π Pacific Institute
for the Mathematical Sciences

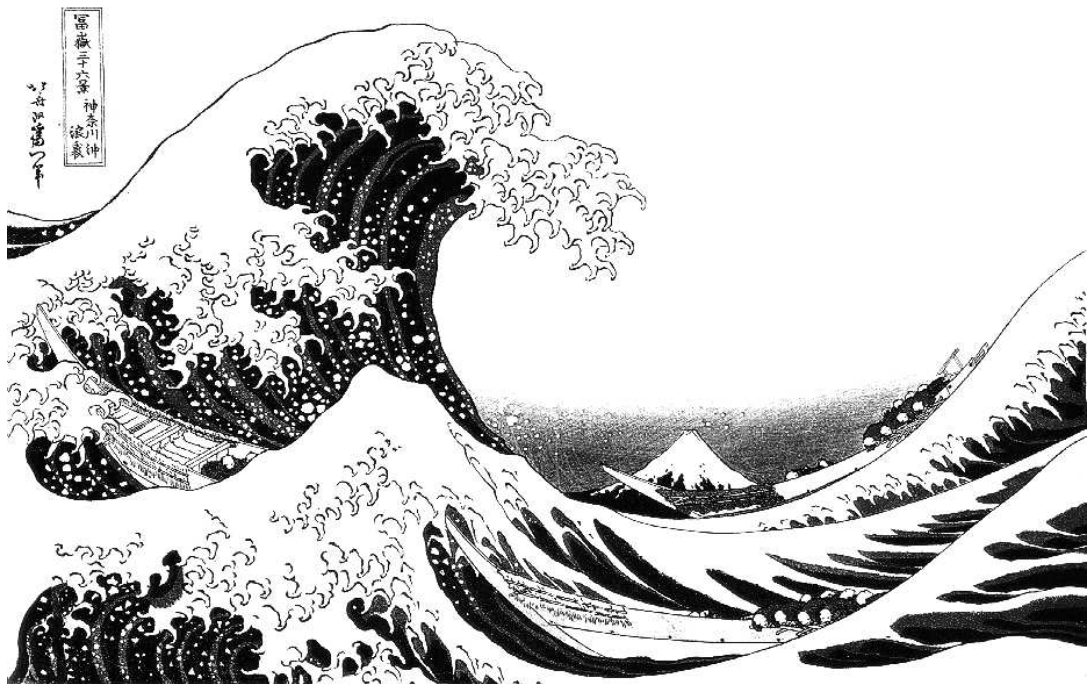# WAVE PHENOMENA IV:
Waves in Fluids from the Microscopic to the Planetary Scale

June 14-18, 2010
Edmonton, Alberta, Canada

The week at a glance

	Monday, June 14	Tuesday, June 15	Wednesday, June 16	Thursday, June 17
8:30	Registration			
9:15	Opening Ceremony and Welcoming Remarks			
	Plenary Session I1	Plenary Session I3	Plenary Session I5	Plenary Session I7
9:30	**J. Bush**	**C. Caulfield**	**G. Swaters**	**P. Weidman**
10:30	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00	**A. Newell**	**R. Grimshaw**	**H.-C. Chang**	**C. Rogers**
12:00	Lunch Break	Lunch Break	Lunch Break	Lunch Break
	Plenary Session I2	Plenary Session I4	Plenary Session I6	Plenary Session I8
1:30	**J. Lister**	**H. Segur**	**M. Ablowitz**	**S. Griffiths**
2:30	**T. Hou**	**R. Craster**	**C. Linton**	**R. Camassa**
3:30	Coffee Break	Coffee Break	Coffee Break	Coffee Break
4:00	Parallel Sessions C1-C3	Parallel Sessions C4-C6	Parallel Sessions C7-C9	Parallel Sessions C10-C12
5:20	Parallel Sessions end	Parallel Sessions end	Parallel Sessions end	Parallel Sessions end
evening	Wave-Breaker Cocktails 6:15-8:30pm in Faculty Club		Conference Banquet 6:00-9:00pm in Faculty Club	Conference ends

Monday June 14th

	9:15 - 9:30	Opening Ceremony and Welcoming Remarks: **Bryant Moodie**

Session I1: Room TL-B1		Session Chair: G.E. Swaters
I1.1	9:30 - 10:30	**John Bush**, *"Walking on Waves"*
	10:30 - 11:00	COFFEE BREAK
I1.2	11:00 - 12:00	**Alan Newell**, *"The Universal Nature of Fibonacci Patterns"*

	12:00 - 1:30	LUNCH BREAK

Session I2: Room TL-B1		Session Chair: H.-C. Chang
I2.1	1:30 - 2:30	**John Lister**, *"Waves and Oscillations in Viscously Dominated Systems"*
I2.2	2:30 - 3:30	**Thomas Hou**, *"Recent Progress on Dynamic Stability of 3D Incompressible Euler and Navier-Stokes Equations"*
	3:30 - 4:00	COFFEE BREAK

Session C1: Room TB-90		Session Chair: B.R. Sutherland
C1.1	4:00 - 4:20	**James Munroe**, P. Odier and T. Dauxois, *"Stability of Forced Vertical Modes in a Rectangular Tank"*
C1.2	4:20 - 4:40	B.K. Gayen and **Sutanu Sarkar**, *"Turbulence during Resonant Generation of Internal Tides"*
C1.3	4:40 - 5:00	**John Grue** and L.K. Brandt, *"Method for Time-Dependent 3D Interfacial Flows Interacting with Variable Topography"*

