Titles and Abstracts
PIMS Workshop on Automata Theory and Symbolic Dynamics

Eugene Asarin

Title: Timed automata: model, languages, dynamics

Timed automata are finite automata working in continuous time; to this aim they are augmented with real-valued "clocks" measuring time lapses between events. They were invented in the early 90s by Alur and Dill as a model of real-time systems for verification purposes. Since then they have become a useful modeling/verification tool and a popular subject of study.

In this talk I will give a gentle introduction to timed automata and will address the following issues:

- what are timed words and timed languages;
- what are timed automata and how to model real-time systems with them;
- timed regular languages and their properties;
- basic algorithms for timed automata and region graph;
- timed automata as dynamical systems, timed sofic shifts;
- notion of entropy of timed automata.

Nathalie Aubrun

Title : Multidimensional effective subshifts

Abstract : This talk will be devoted to multidimensional effective subshifts, that are subshifts that can be defined by a recursively enumerable set of forbidden patterns. This class of subshifts may appear hard to understand at first sight, but we will see that it has very strong connections with the class of SFTs, which is the simplest class from a combinatorial point of view.

Alexis Ballier

Title: Characterizing some limit sets of stable cellular automata

Abstract: A cellular automata (CA) is a dynamical system defined as the iterations of a block map from the full shift to itself. The limit set of a CA consists of the configurations that can be reached in the limit of its evolution. When the limit set is reached after a finite number of iterations, the CA is said to be stable. Stable limit sets of CA are therefore (sofic) factors of a full shift. After A. Maass's work, we know that if a sofic shift factors onto its minimal right-resolving cover then, as soon as these are factors of a full shift, there exists a stable CA with this sofic shift as limit set. As far as we know, this remains the only known technique for constructing stable CA and there is no known characterization of stable limit sets of CA. Nevertheless, we can obtain a characterization of the limit sets of some stable CA. In this talk, we will show that a sofic shift is a factor of a full shift and factors onto its minimal right-resolving cover if and only if it is the stable limit set of a right-continuing almost-everywhere CA (to be defined in this talk).

Nicolas Basset

Title: A maximal entropy stochastic process for a timed automaton

Abstract: We consider the following question: what is the least biased stochastic process over the runs of a given timed automaton? By "least biased", we mean the one that generates runs with a distribution which is closest to uniform (w.r.t. volume).

To answer this question, we first define stochastic processes over runs of timed automata and their entropy, the definition of which is inspired by Shannon's differential entropy rate. We describe an ergodic stationary stochastic process which maximizes the entropy. Its definition is based on the spectral properties of the operator introduced by Asarin and Degorre, which can be seen as a generalization of adjacency matrices for timed graphs. The maximal entropy stochastic process is defined in an analoguous way to the case of irreducible shifts of finite type.

The second main result is an asymptotic equipartition property (AEP) of the runs generated w.r.t. the maximal entropy stochastic process. We finally explain how to achieve a quasi-uniform sampling using this AEP.

Valerie Berthe

Title: Pisot numeration systems and beyond

Properties of symbolic dynamical systems based on beta-numeration under the Pisot hypothesis tend now to be well understood. The aim of this lecture is first to recall the so-called Pisot conjecture and then to discuss current extensions with nonalgebraic parameters by focusing on spectral properties and on finiteness properties.

Mike Boyle

Title: Strong shift equivalence of matrices over a ring

Abstract: Let R be a ring. Two square matrices A,B are elementary strong shift equivalent (ESSE-R) over R if there are matrices U,V over R such that A=RS and B=SR. Strong shift equivalence over (SSE-R) is the equivalence relation generated by ESSE-R. Shift equivalence over R (SE-R) is a tractable equivalence relation which is refined by SSE-R. The refinement is trivial if R= Z (Williams), a principal ideal domain (Effros 1981) or a Dedekind domain (Boyle-Handelman 1993). No results have appeared since 1993.

It turns out that this refinement is captured precisely by a certain quotient group of the group $NK_1(R)$ of algebraic K-theory. It follows that for very many (not all) rings R, the relations SE-R and SSE-R are the same. For the class of nilpotent matrices over R (nilpotent matrices are those shift equivalent to [0]), this quotient group is $NK_1(R)$ itself. When $NK_1(R)$ is not trivial, it is not finally generated. (Farrell, 1977).

This is joint work with Scott Schmieding.

