NONHOLONOMIC SYSTEMS AND THE HAMILTONIZATION PROBLEM.

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Abstract

A nonholonomic system is a mechanical system with constraints in the velocities and thus the equations of motion are not hamiltonian. This fact has consequences in the dynamical behaviour as well as in the geometry underlying the mechanical system. In this talk we will study geometric features of nonholonomic systems and their behaviour after a reduction by a group of symmetries. In particular, we are interested in how Poisson geometry may help in understanding how far these systems are from being hamiltonian.

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ON THE LOCATION OF SATURN F RING: A SIMPLE MODEL

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Abstract

The confinement of planetary narrow rings is understood in terms of the shepherd theory [1]. This theory proposes the existence of two shepherd moons that orbit around the ring, and involves angular momentum transfer mechanisms between them and the ring particles, self-gravity, viscous damping, and assumes the existence of lower-order resonance at the ring boundaries in order to confine the ring. Saturn's F ring is a fascinating example of a narrow eccentric ring that displays a rich dynamical structure: besides its non-zero eccentricity and sharp-edges, it has multiple components entangled in a complicated way which shows a variety of short-time features [2]. The two shepherd moons, Prometheus and Pandora, influence importantly many of the short-time dynamical features observed. Yet, the shepherd theory does not apply to this ring since there are no mean-motion resonances that confine the ring and the mass of the shepherd moons is too small [3]. The confinement of Saturn's F-ring thus remains unexplained.
In this talk, I shall describe our scattering approach to narrow rings [4], illustrating it on a simple billiard system, where the relevant physics can be easily understood. Within this approach, I shall present numerical results based of accurate long-time integrations on a realistic 5-body model for the occurrence and location of Saturn's F ring. Test particles that remain trapped and display some stability properties form a narrow elliptic ring displaying sharp edges. Comparison of our results with the observations shall be provided. This work is in collaboration with Àngel Jorba (U. Barcelona).

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VORTEX DYNAMICS ON SURFACES OF REVOLUTION

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Abstract

Many key features of the motion of satellites, planets, stars, even galaxies can be captured by point mass dynamics. Likewise, many key features of fluid motion such as atmospheric storms, ocean eddies, super fluid vortices, and early stages of mass aggregation in gravitational systems can be captured by point vortex dynamics. Yet, serious mathematical challenges remain. Systems consisting of more than a few points are non-integrable, and complexity increases dramatically with the number of particles. Furthermore, the underlying geometry has a profound influence on the particle motion, as has only just begun to be investigated. Indeed one of today' s challenges is a formulation of the N-vortex dynamics on Riemann surfaces. There are formulations over surfaces with constant Gaussian curvature [1, 4, 5, 6, 9, 10], and lately, for surfaces with not constant Gaussian curvature, conform to the sphere [2, 7]. We present some results about vortex dynamics on surfaces of revolution (among others the sphere and the ellipsoid of rotation) and we show the importance of the curvature in the stability of the relative equilibria

[1, 2]. The stabilization by curvature appears also in the classical N-body problem [5, 8]

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ENERGY TRANSPORT IN DRIVEN HELIUM: LOCALIZATION VS. INTERACTION

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Abstract

Quantum-interference induced suppression of multi-photon ionization of highly excited Rydberg states is a well-studied phenomenon in single electron Rydberg states, known under the name dynamical localisation, and the dynamical analog of Anderson localization of electronic transport across disordered wires. After a short recollection of the basic features of single electron dynamical localisation, we will discuss the impact of electron-electron interaction in the multiphoton ionization of doubly excited states of helium, and disentangle the competing effects of interaction and driving-induced decay channels.

STABILITY RESULTS OF SOME MODELS IN CELESTIAL MECHANICS

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Abstract

I will present some stability results based on KAM theory. I will consider both conservative and dissipative (i.e., conformally symplectic) systems (either maps and flows). The proof is constructive and it provides efficient algorithms to evaluate the breakdown threshold of invariant tori ([2]). For low–dimensional degrees of freedom, KAM theory allows to get stability results. In this context, I will discuss the stability of the restricted 3–body problem (with particular reference to the asteroid 12 Victoria) and the stability of the Moon–Earth synchronous resonance under tidal effects. I will show that computer–assisted proofs may be devised in order to show the existence of invariant tori for realistic values of the parameters ([1], [3], [4]).