Session C2: Room TB-76		Session Chair: T.B. Moodie
C2.1	4:00 - 4:20	**Morris Flynn**, A.R. Kasimov, J.-C. Nave, R.R. Rosales and B. Seibold *"On 'Jamitons', Self-Sustained Nonlinear Waves in Traffic Flow"*
C2.2	4:20 - 4:40	**Aslan Kasimov** *"Analysis and Computation of Self-Sustained Shock Waves"*
C2.3	4:40 - 5:00	**Matthew Emmett** and T.B. Moodie, *"WENO Methods for Sediment Transport via Dam-Break Flows"*

Session C3: Room TB-81		Session Chair: C. Rogers
C3.1	4:00 - 4:20	**Mitsuaki Funakoshi** and K. Tamura, *"Faraday Waves in a Two-Layer Fluid in a Long Container"*
C3.2	4:20 - 4:40	**James Percival** *"The Square Root Depth Equations"*
C3.3	4:40 - 5:00	**Harun Kurkcu**, *"An Integral Representation of the Green's Function for the Three-Dimensional Helmholtz Equation"*

	6:15 - 8:30	WAVE-BREAKER COCKTAILS in Faculty Club

Tuesday June 15th

Session I3: Room TL-B1		Session Chair: A.C. Newell

I3.1	9:30 - 10:30	**Colm-cille Caulfield**, *"When is One Very Much Less Than One? Lee Wave Ray Tracing in Stratified Shear Flow Experiments"*
	10:30 - 11:00	COFFEE BREAK
I3.2	11:00 - 12:00	**Roger Grimshaw**, *"Internal Solitary Waves: Propagation, Deformation and Disintegration"*

	12:00 - 1:30	LUNCH BREAK

Session I2: Room TL-B1		Session Chair: J.W.M. Bush

I4.1	1:30 - 2:30	**Harvey Segur**, *"The Modulational Instability in Real Life"*
I4.2	2:30 - 3:30	**Richard Craster**, *"High Frequency Homogenization and Localized Wave Phenomena in Periodic Media"*
	3:30 - 4:00	COFFEE BREAK

Session C4: Room TB-90		Session Chair: C.P. Caulfield

C4.1	4:00 - 4:20	**Julie Vanderhoff** and J.R. Rottman, *"Energy Exchange of Small-scale Internal Waves with Inertial Waves: A Case Study"*
C4.3	4:20 - 4:40	**Scott Wunsch**, *"Internal Wave Reflection in Shear Flow"*
C4.4	4:40 - 5:00	**Hayley Dosser** and B.R. Sutherland *"Nonlinear Atmospheric Internal Gravity Waves"*

Session C5: Room TB-76		Session Chair: A.B.G. Bush

C5.1	4:00 - 4:20	**Lucy Campbell**, M. Nadon and N. Victor *"Exact Expressions for Transient Forced Waves in some Geophysical Flow Configurations"*
C5.2	4:20 - 4:40	**Gary Klaassen** *"On the Viability of Lagrangian Theories of Internal Wave Spectra"*
C5.3	4:40 - 5:00	**Wenbo Tang**, J.E. Taylor and A. Mahalove, *"Stochastic Lagrangian Mixing In An Inertia-Gravity Wave"*

Session C6: Room TB-81		Session Chair: T.Y. Hou

C6.1	4:00 - 4:20	**John Bowman**, *"Dealiased Convolutions without the Padding"*
C6.2	4:20 - 4:40	**Francois Blanchette**, *"Drop Coalescence in the Inertial Regime"*
C6.3	4:40 - 5:00	**Yuri Antipov** and V.V. Silvestrov, *"Supercavitating Flow in Multiply Connected Domains"*

Wednesday June 16th

Session I5: Room TL-B1		Session Chair: B.R. Sutherland
I5.1	9:30 - 10:30	**Gordon Swaters**, *"The Meridional Flow of Source-Driven Grounded Abyssal Ocean Currents"*
	10:30 - 11:00	COFFEE BREAK
I5.2	11:00 - 12:00	**Hsueh-Chia Chang**, *"Free Surface and Wave Phenomena with Geometric Singularities"*
	12:00 - 1:30	LUNCH BREAK
Session I6: Room TL-B1		Session Chair: H. Segur
I6.1	1:30 - 2:30	**Mark Ablowitz**, *"Asymptotic Reductions of Water and Internal Waves and their Solitary Waves"*
I6.2	2:30 - 3:30	**Chris Linton**, *"Surface Waves"*
	3:30 - 4:00	COFFEE BREAK
Session C7: Room TB-90		Session Chair: G.E. Swaters
C7.1	4:00 - 4:20	**Laura Brandt**, *"Theory and Numerical Simulations of Two-Layer Bores"*
C7.2	4:20 - 4:40	**Alan Tan** and M.R. Flynn, *"Gravity Currents in 2-Layer Stratified Media"*
C7.3	4:40 - 5:00	**Justine McMillan** and B.R. Sutherland, *"Intrusive Gravity Currents and the Solitary Wave Lifecycle in a Cylindrical Geometry"*
C7.4	5:00 - 5:20	**Amber Holdsworth** and B.R. Sutherland *"The Axisymmetric Collapse of a Mixed Patch in Uniformly Stratified Fluid"*
Session C8: Room TB-76		Session Chair: J. Lister
C8.1	4:00 - 4:20	N. Swart and **Susan Allen**, *"Anomalously Strong Tides in the Gully, a Submarine Canyon on the Nova Scotia Shelf"*
C8.2	4:20 - 4:40	**Jan Feys** and S. Maslowe, *"Long Nonlinear Surface Waves Propagating above a Current Varying with Depth"*
C8.3	4:40 - 5:00	**Maryam Namazi** *"Interactions of Equatorially Trapped Waves with a Barotropic Background Zonal Shear"*
Session C9: Room TB-81		Session Chair: R. Craster
C9.1	4:00 - 4:20	**Serge D'Alessio** and J.P. Pascal, *"Film Flow Over Heated Wavy Inclined Surfaces"*
C9.2	4:20 - 4:40	**Hayder Salman**, *"The Nonlinear Schrödinger Equation as a Model of Bose-Einstein Condensate Formation"*
C9.3	4:40 - 5:00	**David Amundsen**, M.P. Mortell and B.R. Seymour, *"Resonant Oscillations Between Two Concentric Spheres"*
	6:00 - 9:00	CONFERENCE BANQUET in Faculty Club

