Workshop on Arithmetic Topology

University of British Columbia
June 10- 14, 2019

Program

Image credit: UBC

Program at a Glance

	Monday 10	Tuesday 11	Wednesday 12	Thursday 13	Friday 14
Location	**All Sessions in** **ESB 1012**	**All Sessions in** **AERL 120**	**All Sessions in** **AERL 120**	**All Sessions in** **AERL 120**	**All Sessions in** **AERL 120**
8:30am	Registration and Refreshments **ESB Atrium** **2207 Main Mall**	Refreshments	Refreshments	Refreshments	Refreshments
8:50am	Welcome From PIMS and Melanie Matchett Wood (on behalf of the Workshop Organizers)				
9:00am - 9:50am	Opening session	**Kirsten Wickelgren** Mini-Course part 1	**Wei Ho** Mini-Course part 1	**Benson Farb** Mini-Course part 2	**Wei Ho** Mini-course part 2
9:50am - 10:15am	Coffee Break				
10:15 am - 11:05am	**Jordan Ellenberg** Mini-Course Part 1	**Benson Farb** Mini-Course part 1	**Ravi Vakil** Mini-Course part 1	**Soren Galatius** Mini-course part 2	**Aaron Landesman presented on behalf of Ravi Vakil** Mini-course part 2
11:05am - 11:30am	Coffee Break				
11:30am -12:20pm	**Jordan Ellenberg** Mini-Course Part 2	**Soren Galatius** Mini-Course part 1	**Kirsten Wickelgren** Mini-Course part 2	**Will Sawin** Talk 4	**Andrew Putman** Talk 6
12:20pm - 2:00pm	Lunch: Self hosted				
2:00pm - 2:50pm	**Ben Williams** Talk 1	**Inna Zakharevich** Talk 2	**Rita Jimenez-Rolland** Talk 3	**Orsola Tommasi** Talk 5	**Daniel Litt** Talk 7
2:50pm - 3:30pm	Coffee break & "Ask the Experts"				
3:30pm - 5:00pm	Junior Participant Talks	Moderated Problem Session 1	Junior participant talks	Moderated Problem Session 2	Moderated problem session - how should people concentrate efforts over next few years
Evening Events 6:00pm	Networking Reception and Dinner **UBC Golf Club** (Directions here)				

Subject to changes and edits

There will be photography throughout this event. PIMS' event photography is used across a variety of our communications platforms including web, print and electronic promotional materials. If, for any reason, you wish not to have your photo taken or used in this manner, please contact the event organizers.

Getting Started

Get connected: Select the "ubcvisitor" wireless network on your wireless device. Open up a web browser, and you will be directed to the login page. You can also log in through "eduroam" if this is available through your university.

FAQs

Q: Where do I check in on Monday Morning?

Check- in and package pick up can be done in the Earth Sciences Building (ESB) Atrium.

Q: Where are the sessions?

- Monday June 10th: All sessions will be in the **Earth Sciences Building (ESB) Room 1012**
- Tuesday June 11- 14: All sessions in **Aquatic Ecosystems Research Lab (AERL 120)**
- You will find a copy of the building floor plan on page 3 and a campus map at the end of the program.

Q: Will the program change? Program changes and updates will be announced at each session.

Q: When should I wear my badge? Please wear your name badges at all times on site so that PIMS Staff recognize you as a guest.

Q: Where can I go for help on site? If you need assistance or have a question, please feel free to talk to us at the registration desk

Q: Where can I get refreshments and meals? For snacks or quick meals, please view the list of UBC eateries online at http://www.food.ubc.ca/feed-me/. Coffee breaks are provided each day of the workshop.

Q: Where can I get a cab to pick me up from the Venue? You can call Yellow Cab (604-681-1111) and request to be picked up at the UBC Bookstore.

Q: How can I get around?