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PERIODIC BRAKE ORBITS IN THE PLANAR ISOSCELES THREE-BODY PROBLEM

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Abstract

The isosceles three body problem is a special case of the three body problem whose triangular configurations describe an isosceles triangle at every instant. A brake orbit is an orbit that starts with zero initial velocity. The purpose of this talk is to find periodic brake orbits in the isosceles three-body problem. We use various topological shooting arguments; we follow a curve of brake initial conditions under the flow until it reaches a suitable surface, and show that the image curve includes a point that corresponds to a periodic orbit. As a result, we find six types of periodic brake orbits.

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CLOSED TRIPLE COLLISION ORBITS IN PHOTOIONIZATION OF THE HELIUM ATOM

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Abstract

The cross section for single-electron photoionization in the helium atom shows fluctuations which decrease in amplitude as the total energy of the atom approaches the double-ionization threshold [1]. Treating the helium atom as a three-body Coulomb problem, we predicted that the decay of the fluctuations can be characterized in terms of a threshold law with an exponent obtained as a combination of Siegel's stability exponents of the triple-collision singularity; the details of the fluctuations were predicted to be linked to a set of infinitely unstable classical orbits starting and ending at the nonregularizable triple collision - so called CTCOs (Closed Triple Collision Orbits) [2]. The validity of the prediction was confirmed by numerical calculations for the *collinear* helium model [3], which was presented at the workshop 'Few-Body Dynamics in Atoms, Molecules, and Planetary Systems' held in 2010. We extend our calculational method to planar helium and analyze the results to confirm the validity of the prediction for the *planar* helium model atom. In addition, it is shown that the CTCOs play the same role in the partial cross sections as well as in the total cross section.

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GENERALIZED HIP–HOP SOLUTIONS OF THE $(N+N)$–BODY PROBLEM

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Abstract

Hip–hop solutions of the equal–mass $2N$–body problem are periodic solutions in which the bodies lie on the vertices of a regular antiprism for all time. A regular antiprism is a polyhedron formed by two congruent regular N–gons lying on parallel planes and such that the orthogonal projection on each other is a regular $2N$–gon.

In [1] and [2] it was shown that there exist families of Hip–hop solutions in the neighbourhood of planar regular $2N$–gon relative equilibria solution and of planar highly eccentric elliptic homographic solutions, the planes containing each group of bodies performing small oscillations along the line joining the centres of the N–gons.

Consider now two regular N–gons, with masses equal within each group, lying on two planes perpendicular to the line joining their centres. It is easily seen that, if the initial velocities within each group are invariant under a $2\pi/N$ rotation around that line, then the bodies remain on such a configuration for all time, provided no collision occurs. A periodic solution of this type with the same mean angular velocity for each group will be called a *Generalized Hip–Hop* solution of the $(N+N)$–body problem.

We show the existence of Generalized Hip–Hop solutions in the case of almost equal mass. These solutions are close to the planar regular $2N$–gon relative equilibria solutions with small vertical oscillations described above.

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RELATIVE EQUILIBRIA OF 3 BODIES IN DIMENSION d

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Abstract

The 3-body problem can be effectively symmetry reduced using invariants [1]. This leads to a Lie-Poisson structure and we analyse the equilibria of this 10-dimensional reduced system with two Casimirs. The Casimirs are the length of the angular momentum vector, and the Gram determinant of four difference vectors. When the spatial dimension d is equal to three then the Gram determinant is zero and the classical relative equilibria of Euler and Lagrange are found. However, when $d > 3$ the Gram determinant may be non-zero, and this allows for additional families of relative equilibria, with shapes that are distinct from the collinear and the equilateral families. These additional relative equilibria where described by Albouy and Chenciner [2] who called them balanced configurations [3]. We will give a detailed description of such relative equilibria and their linear stability.

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SCATTERING MONODROMY IN THE TWO CENTER PROBLEM

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Abstract

The Liouville-Arnold theorem states that the compact preimage of regular value of the energy momentum map of a Liouville integrable system is an n-dimensional torus where n is the number of degrees of freedom, and in the neighbourhood of this torus one can construct action-angle variables. Isolated singular values of the energy momentum map can cause a twist in the torus bundle over the regular image of an energy momentum map. As a result action-angle variables cannot be defined globally. This obstruction to the global construction of action-angle variables has been coined Hamiltonian monodromy, and many prominent examples of Liouville-integrable systems exhibiting this obstruction have been studied in recent years. Through the Bohr-Sommerfeld quantization of action variables monodromy carries over to quantum mechanics where it causes an obstruction to the global existence of quantum numbers. In this talk we discuss the classical and quantum mechanical implications of monodromy for scattering systems, i.e. for Liouville integrable systems where the pre images of the energy momentum map are not compact. As examples we study planar scattering at a repulsive central potential [1] and Euler's problem of two fixed centers [2, 3].