Thursday June 17th

Session I7: Room TL-B1		Session Chair: R. Craster
I7.1	9:30 - 10:30	**Patrick Weidman**, *"Evolution of Solitary Waves in a Two-Pycnocline System"*
	10:30 - 11:00	COFFEE BREAK
I7.2	11:00 - 12:00	**Colin Rogers**, *"Ermakov Structure in 2+1-dimensional Magneto-Gasdynamics"*

	12:00 - 1:30	LUNCH BREAK
Session I8: Room TL-B1		Session Chair: R.H.J. Grimshaw
I8.1	1:30 - 2:30	**Stephen Griffiths**, *"Numerical Modelling of Internal Tides in the Ocean"*
I8.2	2:30 - 3:30	**Roberto Camassa**, *"Settling in Stratified Fluids: A Tortoise-and-Hare Experimental and Mathematical Tale"*
	3:30 - 4:00	COFFEE BREAK

Session C10: Room TB-90		Session Chair: M.R. Flynn
C10.1	4:00 - 4:20	**Tyler Blackhurst** and J.C. Vanderhoff, *"Numerical Investigation of Internal Wave-Vortex Interactions"*
C10.2	4:20 - 4:40	**Alan Brandt** *"Internal Wavefield Generated by a Towed Sphere: Theory/Experiment Comparison"*
C10.3	4:40 - 5:00	E. Ermanyuk, J.-B. Flór and **Bruno Voisin**, *"First and Second Harmonic Internal Waves from a Horizontally Oscillating Sphere"*

Session C11: Room TB-76		Session Chair: M. Ablowitz
C11.1	4:00 - 4:20	**Mona Rahmani**, G.A. Lawrence and B.R. Seymour, *"Pre-saturation Evolution of Kelvin-Helmholtz Instabilities"*
C11.2	4:20 - 4:40	**Ali Mashayek** and W.R. Peltier, *"Shear Instabilities in Stratified Parallel flows and Implications for Mixing Efficiency"*
C11.3	4:40 - 5:00	**Serge D'Alessio** and J.P. Pascal, *"Instability of Inclined Film Flow over Wavy Permeable Surfaces"*
C11.4	5:00 - 5:20	**Kelly Ogden**, S.J.D. D'Alessio and J.P. Pascal, *"The Stability of Liquid Film Flow Over Heated Porous Surfaces"*

Session C12: Room TB-81		Session Chair: P. Weidman
C12.1	4:00 - 4:20	**Stuart King**, *"Stable and Unstable Large Amplitude Internal Solitary Waves"*
C12.2	4:20 - 4:40	**Oleg Derzho** *"Nonlinear Rossby Wave Patterns in Polar Areas"*

I1.1

Walking on Waves

John W. M. Bush[1]
Dept. Mathematics, MIT

Yves Couder and coworkers have made the striking observation that drops on a vibrating fluid interface may bounce indefinitely; moreover, in a certain parameter regime, the interaction between the bouncing droplets and their own wave field allows them to walk steadily across the free surface. These walking droplets are a fluid mechanical example of wave-particle duality, and demonstrate several features reminiscent of quantum systems, including diffraction and quantum tunneling. We here explore a number of problems intended to inform this intriguing system. First, we present the results of a combined experimental and theoretical investigation of droplets bouncing on a vibrating horizontal soap film. A variety of bouncing behaviours were observed, including simple and complex periodic states, multiperiodicity and chaos. A simple theoretical model is developed that captures the essential physics of the bouncing process, reproducing all observed bouncing states. This system is among the very simplest fluid mechanical chaotic oscillators. Second, we explore the dynamics of a ball bouncing on an arbitrary rigid surface, and deduce criteria for a number of walking states. Finally, we attempt to couple the droplet and free surface dynamics in order to yield further insight into Couder's remarkable experiments.

I1.2

The Universal Nature of Fibonacci Patterns

Alan C. Newell[2]
University of Arizona

The universal nature of Fibonacci patterns—The observation that the flowers, bracts, florets on plants lie on families of spirals the number in each family of which belongs to a Fibonacci sequence has intrigued natural scientists for over four hundred years. The goals of the talk will be to explain "why Fibonacci" and to show that Fibonacci patterns are universal objects which will naturally arise in any pattern forming system with broken up-down symmetry in which the pattern is laid down annulus by annulus.

[1]CONTACT: +1 617 253-4387, bush@math.mit.edu, http://www-math.mit.edu/~bush
[2]CONTACT: anewell@math.arizona.edu

Session I2: Monday 1:30-3:30 in TL-B1

I2.1

Waves and Oscillations in Viscously Dominated Systems

John Lister[3]

DAMTP, University of Cambridge

Many familiar types of waves involve inertia and large Reynolds numbers. This talk describes waves and oscillations in three low Reynolds number systems where the dominant forces are viscosity, gravity and surface tension. In each case the system is simple but the dynamics are surprisingly complex. (1) A layer of fluid coating the underside of a horizontal ceiling is stabilised by surface tension and destabilised by gravity to yield pendent drops. A single pendent drop has a self-sustained traveling-wave solution over an uniform film. Collisions between translating drops are governed by their capillary wave fields. (2) A drop of molten glass can be levitated above a porous spherical mould by air injection. Capillary waves on the underside of the drop produce a convoluted set of solution branches for equilibrium shapes and influence stability. (3) It is a common breakfast-table experience that a viscous fluid thread falling a sufficient height onto a stationary horizontal surface will buckle and coil. If the thread falls instead onto a steadily moving horizontal belt then, over a range of belt speeds, the thread can perform various beautiful modes of periodic nonlinear oscillation, leaving 'sewing-machine' patterns on the belt.