Kathleen Carroll

Title: Markov diagrams for some non-Markovian systems

We construct countable-state Markov shifts which represent some minimal subshifts, in particular Sturmian systems and some substitution systems, hoping that the representations can help us to explore the structures and properties of such systems.

This is joint work with Karl Petersen

Nishant Chandgotia

Title : Markov Random Fields and the Pivot Property

Abstract: A Markov random field is a probability measure supported on a shift space satisfying a conditional independence property: given two finite separated sets A and B in Z^d the configurations on A and B are independent conditioned on the configuration on their complement (here, "separated" means that B is contained in the complement of $A \cup \partial A$). If the underlying shift space has a "safe symbol" it is known via the Hammersley-Clifford theorem that the measure is actually a Gibbs measure, that is, the conditional probabilities can be written as a normalized product of weights on the configurations on edges. In this talk we introduce a weakening of the safe symbol property called the pivot property under which the set of conditional probabilities of shift-invariant Markov random fields can be given a finite description. Following Petersen and Schmidt we introduce a reparametrisation of the conditional probabilities in the form of Markov cocycles under which a finite description for the conditional probabilities corresponds to a finite dimensional space of shift-invariant Markov cocycles. We will present a construction for which the pivot property does not hold and the dimension of the space of shift-invariant Markov cocycles is infinite. Time permitting we will discuss some positive results in the context of stationary Markov random fields supported on the space of 3-coloured chessboard.

This is joint work with Tom Meyerovitch

Alfredo Costa

Title: A categorical invariant of flow equivalence of shifts

Abstract: This talk surveys results obtained in joint work with Benjamin Steinberg, concerning the Karoubi envelope of a shift, which is defined as the Karoubi envelope of the syntactic semigroup of the language of blocks of the shift. We prove that, up to natural equivalence of categories, the Karoubi envelope of a shift is invariant under flow equivalence. More precisely, we show that the action of the Karoubi envelope on the Krieger cover of the shift is a flow invariant. An analogous result concerning the Fischer cover of a synchronizing shift is also obtained. From these main results, several flow equivalence invariants — some new and some old — are obtained. Another application concerns the classification of Markov-Dyck shifts: it is shown that, under mild conditions, two graphs define flow equivalent Markov-Dyck shifts if and only if they are isomorphic.

Aldric Degorre

Title: Volume and Entropy of Timed Regular Languages

Abstract: For a timed language, we define a sequence of "volumes" associated to each number of discrete events (and hence the same number of continuous inter-event delays). We also define "entropy" as the logarithm of the asymptotic growth rate of that sequence. In the case of the language accepted by a bounded deterministic timed automaton, we characterize volumes as iterates of some positive linear operator and entropy as the logarithm of its spectral radius.

We exhibit a particular case where this radius is symbolically determined as a zero of some real function derived from volume equations. In the general case, we propose two approximation methods for entropy. In one, we study the accurate spectral properties of the operator and compare its successive iterates. In the other, we discretize the timed automaton and use the standard method of entropy computation for a finite automaton.

Catalin Dima

Title: Entropy of tree automata, joint spectral radii and zero-error coding

Abstract: We associate an entropy to a tree automaton working on infinite trees as the lim sup of the entropy of the trees accepted by the automaton. We show that this entropy equals the joint spectral radius of the set of "adjacency matrices" built from the transitions of the tree automaton, when the tree automaton has a "safety" acceptance condition. This property also holds for tree automata with level constraints, a class of automata which may be used to encode generalizations of the zero-error coding problem. The entropy of "normal" tree automata can be computed as the max of the entropies of the regular trees accepted by the automaton. The same property no longer seems to hold for tree automata with level constraints, as it is related to the finiteness conjecture for the joint spectral radius for integer matrices.

Soren Eilers

Title: Flow equivalence of sofic shifts

Franks resolved in 1984 the fundamental question of when two irreducible shifts of finite type (SFTs) are flow equivalent, offering a complete invariant which is both easy to compute and easy to compare. In joint work with Boyle and Carlsen, we have embarked on the task of trying to similarly classify irreducible sofic shifts up to flow equivalence. It turns out that solving this problem even in the basic case of AFT shifts involves understanding the flow classification both of reducible SFTs and of SFTs with actions of cyclic groups, and I will describe how our results are derived from an extension theorem based on a profound result by Boyle and Krieger.

Felipe Garcia Ramos

Title Measure theoretical equicontinuity and weak convergence for cellular automata.