- **UBC Map link**: Here
- **Public Transit:** Feel free to search and plan your public transport rides by visiting http://www.translink.ca/, where directions, ticket costs and bus schedules are indicated.
- **Parking at UBC:** http://www.parking.ubc.ca/visitor.html

Q: What emergency numbers should I know?

- **Campus security (604-822-2222);**
- **General Emergencies (911);**
- **UBC hospital (604-822-7121).**

Campus Map:

- Earth Sciences Building: Room 1012
- Aquatic Ecosystems Research lab: Room 120

Full UBC Maps are available online here

Speaker Titles and Abstracts (in Alphabetical Order)

Jordan Ellenberg, University of Wisconsin-Madison

Geometric aspects of arithmetic statistics

Arithmetic statistics asks about the distribution of objects arising “randomly” in number-theoretic context. How many A_5-extensions of Q are there whose discriminant is a random squarefree integer? How big is the Selmer group fo a random quadratic twist of an elliptic curve over Q? How likely is the the maximal extension of Q unramified away from a random set of 3 primes to be an extension of infinite degree? Traditionally we ask these questions over Q or other number fields, but they make sense, and are expected to have broadly similar answers, when we replace Q with the function field of a curve over a finite field F_q. Having done this, one suddenly finds oneself studying the geometric properties of relevant moduli spaces over F_q, and under ideal circumstances, the topology of the corresponding moduli spaces over the complex numbers. Under even more ideal circumstances, results in topology can be used to derive theorems in arithmetic statistics over function fields. I’ll give an overview of this story, including some recent results in which the arrow goes the other way and analytic number theory can be used to prove results about the geometry of moduli spaces

Benson Farb, University of Chicago

The general goal of my two talks will be to describe some examples of how phenomena in number theory can be used to discover (and sometimes prove) phenomena in topology, and vice versa.

Talk 1: Point counting and topology

In this first talk I will explain how the machinery of the Weil Conjectures can be used to transfer information back and forth between the topology of a complex algebraic variety and its F_q points. A sample question: How many F_q-points does a random smooth cubic surface have? This was recently answered by Ronno Das using his (purely topological) computation of the cohomology of the universal smooth, complex cubic surface. This is part of a much larger circle of fascinating problems, most completely open.

Talk 2: Coincidences between homological densities, predicted by arithmetic

In this talk I'll describe some remarkable coincidences in topology that were found only by applying Weil's (number field)/(function field) analogy to some classical density theorems in analytic number theory, and then computing directly. Unlike the finite field case, here we have only analogy; the mechanism behind the coincidences remains a mystery. As a teaser: it seems that under this analogy the (inverse of the) Riemann zeta function at (n+1) corresponds to the 2-fold loop space of P^n. This is joint work with Jesse Wolfson and Melanie Wood.

Soren Galatius , University of Copenhagen

E_2 algebras and homology.

Block sum of matrices define a group homomorphism GL_n(R) \times GL_m(R) \to GL_{n+m}(R), which can be used to make the direct sum of H_s(BGL_t(R);k) over all s, t into a bigraded-commutative ring. A similar product may be defined on homology of mapping class groups of surfaces with one boundary component, as well as in many other examples of interest. These products have manifestations on various levels, for example there is a product on the level of spaces making the disjoint union of BGL_n(R) into a homotopy commutative topological monoid. I will discuss how it, and other concrete examples, may be built by iterated cell attachments in the category of topological monoids, or better yet E_2 algebras, and what may be learned by this viewpoint. This is all joint work with Alexander Kupers and Oscar Randal-Williams.

Wei Ho, University of Michigan

Conjectures, heuristics, and theorems in arithmetic statistics

We will begin by surveying some conjectures and heuristics in arithmetic statistics, most relating to asymptotic questions for number fields and elliptic curves. We will then focus on one method that has been successful, especially in recent years, in studying some of these problems: a combination of explicit constructions of moduli spaces, geometry-of-numbers techniques, and analytic number theory.