I2.2

Recent Progress on Dynamic Stability of 3D Incompressible Euler and Navier-Stokes Equations

Thomas Y. Hou[4]

Applied and Computational Mathematics, Caltech

Whether the 3D incompressible Euler and Navier-Stokes equations can develop a finite time singularity from smooth initial data with finite energy has been one of the most long standing open questions. We review some recent theoretical and computational studies which show that there is a subtle dynamic depletion of nonlinear vortex stretching due to local geometric regularity of vortex filaments. The local geometric regularity of vortex filaments can lead to tremendous cancellation of nonlinear vortex stretching, thus preventing a finite time singularity. Our studies also reveal a surprising stabilizing effect of convection for the 3D incompressible Euler and Navier-Stokes equations. Finally, we present a new class of solutions for the 3D Euler and Navier-Stokes equations, which exhibit very interesting dynamic growth property. By exploiting the special structure of the solution and the cancellation between the convection term and the vortex stretching term, we prove nonlinear stability and the global regularity of this class of solutions.

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[3]CONTACT: +44 1223 330888, lister@damtp.cam.ac.uk
[4]CONTACT: 626-395-4546, hou@acm.caltech.edu, http://www.acm.caltech.edu/~hou

C1.1

Stability of Forced Vertical Modes in a Rectangular Tank

James R. Munroe,[5] Philippe Odier, Thierry Dauxois
ENS de Lyon

Internal waves are known to be inherently unstable and evolve in time by transferring energy among different length and time scales. While there has been substantial work exploring internal wave stability theoretically and numerically, there have been comparatively few laboratory studies examining possible instabilities. Using a full-depth wave generator to force a vertical mode-1 wave in a narrow rectangular tank we observe the long term evolution of the wave field. Fluid motions are quantitatively measured using synthetic schlieren and particle image velocimetry. Both standing waves and progressive waves are explored and theoretical predictions are tested. In particular, we investigate the growth rates of resonant standing waves and secondary waves due to parametric sub-harmonic instability.

C1.2

Turbulence during Resonant Generation of Internal Tides

Bishakhdatta Gayen and **Sutanu Sarkar**[6]
Dept Mechanical & Aerospace Engineering, UC San Diego

Hot spots of mixing on topography have been observed in the ocean and are though to be important to ocean mixing. Internal tides are generated as a response to barotropic tidal flow across topography. Three-dimensional direct numerical simulations are performed to examine nonlinear processes during the generation of internal tides on a model continental slope. The excursion number is chosen to be small corresponding to the typical case of a slope that is very long corresponding to the fluid advection during the barotropic tidal oscillation. A strong internal wave is generated in the critical case with slope angle equal to the natural internal wave propagation angle. Wave steepening, that drives spanwise wave breaking via convective instability, occurs and is accompanied by shear instability. Turbulence is present along the entire length of the region with near-critical slope. The dependence of transport and mixing by turbulence on tidal phase has been quantified. The turbulence is found to have a strong effect on the internal wave beam by distorting its near-slope structure. A complicated wave field is found. There are discrete peaks at the fundamental (tidal forcing frequency) and at the second and third harmonics as well as the subharmonic. In addition, there is a broadband frequency spectrum, generated by the turbulent boundary flow, that includes evanescent super-N waves. This work helps understand the connection between boundary turbulence and internal waves during the resonant generation of internal tides.

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[5]CONTACT: james.munroe@ens-lyon.fr, http://perso.ens-lyon.fr/james.munroe
[6]CONTACT: +1 858 534-8243, sarkar@ucsd.edu

C1.3

Method for Time-Dependent 3D Interfacial Flows Interacting with Variable Topography

John Grue[7] and Laura K. Brandt[8]
Department of Mathematics, University of Oslo

We derive, implement and test an interfacial two-layer formulation in three dimensions; the model is fully nonlinear and fully dispersive, where the latter means that the full wavenumber range is resolved. No compromises are made regarding the wavelength, which usually is assumed to be long in relation to the depth of the two-fluid system, see for example existing variants of three-dimensional Boussinesq theories. Another main feature is that the effect of a bottom topography is accounted for. The bottom topography may be fixed or moving, and its vertical variation allowed to be large. The bottom topography may extend in one of the two layers.

The transient formulation is used to integrate forward in time the interfacial wave generation at a three-dimensional, moving geometry, resembling the formation of short internal waves generated by a tidal flow over a three-dimensional bottom variation. With the full resolution of the wavenumber spectrum, the role of dispersion in the formation process of the interfacial waves is highlighted.

There is a recent interest for three-dimensional calculations of nonlinear internal waves, particularly from an ocean modelling point of view. With an aim to resolve short scale phenomena like internal wave motion, internal breaking, motion across the thermoclines, and irreversible mixing processes, non-hydrostatic formulations in three dimensions are today pushed forward. The over all focus is to evaluate and study conversion of energy from low frequency excitation, by the wind and tide, to motion at high frequency, including possible feedback mechanisms. Different codes are used in the investigations. Ocean models are used directly to calculate generation, propagation and shoaling of internal waves. Essential features of the computations include a realistic density stratification and the effect of the three-dimensional bottom topography. The present formulation represents a compliment to the more general but also more complex ocean model calculations.

