Emergent Research:

The PIMS Postdoctoral Fellow Seminar

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Pacific Institute for the Mathematical Sciences

L-Functions of Elliptic Curves Modulo Integers

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ABSTRACT:

Elliptic curves are one of the major objects of study in number theory. Over finite fields, their zeta functions were proven to be rational by F. K. Schmidt in 1931. In 1985, R. Schoof devised an algorithm to compute zeta functions of elliptic curves over finite fields by directly computing the numerators of these rational functions modulo sufficiently many primes. Over function fields of positive characteristic p, we know from the work of A. Grothendieck, M. Artin, J.L. Verdier (1964/1965) and others, that their L-functions are rational. They are even polynomials with integer coefficients if we assume that their j-invariants are nonconstant rational functions, as shown by P. Deligne in 1980 using a result of J.-I. Igusa (1959).

Therefore, we can meaningfully study the reduction of the L-function of an elliptic curve E with nonconstant j-invariant modulo an integer N. In 2003, C. Hall gave a formula for that reduction modulo N, provided the elliptic curve had rational N-torsion.

In this talk, we first obtain, under the assumptions of C. Hall, a formula for the L-function of any of the infinitely many quadratic twists of E. Secondly, without any condition on the rational 2-torsion subgroup of E, we give a formula for the quotient modulo 2 of L-functions of any two quadratic twists of E. Thirdly, we illustrate that sometimes the reduced L-function is enough to determine important properties of the L-function itself. More precisely, we use the previous results to compute the global root numbers of an infinite family of quadratic twists of some elliptic curve and, under extra assumptions, find in most cases the exact analytic rank of each of these quadratic twists. Finally, we use our formulas to compute directly some degree 2 L-functions, in analogy with the algorithm of Schoof.

SPEAKER BIO:

Félix Baril Boudreau obtained his Ph.D. at the University of Western Ontario in August 2022, where he studied arithmetic geometry and specialised in elliptic curves over function fields and their L-functions under the supervision of Dr. Chris Hall. He is currently a PIMS Postdoctoral Fellow at the University of Lethbridge. He works with Dr. Amir Akbary on analytic number theoretic questions involving L-functions of characters over function fields, and with Dr. Andrew Fiori, with whom he explores theoretical and computational aspects of the Langlands program. He also obtained a Master degree at the University of Western Ontario, a Master 1 at the Université Pierre et Marie Curie in France and a Bachelor degree at the Université de Sherbrooke in Québec.

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