# The problem

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Where:

## Definition

An action $\alpha$ of a countable group $G$ on a compact metric space $(X, d)$ has the pseudo-orbit tracing property if for every $\epsilon > 0$ there exists $\delta > 0$ and a finite set $S \subset \Gamma$ such that every $(S, \delta)$ pseudo-orbit is $\epsilon$-traced by an genuine $\alpha$-orbit.

# Pseudo Orbits and tracing

## Definition

Fix $S \subset \Gamma$ and $\delta > 0$. A pseudo-orbit for a $G$ action $\alpha$ on a compact metric space $(X, d)$ is a $\Gamma$-sequence $(x_g)_{g \in \Gamma}$ in $X$ such that $d(\alpha_s(x_g), x_{sg}) < \delta$ for all $s \in S$ and $g \in \Gamma$. We say that a pseudo-orbit $(x_g)_{g \in \Gamma}$ is $\epsilon$-*traced* (or "shadowed") by the orbit of $x \in X$ if $d(\alpha_g(x), x_g) < \epsilon$ for all $g \in \Gamma$.

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**Remark**

The pseudo-orbit tracing property is sometimes called "shadowing property".

# Algebraic actions and "the Pontryagin dictionary"

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- The classical example: Automorphisms of $\mathbb{R}^d/\mathbb{Z}^d$.
- The action (along with all it's dynamical properties) is uniquely determined by the dual $\mathbb{Z}G$-module.

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- There are examples of expansive algebraic actions with a finitely presented dual that do not have the p.o.t property.
- Siddhartha Bhattacharya recently showed an example of an expansive action of a polycyclic group that does not have the p.o.t. property.