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[7]CONTACT: +4722855839, johng@math.uio.no, http://folk.uio.no/johng/
[8]SAIC, 10210 Campus Point Drive, San Diego

C2.1

On 'Jamitons', Self-Sustained Nonlinear Waves in Traffic Flow

M. R. Flynn,[9], A. R. Kasimov[10], J.-C. Nave, R. R. Rosales[11], and B. Seibold[12]
Dept. Mechanical Engineering, University of Alberta

In analogy to gas-dynamical detonation waves, which consist of a shock with an attached exothermic reaction zone, we consider herein nonlinear traveling wave solutions to the hyperbolic ("inviscid") continuum traffic equations. Generic existence criteria are examined in the context of the Lax entropy conditions. Consistent with recent observations, our numerical calculations show that nonlinear traveling waves are attracting solutions, with the time evolution of the system converging towards a wave-dominated configuration. Theoretical principles are elucidated by considering examples of traffic flow on open and closed roadways.

C2.2

Analysis and Computation of Self-Sustained Shock Waves

Aslan Kasimov[13]
KAUST

The talk highlights a class of problems in shock dynamics in which a shock wave is self-sustained due to an acoustic confinement provided by the post-shock transonic flow. Such dynamics are described by a special class of hyperbolic systems with source terms. The range of related phenomena includes continuum models of traffic flow, hydraulic jumps, flow in elastic tubes, and detonation waves. Analytical and computational difficulties in solving these systems will be discussed and illustrated on the examples of detonation waves and hydraulic jumps.

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[9]Dept. Mechanical Engineering, Univ. of Alberta
 CONTACT: 1-780-492-5593, mrflynn@ualberta.ca, http://websrv.mece.ualberta.ca/mrflynn/
[10]Dept. Applied Mathematics and Computational Science, King Abdullah Univ. of Science and Technology
[11]Dept. Mathematics, Massachusetts Inst. Technology
[12]Dept. Mathematics, Temple Univ.
[13]CONTACT: 966-54-470-0076, aslan.kasimov@kaust.edu.sa, http://web.kaust.edu.sa/faculty/AslanKasimov/

C2.3

WENO Methods for Sediment Transport via Dam-Break Flows

Matthew Emmett[14] and Bryant Moodie
Dept. Mathematical and Statistical Sciences, University of Alberta

When a semi-infinite body of homogeneous fluid initially at rest behind a vertical retaining wall is suddenly released by the removal of the barrier the resulting flow is referred to as a dam-break flow. Various asymptotic and numerical approaches must be implemented to solve the resulting depth-averaged shallow-water equations when friction and bed topography are considered. Futhermore, when the bed is no longer stable so that solid particles may be exchanged between the bed and the fluid, the dynamics of the flow become highly complex as the buoyancy forces vary in space and time according to the competing rates of erosion and deposition.

We will present our model and various exact, asymptotic, and numerical results of the model. We will also present some details of our numerical scheme which is based on the Python WENO (PyWENO) implementation of modern Weighted Essentially Non-oscillatory methods.

These models offer insights into the transport of sediment in the worst case scenario of the complete and instantaneous collapse of a dam.

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[14]CONTACT: +1 780 988-6374, memmett@math.ualberta.ca, http://www.math.ualberta.ca/~memmett/

C3.1

Faraday Waves in a Two-Layer Fluid in a Long Container

Mitsuaki Funakoshi[15] and Keita Tamura
Graduate School of Informatics, Kyoto University, JAPAN

We consider the Faraday waves in a two-layer fluid with a free surface in a long container of width W, caused by the resonant vertical oscillation of the container. Using a perturbation method, we derive a nonlinear Schrödinger-type equation that governs the time evolution of complex amplitude of the standing wave of a mode that has the wavelength $2W/n$ in the transverse direction and varies slowly in the longitudinal direction, where n is a positive integer. It is found that the coefficient of the nonlinear term of this equation can take both positive and negative values depending on the ratio of densities of two fluids and on their depths, and that both soliton-like and kink-type Faraday waves are possible. Moreover, model equations that describe also the resonant interaction of the Faraday wave with the other mode are derived. The property of the solutions to these equations is also discussed.

C3.2

The Square Root Depth Equations

James Percival[16]
Dept. Mathematics, Imperial College London

The fully nonlinear, weak dispersive model ascribed variously to Serre (1953), Su & Gardener (1969) or Green & Naghdi (1976) has been used to study wave phenomena in a single fluid layer, with and without variation in topography. These equations may be derived from a columnar motion ansatz in the Lagrangian for the Hamilton's principle for an Eulerian fluid. Unfortunately application of this ansatz in the multilayer Green Naghdi (MGN) equations causes the equations to be ill-posed in the presence of shear. We seek a set of multilayer long wave equations which (a) is both linearly well-posed and Hamiltonian; (b) preserves the MGN linear dispersion relation for fluid at rest; (c) has the same travelling wave solutions as MGN in the absence of imposed background shear. We find the new square root depth equations satisfy these conditions. The key step is to modify the formula for the kinetic energy of vertical motion by using the convective velocity instead of the material representation.

