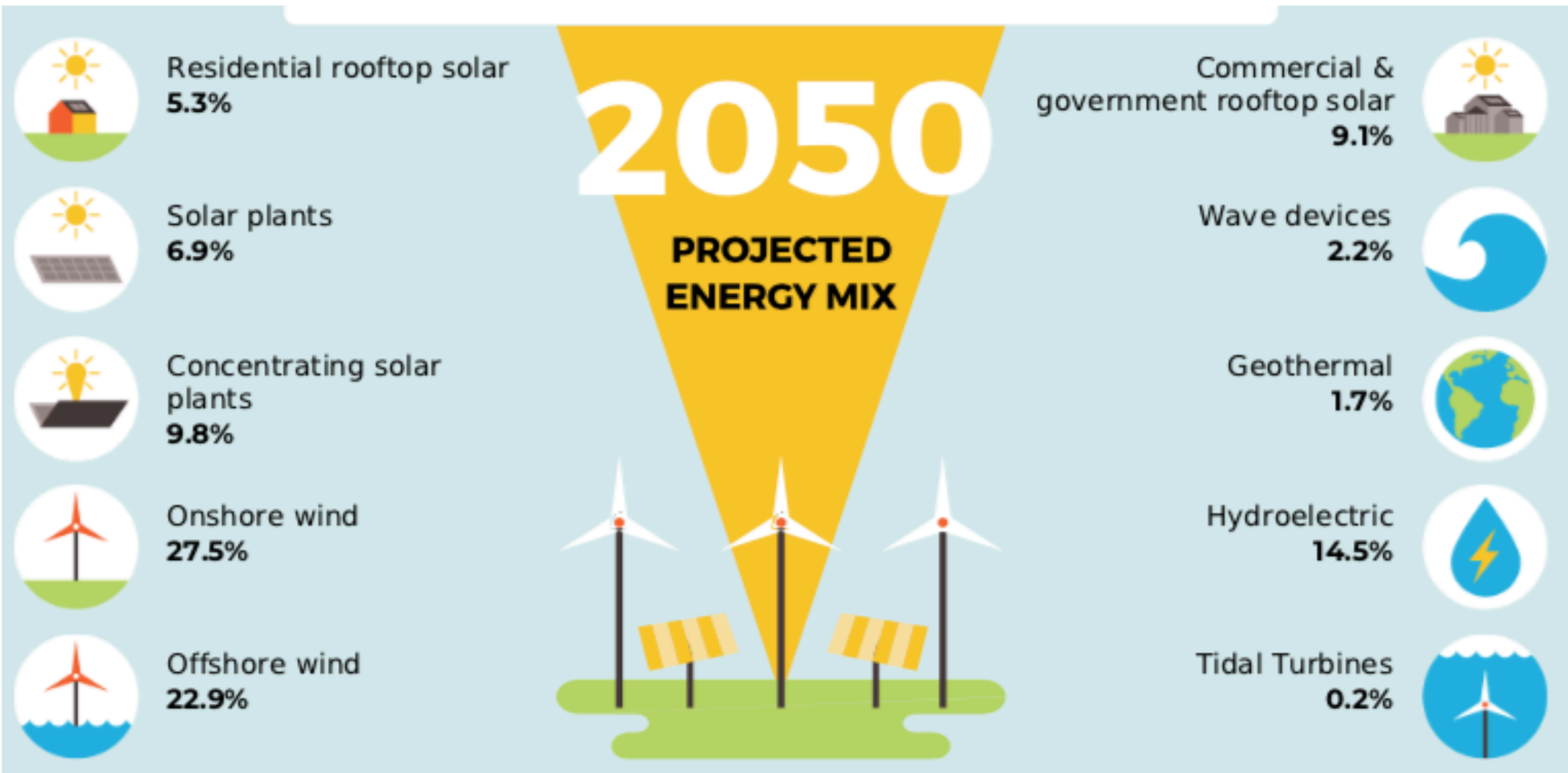
A Vision to Transition to 100% Wind, Water & Solar Energy in Canada

A group of U.S. civil engineering has calculated that Canada could be completely powered by renewable energy, if we just decide to do it.

They say that would save \$110.1 billion on health care costs every year and prevent $9,884$ annual air pollution deaths.