Rita Jimenez-Rolland, IM-UNAM Oaxaca

Representation stability and asymptotic stability of factorization statistics

In this talk we will consider some families of varieties with actions of certain finite reflection groups — such as hyperplane complements or complex flag manifolds associated to these groups. The cohomology groups of these families stabilize in a precise representation-theoretic sense. Our goal is to explain how these stability patterns manifest, and can be recovered from, as asymptotic stability of factorization statistics of related varieties defined over finite fields.

Daniel Litt, Institute for Advanced Study

Geometricity and Galois actions on fundamental groups

Which local systems on a Riemann surface X arise from geometry, i.e. as (subquotients of) monodromy representations on the cohomology of a family of varieties over X? For example, what are the possible level structures on Abelian schemes over X? We describe several new results on this topic which arise from an analysis of the outer Galois action on etale fundamental groups of varieties over finitely generated fields.

Andrew Putman, Notre Dame

The stable cohomology of the moduli space of curves with level structures

I will prove that in a stable range, the rational cohomology of the moduli space of curve with level structures is the same as the ordinary moduli space of curves: a polynomial ring in the Miller-Morita-Mumford classes.

Will Sawin, Columbia University

The circle method and the cohomology of moduli spaces of rational curves.

The cohomology of the space of degree d holomorphic maps from the complex projective line to a sufficiently nice algebraic variety is expected to stabilize as d goes to infinity. The limit is expected to be the cohomology of the double loop space, i.e. the space of degree d continuous maps from the sphere to that variety. This was shown for projective space by Segal, and there has been further subsequent work. In joint work with Tim Browning, we give a new approach to the problem for smooth affine hypersurfaces of low degree (which should also work for projective hypersurfaces, complete intersections, and/or higher genus curves), based on methods from analytic number theory. We take an argument of Birch that solves the number-theoretic analogue of this problem and translate it, step by step, into the language of ell-adic sheaf theory using the sheaf-function dictionary. This produces a spectral sequence that computes the cohomology, whose degeneration would imply that the rational compactly-supported cohomology matches that of the double loop space.

Orsola Tommasi, University of Padua

Stable cohomology of complements of discriminants

The discriminant of a space of functions is the closed subset consisting of the functions which are singular in some sense. For instance, for complex polynomials in one variable the discriminant is the locus of polynomials with multiple roots. In this special case, it is known by work of Arnol'd that the cohomology of the complement of the discriminant stabilizes when the degree of the polynomials grows, in the sense that the k-th cohomology group of the space of polynomials without multiple roots is independent of the degree of the polynomials considered. A more general set-up is to consider the space of non-singular sections of a very ample line bundle on a fixed non-singular variety. In this case, Vakil and Wood proved a stabilization behaviour for the class of complements of discriminants in the Grothendieck group of varieties. In this talk, I will discuss a topological approach for obtaining the cohomological counterpart of Vakil and Wood's result and describe stable cohomology explicitly for the space of complex homogeneous polynomials in a fixed number of variables and for spaces of smooth divisors on an algebraic curve.

Ravi Vakil, Stanford University

The Grothendieck ring of varieties, and stabilization in the algebro-geometric setting

A central theme of this workshop is the fact that arithmetic and topological structures become best behaved "in the limit". The Grothendieck ring of varieties (or stacks) gives an algebro-geometric means of discovering, proving, or suggesting such phenomena

In the first lecture of this minicourse, Ravi Vakil will introduce the ring, and describe how it can be used to prove or suggest such stabilization in several settings.

In the second lecture of the minicourse, Aaron Landesman will use these ideas to describe a stability of the space of low degree covers (up to degree 5) of the projective line (joint work with Vakil and Wood). The results are cognate to Bhargava's number field counts, the philosophy of Ellenberg-Venkatesh-Westerland, and Anand Patel's fever dream**.**

Kirsten Wickelgren, Georgia Institute of Technology

A^1 enumerative geometry: counts of rational curves in P^2

We will introduce A^1 homotopy theory, focusing on the A^1 degree of Morel. We then use this theory to extend classical counts of algebraic-geometric objects defined over the complex numbers to other fields. The resulting counts are valued in the Grothendieck--Witt group of bilinear forms, and weight objects using certain arithmetic and geometric properties. We will focus on an enrichment of the count of degree d rational plane curves, which is joint work with Jesse Kass, Marc Levine, and Jake Solomon.