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[15]CONTACT: 81-75-753-5881, mitsu@acs.i.kyoto-u.ac.jp
[16]CONTACT: +44 20 75 941534, j.percival@imperial.ac.uk

C3.3

An Integral Representation of the Green's Function for the Three-Dimensional Helmholtz Equation

Harun Kurkcu[17]

Dept. Mathematics, Simon Fraser University

A difficulty that arises in the context of infinite, d-periodic rough-surface scattering relates to the effective numerical evaluation of the corresponding "quasi-periodic Green function". Due to its relevance in a variety of applications, this problem has generated significant interest over the last few years, and a variety of numerical methods have been devised for this purpose. Here we present a new integral representation of the Green's function for the three-dimensional Helmholtz equation for linear array of point sources which is derived from the basic form using geometric series expansions and the inverse Laplace transform. We include a variety of numerical results that, our algorithm compares favorably with every classical algorithm both in low and high frequency regimes.

[17]CONTACT: +1 778 782-6656, hka50@sfu.ca

I3.1

When is One Very Much Less Than One? Lee Wave Ray Tracing in Stratified Shear Flow Experiments

Colm-cille Caulfield[18], Stuart Dalziel[19] and Michael Patterson[20]
BP Institute & DAMTP, University of Cambridge

Lee waves, which develop as a stratified fluid flows over topography, are very important in both the atmosphere and the oceans, particularly as they are key mechanisms by which momentum is redistributed within the flow. Their dynamics are both rich and complicated if either the stratification, or the flow velocity vary with height, and it is well-known that wave packets may be either reflected or absorbed depending on the particular structure of the velocity distribution. Mathematical description of this behaviour can be made using a WKBJ approach, relying on the fundamental assumption that the ambient flow properties are "slowly varying" relative to the waves as they propagate along a ray.

In this talk, I describe a novel lee wave experimental dataset, obtained using synthetic schlieren in a recirculating stratified shear flow tank. Our approach allows the quantitative analysis of lee wave fields which are stationary in the laboratory frame. I demonstrate how linear wave ray tracing can be useful to describe fully nonlinear lee waves, even where the ambient flow is varying sufficiently rapidly for formal breakdown of the WKBJ assumption. I also describe quantitatively the interaction of the lee wave field with both wake turbulence and flow boundary layers.

I3.2

Internal solitary waves: propagation, deformation and disintegration

Roger Grimshaw[21]
Department of Mathematical Sciences, Loughborough University

Internal solitary waves are a common feature of the coastal ocean. In this talk I will describe, using model equations of the Korteweg-de Vries type, how these waves are affected by variable bottom topography. The scenarios range from adiabatic deformation, to disintegration and transformation into wave trains with very different features. Although the context is that of internal waves in the coastal ocean, the mechanisms involved are generic and can arise in many other physical contexts.

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[18]CONTACT: +44 1223 765710, c.p.caulfield@bpi.cam.ac.uk, http://www.bpi.cam.ac.uk/users/colm
[19]DAMTP, University of Cambridge
[20]DAMTP, University of Cambridge; Department of Architecture & Civil Engineering, University of Bath
[21]CONTACT: +44 1509-223480, R.H.J.Grimshaw@lboro.ac.uk,
http://www.lboro.ac.uk/departments/people/grimshaw.html

I4.1

The Modulational Instability in Real Life

Harvey Segur[22]
Dept. Applied Mathematics, University of Colorado

The modulational instability has been a cornerstone of the theory of nonlinear wave propagation ever since it was discovered in the 1960s by several people, working in different scientific fields and in different countries. [For a historical review of this interesting event, see Zakharov & Ostrovsky, Physica D, 238 (2008).] The instability has now been found in Hamiltonian models of water waves, nonlinear optics, plasmas and elsewhere. Because of it, a uniform train of nearly monochromatic, oscillatory waves of moderate amplitude is often unstable to small perturbations in non-dissipative media.

Even so, Segur et al. (2005) showed that while modulational instabilities occur in nonlinear media without dissipation, even small amounts of dissipation can stabilize the instability. Their conclusion has been controversial ever since it was first proposed, even though they supported their claims with both mathematical proof and experimental data. This lecture reviews that controversy, including the role of nearby topics that have appeared in these arguments: nonlinear Schrödinger-type models, Fermi-Pasta-Ulam recurrence, frequency downshifting and more.

I4.2

High Frequency Homogenization and Localized Wave Phenomena in Periodic Media

Richard Craster[23], Julius Kaplunov, Aleksey Pichugin and Julia Postnova
Dept. Mathematical and Statistical Sciences, University of Alberta

It has long been known, and understood, how to "average" complex, inhomogeneous media, at low frequencies, to obtain equivalent homogenized continuum equations. The waves are usually assumed long relative to the inhomogenities (the microscale) and the resulting governing equations involve only the macroscale variables, and material properties are averaged ones. Far less is known about how to achieve high frequency homogenized models valid where the wavelength and microstructure are of the same order.

In this talk I will describe a general asymptotic theory that creates high frequency homogenized equations for doubly or singly periodic media. The asymptotic approach will be illustrated on a wide variety of media: waves passing through an electromagnetic medium containing a doubly-periodic array of holes, waves in discrete lattice models from solid state physics and waves in structural dynamics. In each case the asymptotic approach gives rise to continuum PDE approximations that can be compared numerically with results from the origin system. Interesting phenomena such as localization due to defects and wave trapping are accurately predicted by the homogenized equations and they provide a general framework for exploring multiscale phenomena.

[22]CONTACT: +1 303 492-0592, segur@colorado.edu
[23]CONTACT: craster@ualberta.ca

C4.1

Energy Exchange of Small-scale Internal Waves with Inertial Waves: A Case Study

Julie Vanderhoff[24] and James Rottman[25]
Brigham Young University

The interaction of small and large-scale internal gravity waves potentially leads to the breaking of the small-scale waves and to modification of the larger scale motions. Here, this interaction is investigated numerically using ray theory and fully-nonlinear simulations. Simulations are initiated with observations from the HOME Nearfield experiment, sited at the southern lip of the Kaena Ridge, west of Oahu, Hawaii, where a distinct modulation of short wave variance by the inertial motion is seen. Energy transfer between the large-scale inertial and small-scale internal waves is independent of azimuth, depending only on relative vertical propagation direction of these waves. Results support a cascade of energy from the large scale inertial motions directly to the smallest internal wave scales.