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INTERSECTION OF CURVES AND ORBIT DETERMINATION

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Abstract

We review some recent results concerning orbit determination problems which are reduced to the investigation of intersections of algebraic curves.
First we show a generalization of Charlier's theory [1], giving an interpretation of the occurence of alternative solutions in the computation of preliminary orbits by Gauss' and Laplace's methods.
Then we review two recent orbit determination methods [2], [3], useful for the linkage of two short arcs of observations. Here the integrals of the two-body motion are used to write polynomial equations for preliminary orbits. We also show some applications of these methods to real data of asteroids and space debris.

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PHASE SPACE STRUCTURE OF THE WANNIER RIDGE OF DRIVEN HELIUM

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Abstract

The three-body physics of helium in a strong driving field has caught a lot of attention since the famous "helium knee" showed the importance of electronic correlation in the double ionisation process [1]. Nowadays, there still exist many open questions on the role of electron-electron interaction in non-sequential ionisation [2, 3]. We study the Wannier ridge sub-manifold of the classical helium phase space for vanishing total angular momentum (i.e. planar configurations). The Wannier ridge corresponds to a completely symmetric and therefore correlated motion of the electrons. We prove that this configuration is invariant in the presence of periodic driving, if and only if the field is linearly polarised along the symmetry axis. Thus, correlated field-induced double ionisation can occur through the Wannier ridge, at least at the classical level. We use a time-frequency representation of orbits given by wavelet analysis to study the 5-dimensional phase space. Although the field mostly destroys the regular dynamics, we observe that it is also able to stabilise some regions of phase space for certain field frequencies and intensities [4].

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RECOLLIDING PERIODIC ORBITS IN ATTOSECOND PHYSICS

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Abstract

Can an ionized electron be driven back to the core by an ultrastrong laser pulse? This is a high-stakes issue in attosecond physics since the returning electron, by carrying back the energy it has absorbed from the laser, can act as the agent of many key processes in intense laser physics, including the ultrafast imaging of macromolecules and the design of new light sources through generation of ultra-high harmonics [1, 2]. When the laser is linearly polarized, the "recollision" (or "rescattering") model [3, 4] has been immensely successful in both interpreting current experiments and devising new ones. However the picture is much less clear for all other polarizations [5, 6]. The very same recollision model predicts that returns must be suppressed in circularly polarized laser pulses because ionized electrons tend to spiral away from the core. In contrast, we present evidence that electrons return to the core along specific families of periodic orbits which shuttle electrons towards and away from the core. We show that these special periodic orbits are the key agents of attosecond processes in circularly polarized fields.

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TRAIN CORRESPONDENCES FOR CELESTIAL BODIES

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Abstract

Joint with Richard Montgomery (UC Santa Cruz); to appear.

By kinematics of scattering we mean motion of n particles on line segments of $\mathbb{R}^d$, with two-body collisions at the end points, preserving momentum and energy. It is known (see [1]) that the number of collisions is finite.

We analyze the kinematics of scattering. Then, adapting techniques from [2], we show that every transversal kinematical scattering process can be approximated by the dynamics of particles with Coulomb interaction (attractive or repulsive), in the limit of large energy or large spatial extensiveness.

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CHAOTIC DIFFUSION AND LARGE SCALE CHAOS OF THE PLANETARY MOTION IN THE SOLAR SYSTEM

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Abstract

The motion of the planets in the Solar System is chaotic. This chaotic behavior is due to multiple resonances in the precessing motion of the orbits of the planets which time scales range from 46 kyr to a few Myr. The main effect of this chaotic behavior is to induce an exponential divergence of nearby orbits that multiplies by 10 the initial error in the model or parameters every 10 Myr. Due to the interactions of the asteroids which have a chaotic motion on an ever shorter time scale, the possibility of prediction of the precise motion of the planets is strictly limited to 60 Myr. Over longer time scale, the chaotic diffusion of the orbits can drive the precession motion of Mercury in resonance with the precessing motion of Jupiter, leading to macroscopic instabilities in the Solar System with the possibility of planetary collisions. Although the full system is complex, the main features of this process can be understood with the help of a one degree of freedom Hamiltonian system.