Abstract: Different notions of measure theoretical equicontinuity (μ-equicontinuity) have been studied for topological dynamical systems, notably cellular automata. Here, the measure μ is typically not invariant. We will characterize some of these

notions and give results on weak convergence and weak Cesaro convergence of the orbit of μ under the dynamics of a μ-equicontinuous cellular automaton.

Benjamin Hellouin de Menibus

Title: Characterizing typical asymptotic behaviours of cellular automata.

Abstract: A cellular automaton (CA) is a discrete dynamical system that can be seen as a physical or a computational model. Computer simulation of various CA show that, starting from a random configuration, they can exhibit a wide variety of typical asymptotic behaviours. In this talk, I will characterize the behaviours that can be observed in this way, following the trend of recent results characterizing possible parameters of discrete dynamical systems by computability conditions. More precisely, typical asymptotic behaviour is well-described by the limit probability measure(s). When the initial measure is computable, computability obstructions appear on the limit measure(s). Conversely, every measure satisfying those obstructions can be reached at the limit by a cellular automaton that we build explicitly, using auxiliary states. This result has consequences in decidability, and this method can be further extended so that the limit measures depend on the initial measure, considered as an argument or an oracle. Furthermore, additional hypotheses allow us to remove the need for auxiliary states in some of those results.

Pavel Heller

Title: k-block versus 1-block parallel addition in non-standard numeration systems

Abstract: A positional numeration system is given by a base β in $\mathbb{C}$, $|\beta| > 1$, and a finite alphabet $\mathcal{A}$ of contiguous integers containing 0. We focus on the question whether, for a given numeration system, there exists a parallel algorithm performing addition of numbers with finite $(\beta, \mathcal{A})$-representations. By parallel algorithms we mean algorithms which perform the addition $x + y$ in constant time, independently of the lengths of the representations of x and y. This is equivalent to say that addition is a local function (or a sliding block code) from the alphabet $\mathcal{B} = \mathcal{A} + \mathcal{A}$ to $\mathcal{A}$. Recently, it has been shown that for any algebraic number β, $|\beta| > 1$, which has no conjugates of modulus 1, there exists an alphabet $\mathcal{A}$ allowing parallel addition. In general, the cardinality of $\mathcal{A}$ is unnecessarily large. In 1999, Kornerup suggested to consider a more general type of parallel algorithms, which, instead of treating each digit separately, manipulate blocks of digits of length $k \geq 1$. In that setting addition is a local function from $\mathcal{B}^k$ to $\mathcal{A}^k$.

In this talk we present an easy-to-check property of $(\beta, \mathcal{A})$ which guarantees the possibility of block parallel addition. We apply this result to the bases β which are Parry numbers, i.e., numbers whose Rényi expansion of unity $d_\beta(1) = t_1 t_2 t_3 \dots$ is finite or eventually periodic. We show that if β additionally satisfies the property (F) or (PF), then block parallel addition is possible on the alphabet $\{0, \dots, 2t_1\}$ or $\{-t_1, \dots, t_1\}$. Specifically, we prove the usefulness of this concept on the d-bonacci base, where $\beta > 1$ is a root of the polynomial $f(X) = X^d - X^{d-1} - X^{d-2} - \dots - X - 1$, by showing that k-block parallel addition is possible on the alphabets $\{0, 1, 2\}$ and $\{-1, 0, 1\}$ for some convenient k. However, if we require $k = 1$ (i.e.,

the standard parallel algorithm working with single digits), the cardinality of any alphabet allowing parallel addition in the d-bonacci base must be at least $d+1$.

This is joint work with Christiane Frougny, Edita Pelantová, and Milena Svobodová.

Emmanuel Jeandel

Title: Computability in Multidimensional Symbolic Dynamics

Abstract: While the theory of one-dimensional symbolic dynamics is instrinsically linked with finite automata theory, the study in higher dimensions entails computability theory, as even the most basic problem (is a SFT empty?) becomes undecidable in dimension 2. For a long time this has been seen as a hurdle in the study of dynamical properties, but recent results have shown that it is possible to obtain precise statements and characterizations of invariants in terms of well known computability concepts. This talk will give an overview of this recent trend.