C4.2

Internal Wave Reflection in Shear Flow

Scott Wunsch[26]
Applied Physics Laboratory, Johns Hopkins University

Internal waves propagating through a shear flow can exchange energy with the mean flow. Waves may be reflected or transmitted through the shear, gaining or losing energy by exchange with the mean flow field. This effect is most pronounced at a 'critical level', a depth where the wave horizontal phase velocity matches the local mean flow speed. Laboratory experiments are underway to study internal wave reflection off shear and BV gradients using the synthetic schlieren measurement technique. Particular interest is on conditions leading to internal wave amplification, which based on theory should occur when the Richardson number is less than $1/4$. Experimental results indicate that, as the stratified shear flow is subject to instabilities as Richardson number approaches $1/4$, internal wave interactions are more complex than the idealized theories predict.

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[24]CONTACT: +1 801 422-2232, jvanderhoff@byu.edu, http://me.byu.edu/faculty/julievanderhoff
[25]SAIC
[26]CONTACT: 240-228-1859, scott.wunsch@jhuapl.edu

C4.3

Nonlinear Atmospheric Internal Gravity Waves

Hayley Dosser[27] and Bruce R. Sutherland
Dept. Physics, University of Alberta

General circulation models parametrize the transport and deposition of momentum by atmospheric internal gravity waves. The breaking levels of such waves are typically predicted through the use of linear theory. The purpose of this work is to examine the weakly nonlinear dynamics of internal gravity waves in a stationary, uniformly stratified anelastic fluid, and to study how these nonlinear effects significantly change the breaking height of the waves. An initially small-amplitude internal gravity wave propagating vertically upward will increase in amplitude and weakly nonlinear effects will develop due to interactions with an induced horizontal mean flow. A new derivation for this wave-induced mean flow is presented and the nonlinear Schrödinger equation describing the weakly nonlinear evolution of these waves in an anelastic gas is derived. The solutions of this equations are compared with fully nonlinear numerical simulations. It is found that interactions with the wave-induced mean flow are the dominant mechanism for wave evolution. High frequency, non-hydrostatic waves experience enhanced amplitude growth due to the weakly nonlinear effects of modulational instability, and so begin to overturn well below the breaking level predicted by linear theory. Low frequency, hydrostatic waves are modulationally stable, however they become unstable and break below the level predicted by linear theory due to high-order dispersion terms in the Schrödinger equation. This seems to indicate that current GCMs may be placing momentum attributed to internal gravity wave breaking at inaccurate heights in the atmosphere.

[27]CONTACT: (780) 492-7735, hdosser@ualberta.ca

C5.1

Exact Expressions for Transient Forced Waves in some Geophysical Flow Configurations

Lucy Campbell[28], M. Nadon and N. Victor
School of Mathematics and Statistics, Carleton University

We discuss the derivation of exact solutions describing the propagation of transient forced internal gravity waves and Rossby waves in two-dimensional geophysical fluid flows. The configurations studied are simplified by the assumptions of zero aspect ratio and constant mean velocity. In each case, the solution consists of a part with steady amplitude and a transient part in the form of an infinite series that goes to zero in the limit of infinite time. Because of the exact nature of these solutions, they can be used as a starting point for further analytical and numerical studies of wave propagation.

C5.2

On the Viability of Lagrangian Theories of Internal Wave Spectra

Gary Klaassen[29]
Dept. Earth & Space Science & Engineering, York University

A growing body of literature has been built on the premise that kinematic advection produced by superpositions of low-wavenumber sinusoidal Lagrangian gravity waves can provide an explanation for the high-wavenumber Eulerian spectral tails commonly found in oceanic and atmospheric measurements. My own calculations indicate that the approximations employed in those Lagrangian theories are invalid for wave parameters typical of the middle atmosphere. Furthermore, these theories only obtain average vertical wavenumber spectra close to those revealed by atmospheric measurements when the Lagrangian to Eulerian transformation is approaching singular conditions. The results have implications for our understanding of the nature of geophysical internal wave fields, and for the Doppler-spread parameterization of gravity wave drag.

C5.3

Stochastic Lagrangian Mixing In An Inertia-Gravity Wave

Wenbo Tang,[30] Jay E. Taylor & Alex Mahalov
School of Mathematical & Statistical Sciences, Arizona State University

The identification of Lagrangian Coherent Structures (LCS) in nonlinear, deterministic fluid flows has been a heated topic on studies of chaotic mixing and transport patterns in recent years. The Finite-Time Lyapunov Exponent is discovered to be one of the most efficient tools in characterizing Lagrangian mixing structures. In this talk, we discuss extensions of LCS identification with flows embedded in stochastic environments. Specifically, we examine the mixing patterns of an inertia-gravity wave arising from the stochastic mean flow, and discuss the long time-evolution of its statistics. Note that any mixing pattern arising from the mean flow indicate a deviation from determinism, as no Lagrangian mixing can happen inside an idealized, stable inertia-gravity wave.