CORRELATED TWO-ELECTRON WAVE-PACKETS IN HELIUM

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Abstract

A natural candidate to investigate the possibility of manipulating the correlated electronic dynamics with the help of external fields is the helium atom. In this theoretical contribution we explore two different scenarios: on the one hand, under near-resonant periodic driving certain highly doubly excited helium states might transform in two-electron nondispersive wave packets, i.e. robust quantum objects that evolve along classical trajectories. On the other hand, we explore to what extend the time delay in attosencond transient absorption experiments of helium dressed by a few-cycle visible pulse can be used to control the shape of two-electron correlated wave functions.

INVARIANT TORI IN THE SPATIAL 3-BODY PROBLEM BY AVERAGING AND REDUCTION

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Abstract

We study the spatial three-body problem, which is a system of nine degrees of freedom. In order to apply perturbation theory we need to establish the possible regimes where the Hamiltonian of the three-body problem can be split into two parts: the unperturbed Hamiltonian composed of two Keplerian terms and the perturbation, which is supposed to be small with respect to the principal part.

Our approach is based on a combination of averaging techniques with reduction theory with the aim of building a reduced Hamiltonian and a reduced phase space as simple as possible. The reduction process takes into account all possible continuous symmetries of the problem, including the symmetry generated by the two approximate integrals obtained after performing the normalisation with respect to the two fast angles and truncating the higher-order terms.

Once the reduction process is completed, the averaged Hamiltonian defines a system of one degree of freedom in the fully reduced space whose dimension is two and which is embedded in $\mathbb{R}^3$. This phase space is a surface that depends on three parameters, it is parametrised using Deprit's elements [1] and may have zero, one, two or three singular points. Our approach is global in the sense that we deal with the flow of the fully-reduced system in the whole fully-reduced space.

The next step is the discussion of the occurrence of the different relative equilibria of the reduced Hamiltonian system. This is done in terms of the invariants and the fundamental constraint that define the fully-reduced phase space. There are two basic parameters to perform the analysis, namely the modulus of the total angular momentum vector and the modulus of the angular momentum vector of the outer ellipse. They generate the plane of parameters which is divided into six different regions and presents five bifurcation lines. Each region has a different number of relative equilibria, ranging from two to six. The number and stability of the equilibria change when crossing the different lines [3].

We consider every elliptic point in the reduced phase space and reconstruct it step by step to compute the corresponding KAM invariant tori that persist in the original space $\mathbb{R}^{12}$. In the cases where the equilibria are placed on the singular points of the two-dimensional reduced phase space we need to make a local analysis around the singularities, passing to a surface with no singular points.

In the process of reconstruction we define intermediate spaces and sets of variables depending on the type of motions under consideration. Due to the scale in the perturbation, the Hamiltonian is too degenerate to apply classical results of KAM theory. Thus, we use a specific result by Han, Li and Yi [2] for this kind of systems.

We find Cantor families of five-dimensional KAM tori for the spatial three-body problem.

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QUANTUM AND CLASSICAL MOTION OF TWO ELECTRONS IN ATOMS AND LASER FIELDS

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Abstract

In atomic physics, the few-body problem arises already for the second element in the periodic table, helium (He), exhibiting two electrons "orbiting" inside the $1/r$ Coulomb potential around the doubly-charged nucleus. Any atom or ion besides hydrogen (H) or H-like systems is thus in general not analytically solved and theoretical description relies on approximation or numerics that need to be tested.

Here, experiments are presented on measurements and the control of two-electron dynamics in atoms by applying laser/electric fields, on time scales of femtoseconds to attoseconds. Quantum-mechanical correlated wave packets of two excited electrons in helium can be observed and controlled [1]. In stronger laser fields, quasi-classical recollision occurs, where a single electron is freed, accelerated in the laser field and returns to its parent ion where it can further excite other electrons. Experiments show that a two-electron (doubly-) excited state can also be formed in this process [2]. Comparing measurements with classical trajectory simulations, we find that two electrons are then removed from the atom sequentially, but within a fraction of a laser-optical cycle (∼200 as) even when initially bound to a fully symmetric orbit.

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BIFURCATION ANALYSIS OF SYMMETRIC HAMILTONIAN RESONANCES

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Abstract

We present a general analysis of the bifurcation sequences of periodic orbits in general position of resonant Hamiltonian normal forms invariant under $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. The low-order cases 2:2 and 2:4 show rich structure [1, 2, 3] investigated both with geometric methods and a singularity theory approach.