Ben Williams, UBC

A1-homotopy of the general linear group and a conjecture of Suslin

Following work of Röndigs-Spitzweck-Østvær and others on the stable A1-homotopy groups of the sphere spectrum, it has become possible to carry out calculations of the n-th A1-homotopy group of BGLn for small values of n. This group is notable, because it lies just outside the range where the homotopy groups of BGLn recover algebraic K-theory of fields. This group captures some information about rank-n vector bundles on schemes that is lost upon passage to algebraic K-theory. Furthermore, this group relates to a conjecture of

Suslin from 1984 about the image of a map from algebraic K-theory to Milnor K-theory in degree n. This conjecture says that the image of the map consists of multiples of (n-1)!. The conjecture was previously known for the cases n=1, n=2 (Matsumoto's theorem) and n=3, where it follows from Milnor's conjecture on quadratic forms. I will establish the conjecture in the case n=4 (up to a problem with 2-torsion) and n=5 (in full). This is joint work with Aravind Asok and Jean Fasel.

Inna Zakharevich, Cornell

Quillen's Devissage in Geometry

In this talk we discuss a new perspective on Quillen's devissage theorem. Originally, Quillen proved devissage for algebraic K-theory of abelian categories. The theorem showed that given a full abelian subcategory $\mathcal{A}$ of an abelian category $\mathcal{B}$, $K(\mathcal{A})\simeq K(\mathcal{B})$ if every object of $\mathcal{B}$ has a finite filtration with quotients lying in $\mathcal{A}$. This allows us, for example, to relate the K-theory of torsion $\mathbf{Z}$-modules to the K-theories of $\mathbf{F}_p$-modules for all p. Generalizations of this theorem to more general contexts for K-theory, such as Walhdausen categories, have been notoriously difficult; although some such theorems exist they are generally much more complicated to state and prove than Quillen's original. In this talk we show how to translate Quillen's algebraic approach to a geometric context. This translation allows us to construct a devissage theorem in geometry, and prove it using Quillen's original insights.

Short Talks Titles and Abstracts (in Alphabetical Order)

Santiago Arango, Universidad de Los Andes

Arithmetic Equivalence

Two number fields are said to be arithmetically equivalent if their zeta functions are equal. Gassman established a group theoretic notion directly related to arithmetic equivalence, and since then, the correspondence of Galois theory with the theory of covering spaces has been exploited to characterize other types of equivalence in several areas of mathematics; including Riemannian Geometry, Riemann Surfaces, Graph Theory and Algebraic Geometry.

Oishee Banerjee, University of Chicago

Cohomology of the space of polynomial morphisms on \A^1 with prescribed ramifications.

In this talk we will discuss the moduli spaces Simp^m_n of degree n+1 morphisms \mathbb{A}^1_{K} \to \mathbb{A}^1_{K} with "ramification length < m" over an algebraically closed field K. For each m, the moduli space Simp^m_n is a Zariski open subset of the space of degree n+1 polynomials over K up to Aut (\mathbb{A}^1_{K}). It is, in a way, orthogonal to the many papers about polynomials with prescribed zeroes -- here we are prescribing, instead, the ramification data. We will also see why and how our results align, in spirit, with the long standing open problem of understanding the topology of the Hurwitz space.