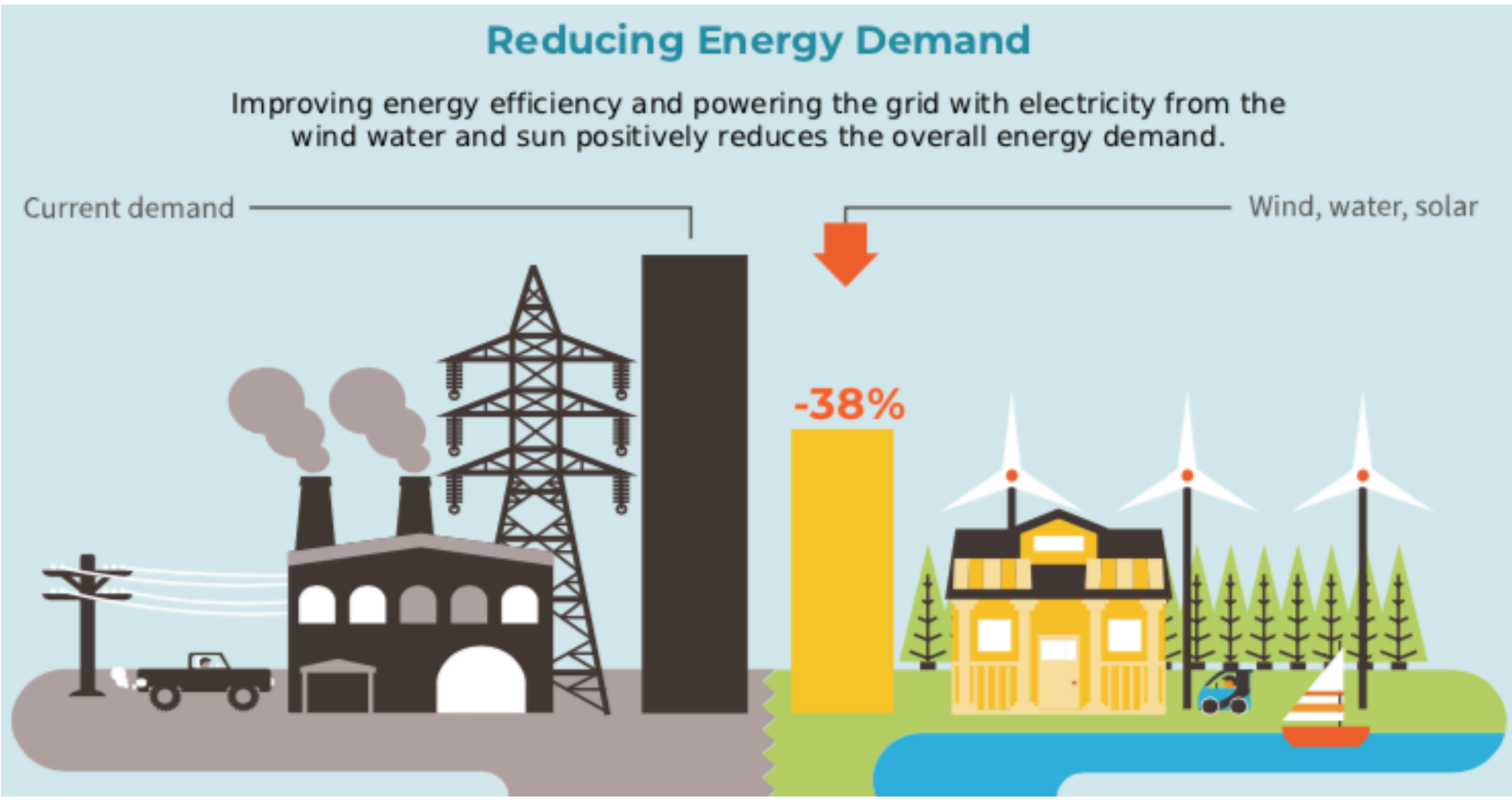
Their research is available at thesolutionsproject.org.

A Vision to Transition to 100% Wind, Water & Solar Energy in Canada



A Vision to Transition to 100% Wind, Water & Solar Energy in Canada

A Vision to Transition to 100% Wind, Water & Solar Energy in Canada



A Vision to Transition to 100% Wind, Water & Solar Energy in Canada: Health Cost Savings

Health Cost Savings

Avoided health costs per year:

3.94% of country GDP

Lives lost to air pollution that we could save each year:

9,884

The transition pays for itself in as little as **4.1 years** from air pollution and climate cost savings alone