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STABILITY OF RELATIVE EQUILIBRIA IN THE PLANAR N-VORTEX PROBLEM: A TOPOLOGICAL APPROACH

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Abstract

In the weather research and forecasting models of certain hurricanes [1], vortex crystals are found within a polygonal-shaped eyewall. These special configurations can be interpreted as relative equilibria (rigidly rotating solutions) of the point vortex problem introduced by Helmholtz. Their stability is thus of considerable importance. Adapting the approach of Moeckel [2] for the companion problem in celestial mechanics, we present some theory and results on the linear and nonlinear stability of relative equilibria in the planar N-vortex problem [3]. A topological approach is taken to show that for the case of positive circulations, a relative equilibrium is linearly stable if and only if it is a nondegenerate minimum of the Hamiltonian restricted to a level surface of the angular impulse (moment of inertia). Using a criterion of Dirichlet's, this implies that any linearly stable relative equilibrium with positive vorticities is also nonlinearly stable. Two symmetric families, the rhombus and the isosceles trapezoid, are analyzed, with stable solutions found in each case.

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STATIC & RIGID ROTOR CONFIGURATIONS OF THREE CLASSICAL 12-6-LENNARD-JONES PARTICLES

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Abstract

Motivated by the continuous search for stable geometric configurations of atoms, we consider the planar evolution of two freely movable point particles around a third immovable one with respect to the 12-6-Lennard-Jones potential. Here, all three considered point particles may have different Lennard-Jones parameters which breaks the usual symmetry conditions imposed on such systems. By means of a classical non-regularized Hamiltonian description of our restricted three particle system, we study the existence of genuine equilibria and rigid rotor solutions (central configurations). We prove, depending on the choice of the Lennard-Jones parameters, that for these genuine and relative equilibria collinear alignments as well as triangle shaped configurations can occur. Finally, the spectral stability of these configurations is discussed.

REGULARIZATION OF THE KEPLER PROBLEM ON THE SPHERE

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Abstract

In this talk I will present some results concerning the Kepler problem on the three-sphere S^3, that I obtained in collaboration with Shengda Hu [1]. I will show how to perform a Moser-type regularization and how to adapt the Ligon-Schaaf regularization to the problem under consideration. Then, I will explain the relationship between these regularizations and the corresponding regularizations for the Kepler problem in $\mathbb{R}^3$. This will be done by showing that the Moser regularization and the Ligon-Schaaf map we obtained can be understood as the composition of the corresponding maps for the Kepler problem in Euclidean space, and the gnomonic transformation.

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THE STABILITY AND CHAOTIC BEHAVIOURS OF THE CALEDONIAN SYMMETRIC FOUR AND FIVE BODY PROBLEMS

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Abstract

The Caledonian Symmetric Four and Five Body Problems (CS4BP and CS5BP) (Steves and Roy, 2001; Szell, Steves and Erdi, 2004; and Shoaib, Steves and Szell, 2008) are restricted four and five body gravitational models relevant to the study of the stability and evolution of symmetric quadruple/quintuple stellar clusters and exoplanetary systems of two planets orbiting a binary/triplet of stars. Recently we have developed a regularization algorithm to study close encounters and collision events occurring in the CS4BP and CS5BP (Sivasankaran, Steves and Sweatman, 2010). We describe an analytical criterion for hierarchical stability of the Caledonian symmetric few body problems, verify the criterion through numerical investigations and show an interesting link between the hierarchical stability and the chaotic behaviour of the symmetric systems. The critical value C_{crit} at which the system becomes hierarchically stable for all time depends only on the mass ratios mu_0 and mu_1 of the five body system.

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NORMAL FORMS FOR ROTATIONALLY-INVARIANT SYSTEMS

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Abstract

Rotationally invariant cotangent-bundle systems and the particular the case of n-mass point systems are studied using a recently-obtained parametrisation of the phase space. This parametrisation, in which the Marsden-Weinstein symplectic reduced space becomes a vector space with a canonical symplectic form, is beneficial in the study of dynamics near relative equilibria. In particular, it can be shown that Nekhoroshev's estimates near an elliptic equilibrium point in the reduced space induce similar estimates in the full phase-space.

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INVARIANT TORI IN THE SPATIAL 3-BODY PROBLEM BY AVERAGING AND REDUCTION

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Abstract

We study the spatial three-body problem, which is a system of nine degrees of freedom. In order to apply perturbation theory we need to establish the possible regimes where the Hamiltonian of the three-body problem can be split into two parts: the unperturbed Hamiltonian composed of two Keplerian terms and the perturbation, which is supposed to be small with respect to the principal part.