Thomas Brazelton, University of Pennsylvania

An Enriched Degree of the Wronski Map

This talk will describe progress towards providing an enriched degree of the Wronski map. As a result, we compute an arithmetic count of p-planes intersecting mp general m-planes which holds over any perfect field. These computations recover familiar enumerative geometry results such as four lines in 3-space, as well as the complex degree of the Wronski as computed by Schubert, and the real degree as computed by Eremenko and Gabrielov. This work is part of the ongoing program in A^1-enumerative geometry, developed by Wickelgren, Kass, Levine, and others, to use Voevodsky's machinery of A^1-homotopy theory to enrich results in classical enumerative geometry over a wider range of fields.

Lei Chen, CalTech

Section problems for configurations of points on the Riemann sphere

I will talk about a suite of results concerning the problem of adding m distinct new points to a configuration of n distinct points on the Riemann sphere, such that the new points depend continuously on the old. Altogether, the results of the paper provide a complete answer to the following question: given $n \neq 5$, for which m can one continuously add m points to a configuration of n points? For $n \geq 6$, we find that m must be divisible by $n(n-1)(n-2)$, and we provide a construction based on the idea of cabling of braids. For n=3,4, we give some exceptional constructions based on the theory of elliptic curves.

Weiyan Chen, University of Minnesota

Choosing distinct points on curves

Every smooth cubic plane curve has 9 inflection points, 27 sextatic points, and 72 "points of type nine". Motivated by these classical algebro-geometric constructions, we study the following topological question: for a fixed n, is it possible to continuously choose n distinct unordered points on each smooth cubic plane curve? We prove that the answer is no unless n is a multiple of 9.

Ronno Das, University of Chicago

Points and lines on cubic surfaces

The Cayley-Salmon theorem states that every smooth cubic surface in CP^3 has exactly 27 lines; marking a line on each cubic surface produces a 27-sheeted cover of the moduli space M of smooth cubic surfaces. Similarly, marking a point produces a 'universal family' of cubic surfaces over M. I will describe the rational cohomology of these spaces. These purely topological computations have purely arithmetic consequences: the typical smooth cubic surface over a finite field F_q contains 1 line and $q^2 + q + 1$ points.

Kevin Kordek, Georgia Institute of Technology

Representation stability in the level 4 braid group

The level 4 braid group is the kernel of the mod 4 reduction of the integral Burau representation. It can also be described as the fundamental group of the universal 2-torsion abelian cover of the complement of the braid arrangement or, alternatively, as the subgroup of the pure braid group generated by squares of Dehn twists. This group arises naturally in topology, geometric group theory, algebraic geometry, and number theory. Its homology supports various enriched structures, such as a mixed Hodge structure, and possesses a large group of natural symmetries. In this talk I will present a recent theorem with Dan Margalit which states that its first homology satisfies uniform representation stability and give several applications.

Aaron Landesman, Stanford University

Homological Stability for Selmer Spaces?

The minimalist conjecture predicts that 0% of elliptic curves have rank 2 or more. We are able to prove a version of this conjecture over function fields $F_q(t)$ after first taking a large q limit.

Our method of proof is to show that the zeroth homology of a sequence of ""Selmer spaces"" stabilizes.

In this talk, we describe these Selmer spaces. If one could show the higher homologies of Selmer spaces stabilize, it would likely imply that 0% of elliptic curves over a fixed function field have rank 2 or more.

Stephen McKean, Georgia Tech

Enriching Bézout's Theorem

Bézout's Theorem is a classical result from enumerative geometry that counts the number of intersections of projective planar curves over an algebraically closed field. Using a few tools from A1-homotopy theory, we enrich Bézout's Theorem for perfect fields. Over non-algebraically closed fields, this enrichment imposes a relation on the tangent directions of the curves at their intersection points. As concrete examples, we will discuss Bézout's Theorem over the reals and finite fields.

Gregory Michel, University of Minnesota

Hurwitz Space Statistics and the Dihedral Nichols Algebras

Recent work by Ellenberg, Tran, and Westerland has established a connection between various counting statistics related to Hurwitz spaces and cohomology calculations involving Nichols algebras. In this talk, we'll briefly explore these connections and explain preliminary results of these calculations when the group underlying the Nichols algebra is a dihedral group.