Natasha Jonoska

Title: Regular Splicing Languages

Restriction enzymes act on double stranded DNA molecules by cleaving certain recognized segments leaving short single stranded overhangs. Molecules with the same overhangs can be joined (in a cross-over fashion) in presence of a ligase. A splicing system is a formal model introduced by T. Head in 1986 that formalizes these biomolecular processes. The system generates languages, called splicing languages, by using a contextual cross-over operation over words. In spite of wide investigations of finite splicing systems, basic questions, such as their characterization, remain unsolved. Two recent results provide better understanding of this class of languages. The first result proves the conjecture that a regular splicing language must have a constant in the sense of Schutzenberger. The second result shows that for a given regular language, there is a bound on the string length of the splicing rules, implicitly proving that it is decidable whether a given regular language is a splicing language. In this talk we give a brief survey on these and other results.

Jarkko Kari

Title: A survey on cellular automata

Cellular automata are discrete dynamical systems that are of interest in both automata theory and symbolic dynamics. Due to the simplicity and purity of the concept, they have found applications in modeling various physical systems. Cellular automata can also be viewed as massively parallel computers consisting of a uniform array of very simple computing elements with local interconnections. In this talk we review classical results such as the Garden-of-Eden-theorem and Hedlund's theorem. Reversibility and surjectivity of cellular automata are of particular interest. We recall several undecidable questions concerning cellular automata, and survey results on topological dynamics aspects such as sensitivity and mixing properties. The aim is to provide an overview of the history and the present state of the theory of cellular automata.

Wolfgang Krieger

Title: Contextfreeness in symbolic dynamics

Abstract: To a sufficiently synchronizing subshift X there is associated a labelled directed graph that presents X (see W. K., Presentations of symbolic dynamical systems by labelled directed graphs (Notes for a "mini-cours," SDA2, Paris 4-5 October 2007), arXiv:1209.1872 math.DS (2012)). We consider the situation that this graph is context-free in the sense of Muller and Schupp. We discuss special cases.

Petr Kůrka

Title: Expansion subshifts of Möbius number systems

Abstract: Möbius transformations $M(x) = (ax+b)/(cx+d)$ act on the extended real line $\overline{R} = R \cup \{\infty\}$. A Möbius number system (F, W) consists of Möbius transformations $(F_a)_{a\in A}$ indexed by a finite alphabet A and of open intervals $W_a \subseteq V(F_a)$ with $\cup_{a\in A}\overline{W_a} = \overline{R}$, where $V(F_a)$ consists of points, where F_a^{-1} is expansive with respect to the chord metric $d(x, y) = |x-y|/\sqrt{(x^2+1)(y^2+1)}$. The expansion graph has labelled edges $x \xrightarrow{a} F^{-1}(x)$, where $x \in W_a$. For a word $u \in A^*$ of length $n+1$ denote by $W_u = W_{u_0} \cap F_{u_0}W_{u_1} \cap \cdots \cap F_{u_0} \dots F_{u_{n-1}}W_{u_n}$ the set of points with expansion u. The expansion subshift $S_{F,W} = \{u \in A^N : \ \forall n, W_{u_{[0,n)}} \neq \emptyset\}$ consists of all infinite expansions. We characterize Möbius number systems whose expansion subshifts are of finite type and sofic. In these systems, arithmetical algorithms simplify substantially.

Sylvain Lombardy

Title: Conjugacy of Transducers

Abstract: We extend the definition of conjugacy to transducers. We show how this algebraic relation is related to coverings and co-coverings, as for weighted automata. We present a pseudo-sequentialisation procedure that turns every functional transducer into an equivalent unambiguous one. Moreover, this operation can be interpreted as a conjugacy and, as a result, two equivalent functional transducers are linked by conjugacies. Just as for weighted automata, this latter case implies that there exists a third transducer that can be obtained from any of the first ones by (co-)coverings and pushing of output.

This work is in collaboration with M.-P. Béal and J. Sakarovitch

Victor Marsault

Title: Autosimilarities in a rational base number system

Abstract: This work proves that, in a rational base number system, several structures are computable locally from each other, hence the term 'autosimilar'. More precisely, it refutes the idea that a minimal word contains all the information of $L_{\frac{p}{q}}$; proves that computing the minimal word of $n+1$ from the minimal word of n requires a structure similar to $L_{\frac{p}{q}}$; and presents a tangible differentiation between classes of rational base number systems.

Ronnie Pavlov

Title: Approximation methods for entropy of multidimensional shifts of finite type

Abstract: In this talk, we'll discuss the approximation/computation of entropies of shifts of finite type and certain measures on them. We'll cover the types of hypotheses which are needed to make any meaningful approximations at all (for instance, there exist SFTs with entropies which are not even computable), as well as some known techniques and their rates of approximation.