Our approach is based on a combination of averaging techniques with reduction theory with the aim of building a reduced Hamiltonian and a reduced phase space as simple as possible. The reduction process takes into account all possible continuous symmetries of the problem, including the symmetry generated by the two approximate integrals obtained after performing the normalisation with respect to the two fast angles and truncating the higher-order terms.

Once the reduction process is completed, the averaged Hamiltonian defines a system of one degree of freedom in the fully reduced space whose dimension is two and which is embedded in $\mathbb{R}^3$. This phase space is a surface that depends on three parameters, it is parametrised using Deprit's elements [1] and may have zero, one, two or three singular points. Our approach is global in the sense that we deal with the flow of the fully-reduced system in the whole fully-reduced space.

The next step is the discussion of the occurrence of the different relative equilibria of the reduced Hamiltonian system. This is done in terms of the invariants and the fundamental constraint that define the fully-reduced phase space. There are two basic parameters to perform the analysis, namely the modulus of the total angular momentum vector and the modulus of the angular momentum vector of the outer ellipse. They generate the plane of parameters which is divided into six different regions and presents five bifurcation lines. Each region has a different number of relative equilibria, ranging from two to six. The number and stability of the equilibria change when crossing the different lines [3].

We consider every elliptic point in the reduced phase space and reconstruct it step by step to compute the corresponding KAM invariant tori that persist in the original space $\mathbb{R}^{12}$. In the cases where the equilibria are placed on the singular points of the two-dimensional reduced phase space we need to make a local analysis around the singularities, passing to a surface with no singular points.

In the process of reconstruction we define intermediate spaces and sets of variables depending on the type of motions under consideration. Due to the scale in the perturbation, the Hamiltonian is too degenerate to apply classical results of KAM theory. Thus, we use a specific result by Han, Li and Yi [2] for this kind of systems.

We find Cantor families of five-dimensional KAM tori for the spatial three-body problem.

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RECOLLIDING PERIODIC ORBITS IN ATTOSECOND PHYSICS

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Abstract

Can an ionized electron be driven back to the core by an ultrastrong laser pulse? This is a high-stakes issue in attosecond physics since the returning electron, by carrying back the energy it has absorbed from the laser, can act as the agent of many key processes in intense laser physics, including the ultrafast imaging of macromolecules and the design of new light sources through generation of ultra-high harmonics [1, 2]. When the laser is linearly polarized, the "recollision" (or "rescattering") model [3, 4] has been immensely successful in both interpreting current experiments and devising new ones. However the picture is much less clear for all other polarizations [5, 6]. The very same recollision model predicts that returns must be suppressed in circularly polarized laser pulses because ionized electrons tend to spiral away from the core. In contrast, we present evidence that electrons return to the core along specific families of periodic orbits which shuttle electrons towards and away from the core. We show that these special periodic orbits are the key agents of attosecond processes in circularly polarized fields.

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A SIMPLE METHOD TO CONSTRUCT LOCAL EQUILIBRIUM FUNCTION

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Abstract

We have developed a simple method to construct local equilibrium function for lattice Boltzmann method (LBM). This new method can make LBM model satisfy compressible flow with a flexible specific-heat ratio. Test cases, including the one dimensional Sod flow, one dimensional Lax flow and thermal Couette flow are presented. Good results obtained using proposed new method, indicate that the proposed method is potentially capable of constructing of the local equilibrium function for LBM.

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THE RELATIONSHIPS BETWEEN REGULAR POLYGONAL CENTRAL CONFIGURATIONS AND MASSES FOR NEWTONIAN N-BODY PROBLEMS

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Abstract

This is a joint work with Li Wei.

In 1985, Perko and Walter ([5]) proved that the N($N \geq 4$) point masses that locate at the vertices of a regular N-polygon form a central configuration if and only if all the masses are equal. In this talk, we will give a simpler proof of their theorem and show that if the N point masses locate at the vertices of a regular N+1-polygon, they can't form a central configuration for any masses.

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LONG-LIVED ORBITS WITH NON-VANISHING ANGULAR MOMENTUM IN DOUBLY-EXCITED PLANAR HELIUM

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Abstract

Helium as a microscopic embodiement of the 3-body problem is known to be chaotic. Early studies ([1, 2]) have shown that there are (quasi-)stable submanifolds, which support, e.g., the frozen planet configuration. We present some new long-lived classical orbits that exhibit surprisingly long life times and unusual shapes. These orbits were inferred from a spectral analysis, including (auto-)ionization rates, of the quantum problem.

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