Ben O'Connor, University of Chicago

Configurations of noncollinear points

Let X_n be the space of n distinct points in the projective plane P^2, no 3 of which are collinear. The space X_6 is closely related to the moduli space of smooth cubic surfaces. For n = 4, 5, 6, we compute the rational cohomology of X_n as an S_n-representation via the Grothendieck--Lefschetz trace formula and certain twisted point counts. We explain why the problem becomes significantly more difficult for n=7, and for large n eventually encodes algebraic singularities of every type. This is joint work with Ronno Das.

Sabrina Pauli, University of Oslo

Types of Lines on Quintic Threefolds and Beyond

A classical result says that there are 27 complex lines on a complex smooth cubic surface.There are two types of lines on real cubic surfaces called hyperbolic and elliptic. It is known that the number of real hyperbolic lines minus the number of real elliptic lines on a real smooth cubic surface is equal to 3.Recently, Kass and Wickelgren defined the type of a line on a cubic surface over an arbitrary field of characteristic not equal to 2 to be the A^1degree of an associated involution. They showed that the type of the line coincides with its local A^1-degree. Using this they counted lines on cubic surfaces over arbitrary fields of characteristic not equal to 2 generalizing the classical counts over the complex and real numbers. In my short talk I will explain how to define the type of a line on a degree 2n-3 hypersurface in P^n over an arbitrary field of characteristic not equal to 2 and show that it is equal to its local A^1-degree. This generalizes the definition of the type of a real line on a real degree 2n-3 hypersurface in P^n by Finashin and Kharlamov.

Mark Shusterman, University of Wisconsin Madison

Nonvanishing of hyperelliptic zeta functions over finite fields

We combine genus theory with the methods of Ellenberg-Venkatesh-Westerland to obtain non-vanishing results (on the critical line) for Dirichlet L-functions over function fields. This is a joint work with Ellenberg and Li.

Hunter Spink, Harvard University

Incidence Strata of Affine Varieties with Complex Multiplicities (Joint with D. Tseng)

Given an affine variety X, what do the closed (1,2,4)-incidence strata in Sym^7(X) and the closed (5,9,100)-incidence strata in Sym^114(X) have in common? We show how to canonically interpolate the multiplicities to arbitrary complex numbers by hand, and time permitting, by the Deligne categories Rep(S_m). Over the integers and characteristic p this yields finite-type schemes which interpolate between fixed residual strata in spaces of rational functions with simple poles.

Philip Tosteson, University of Michigan

Representation Stability and Deligne-Mumford compactifciations

We use techniques from representation stability to study the asymptotic behavior of $H_i(\bar M_{g,n})$ for $n >> 0$.

Bena Tshishiku, Harvard University

Arithmetic groups and characteristic classes of manifold bundles

In this talk we'll describe how to obtain new characteristic classes of manifold bundles from the unstable cohomology of arithmetic groups. This combines an old technique of Millson-Raghunathan with more recent work of Berglund-Madsen.

Paul Tsopméné, University of Regina

Euler characteristics for spaces of string links and the modular envelope of L∞

We make calculations in graph homology which further understanding of the topology of spaces of string links, in particular calculating the Euler characteristics of finite-dimensional summands in their homology and homotopy. In doing so, we also determine the supercharacter of the symmetric group action on the positive arity components of the modular envelope of L∞.

Isabel Vogt, Stanford

An enriched count of bitangents

It is a classical fact that every smooth plane quartic over an algebraically closed field has exactly 28 bitangents. This enumerative result is intimately related to the 27 lines on a smooth cubic surface. Using recent work of Kass--Wickelgren on an enriched count of the lines on a cubic surface over a non-algebraically closed field, we explore enrichment of the bitangents to a smooth plane quartic. Subtleties arise regarding (lack of) a relative orientation. This is joint work with Hannah Larson.

Organizing Committee

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Jesse Wolfson (University of California, Irvine)

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