Parts of this talk are based on joint work with Brian Marcus.

Dominique Perrin

Title: Subshifts of linear complexity, Rauzy induction and subgroups of finite index of free groups

Abstract: We investigate properties of subshifts of linear complexity concerning words of first return. In particular, we show that, in a regular interval exchange transformation, the first return words to a given one form a basis of the free group. The proof uses a generalization of Rauzy induction to a two-sided induction.

This is joint work with Valerie Berthé, Clelia De Felice, Francesco Dolce, Giuseppina Rindone and Christophe Reutenauer.

Mathieu Sablik

Title: Speed of convergence of the realization of an effective subshift by a multidimensional SFT or Sofic

Jacques Sakarovitch

Title: On equivalence and conjugacy of weighted automata

Abstract: Decidability of equivalence of weighted automata is classically based upon linear algebra techniques. The same result is revisited with the notions of conjugacy and automata covering, leading to a combinatorial description of the equivalence, up to a 1-1 correspondence between computations of equivalent automata when the weights are taken in suitable semirings.

This is joint work with Marie-Pierre Béal and Sylvain Lombardy.

Klaus Schmidt

Title: The role of homoclinic points for algebraic actions

Michael Schraudner

Title: Entropy minimality of multidimenstional shifts of finite type

Abstract: An irreducible $\mathbb{Z}$ shift of finite type X is always entropy minimal, i.e. every proper subsystem of X has strictly smaller entropy. While this result has a lot of applications in the theory of one-dimensional subshifts, it does not extend to the class of $\mathbb{Z}^d$ subshifts for $d > 1$.

In the multidimensional setting only very strong uniform mixing conditions (e.g. UFP) guarantee entropy minimality, whereas non-entropy minimal, uniformly mixing (block or corner gluing) examples exist. We will show some of these non-entropy minimal examples, analyze the mechanism behind this phenomenon, and we will give a necessary and sufficient condition characterizing entropy minimality of $\mathbb{Z}^d$ shifts of finite type.

This is joint work with Samuel Lightwood.

Pascal Vanier

Title : Hardness of Conjugacy, Embedding and Factorization of multidimensional SFTs.

Abstract: Subshifts of finite type are sets of colorings of the plane defined by local constraints. They can be seen as a discretization of continuous dynamical systems. We investigate here the hardness of deciding factorization, conjugacy and embedding of subshifts of finite type (SFTs) in dimension $d > 1$. In particular, we prove that the factorization problem is Σ_3^0-complete and that the conjugacy and embedding problems are Σ_1^0-complete in the arithmetical hierarchy. If time permits, we will also give some results for effective and sofic shifts.

Tomas Vavra

Title: "Confluence" in Ito-Sadahiro number systems

Abstract: In positive base number systems many properties are specific for the class of confluent Pisot bases, i.e. zeros of $x^k - mx^{k-1} - mx^{k-2} - \cdots - mx - n$, where $k \geq 1$, $m \geq n \geq 1$. The main aspect is that any integer combination of non-negative powers of the base with coefficients in $\{0, 1, \ldots, \lceil \beta \rceil - 1\}$ is a β-integer, although the sequence of coefficients may be forbidden in the corresponding number system. Confluent Pisot bases are also among the only cases where the Erdos-Komornik-Joo problem has been solved. We concentrate on the question of confluence in negative base systems, introduced by Ito and Sadahiro. We show that any integer combination of non-negative powers of the base with coefficients in $\{0, 1, \ldots, \lfloor \beta \rfloor\}$ is a $(-\beta)$-integer if and only if β is a zero of the above polynomial satisfying $m = n$ when k is even. It turns out that these are also precisely the bases, for which the infinite word $u_{-\beta}$ coding $(-\beta)$-integers has the same language as that of u_β. As a consequence of our result, one can solve an instance of the Erdos-Komornik-Joo problem generalized to negative bases.

Reem Yassawi

Title: A characterization of p-automatic sequences as columns of linear cellular automata

Abstract: A p-automatic sequence is one that is generated by a finite state machine. Cobham's theorem characterizes these sequences as the letter to letter projections of fixed points of length p substitutions. We show that a sequence over a finite field F_q of characteristic p is p-automatic if and only if it occurs as a column of the spacetime diagram, with eventually periodic initial conditions, of a linear cellular automaton with 'memory' over F_q. As a consequence, the subshift

generated by a length p- substitution can be realized as a topological factor of a linear cellular automaton.

This is joint work with Eric Rowland.