

Automorphisms of p -completed classifying spaces of groups of Lie type

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For any space X , let $\text{Out}(X)$ be the group of homotopy classes of self homotopy equivalences of X . If G is a finite group, then elementary properties of covering spaces show that the natural homomorphism $\text{Out}(G) \rightarrow \text{Out}(BG)$ is an isomorphism. However, if p is a prime and BG_p^\wedge denotes the Bousfield-Kan p -completion of BG , then $\text{Out}(BG_p^\wedge)$ can be very different from $\text{Out}(BG) \cong \text{Out}(G)$.

In joint work with Carles Broto and Jesper Møller, we were surprised to find that when G is a finite simple group of Lie type, there is a close relation between these two groups. If p is the defining characteristic of G , then $\text{Out}(BG_p^\wedge) \cong \text{Out}(G)$ (the natural homomorphism is an isomorphism) with just one exception: $G = SL_3(2)$. If p is different from the defining characteristic, then the natural homomorphism is in general neither surjective nor injective, but there is always another finite group G^* such that $BG_p^{*\wedge} \simeq BG_p^\wedge$, and the natural map $\text{Out}(G^*) \rightarrow \text{Out}(BG_p^{*\wedge})$ is split surjective.

The methods used to prove this are partly homotopy theoretic and partly algebraic.