**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:
Land Usage**

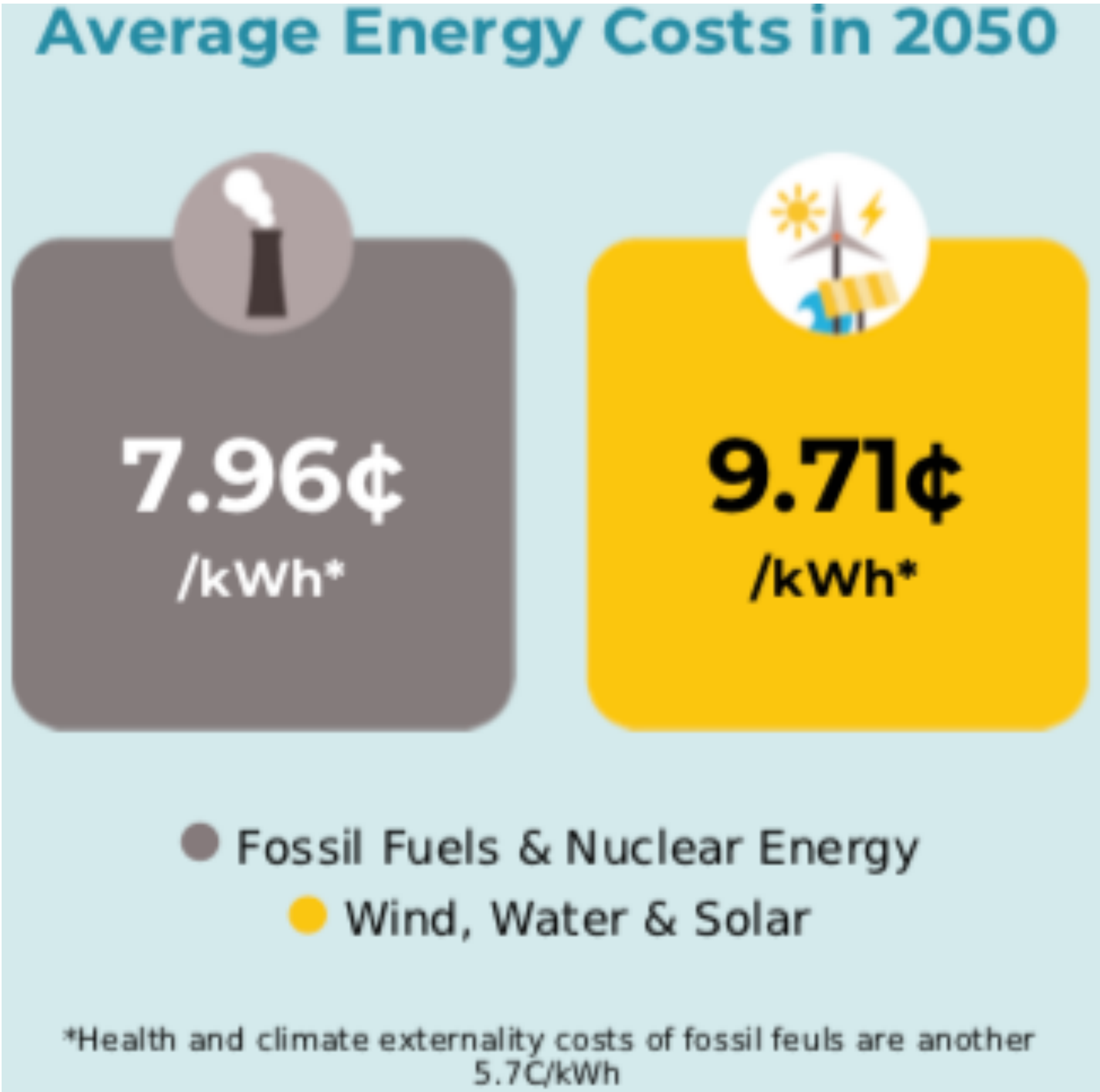
Land Usage

Percentage of Canada Land Needed for All New Wind, Water & Solar Generators

0.03%
Footprint Area

0.25%
Spacing Area

A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:
Average Energy Costs in 2050


Average Energy Costs in 2050
7.96¢
/kWh*
9.71¢
/kWh*
Fossil Fuels & Nuclear Energy
Wind, Water & Solar
*Health and climate externality costs of fossil feuls are another 5.7C/kWh

**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:
Money in Your Pocket**

Money in your Pocket

Energy cost savings per person:

$40.70

Energy, health, and climate cost savings per person:

$10,618

References

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Conclusion

In this talk we overviewed my recent results in energy market modelling, including:

- option pricing formula for a mean-reversion asset,
- variance and volatility swaps in energy markets,
-applications of weather derivatives in energy markets,
- pricing crude oil options using Lévy processes,
-energy contracts modelling with delayed and jumped volatilities.

I also talked about the clean renewable energy prospective, and a vision to transition to 100% wind, water & solar energy in Canada.

The End

Thank You!

Q&A time!