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[28]CONTACT: +1 613 520-2600 x1208, campbell@math.carleton.ca, http://www.math.carleton.ca/~campbell
[29]CONTACT: gklaass@yorku.ca
[30]CONTACT: 1-480-965-1476, wenbo.tang@asu.edu, http://math.asu.edu/~wtang

C6.1

Dealiased Convolutions without the Padding

John C. Bowman[31]

Dept. Mathematical & Statistical Sciences, University of Alberta

An algorithm is described for calculating dealiased linear convolution sums without the expense of conventional zero-padding or phase-shift techniques. For one-dimensional in-place convolutions, the memory requirements are identical with the zero-padding technique, with the important distinction that the additional work memory need not be contiguous with the data. This decoupling of the data and work arrays dramatically reduces the memory and computation time required to evaluate higher-dimensional in-place convolutions. The technique also allows one to efficiently dealias the hyperconvolutions that arise on Fourier transforming cubic and higher powers. Implicitly dealiased convolutions can be built on top of state-of-the-art fast Fourier transform libraries: vectorized multidimensional implementations for the complex and centered Hermitian (pseudospectral) cases have been implemented in the open-source software fftw++.

C6.2

Drop Coalescence in the Inertial Regime

Francois Blanchette[32]

School of Natural Sciences, UC Merced

We present results from a combination of numerical and experimental study of drop coalescence, focusing on the regime where inertial effects are dominant. We first consider the coalescence of drops of identical composition and review conditions under which partial coalescence may occur. A mechanism allowing partial coalescence is presented, and the importance of capillary waves is underlined. We then extend those results to fluids of different surface tension and examine the importance of the Marangoni effect and its effect on capillary waves.

The second part of the talk is concerned with the mixing resulting from the coalescence of two drops, mostly in the context of microfluidic devices. We examine the mixing resulting from the coalescence of drops of different sizes and composition, and determine the conditions under which mixing is optimized. We compare results for drops in a stationary fluid and drops moving at a steady velocity in a capillary. We introduce a simple and effective measure of the extent of the mixing in a fluid.

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[31]CONTACT: +1 780 492-0532, bowman@math.ualberta.ca, http://www.math.ualberta.ca/~bowman
[32]CONTACT: 209-228-4062, fblanchette@ucmerced.edu, http://faculty.ucmerced.edu/fblanchette/

C6.3

Supercavitating Flow in Multiply Connected Domains

Yuri A. Antipov[33] and Vasily V. Silvestrov[34]
Dept. Mathematics, Louisiana State University

Mathematical models of cavitation in fluids and quick and accurate numerical predictions are essential at different stages in the design process and the evaluation of performance and cavitation patterns. A question of particular interest is how to extend the traditional hodograph method used for simply connected flows to more complicated cavitation models. These models are nonlinear and include flows in multiply connected domains. We present two approaches for the construction of a conformal map from a canonical parametric domain into an n-connected domain of supercavitating flow. The first approach works when $n \leq 3$, and the canonical domain is the exterior of n slits along the real axis. The map is given by quadratures and expressed through the solutions of two Riemann-Hilbert problems on a symmetric hyperelliptic Riemann surface. The second method is applicable for any n, and the parametric domain is the exterior of n circles. To reconstruct the map, it is required to solve two Riemann-Hilbert problems for piece-wise meromorphic G-automorphic functions (G is a Schottky group). This method leads to a series-form solution and does not need the solution to a Jacobi inversion problem (for the first method, it cannot be bypassed).

[33]CONTACT: +1 225 578 1567, antipov@math.lsu.edu, http://www.math.lsu.edu/~antipov/

[34]v_silvestrov@mail.ru, Dept. Mathematics, Gubkin Russian State University of Oil and Gas

I5.1

The Meridional Flow of Source-Driven Grounded Abyssal Ocean Currents

Gordon E. Swaters[35]

Dept. Mathematical and Statistical Sciences, University of Alberta

An important component in Earth's climate system is the Meridional Overturning Circulation (MOC), which is the planetary-scale process by which incoming solar heating is distributed poleward and vertically in the ocean. Observations suggest that, at lowest order, the deep, or abyssal, branch of this circulation is largely organized as a geostrophic topographically-steered density-driven gravity current. Unfortunately, even the most highest resolution numerical global climate models do not have sufficient spatial resolution to resolve these flows. Thus, at present, a detailed theoretical understanding of the dynamics of these currents is accomplished either, regionally, by specialized very high resolution numerical models or, hemispherically, by process models of reduced complexity. In this talk I will provide an overview of our efforts to develop a process model of reduced complexity that is capable of describing the sub-inertial evolution of these currents on a mid-latitude beta-plane including their meridional flow along the continental slope boundary, interaction with the overlying ocean and their transition to baroclinic instability.

I5.2

Free Surface and Wave Phenomena with Geometric Singularities

Hsueh-Chia Chang[36]

Dept. Chemical and Biomolecular Engineering, University of Notre Dame

We examine several free surface and wave phenomena involving geometric singularities - all related to important analytical and diagnostic technologies.

In the presence of a DC field, a liquid drop is known to deform into a Taylor cone with a 49 degree half angle such that the singular Maxwell electric stress balances the singular azimuthal capillary pressure. We have recently found that, under an AC field, a different cone with a universal half angle of 11 degrees develops when net charge accumulation occurs within the cone. With free space charges in place of the induced surface charges in the DC case, the AC cone cannot be described by the spherical harmonics of the Laplace equation. Instead, we formulate a variational approach to describe the Coulomb energy due to repulsion of the space charge and show that the AC cone angle can be accurately estimated by evaluating a singular elliptic integral.

The dominant spherical harmonics of the Laplace equation outside the DC and AC cones are analyzed to show that electric field lines emitted from a conducting cone exhibit maxima in intensity at several azimuthal angles. These discrete cone angles are used to sort nanocolloids and molecules sprayed from the cone.

Under specific conditions, the AC polarization at a three-phase contact line formed by two Maxwell-Wagner liquids, with complex permittivies, on insulating solid is shown to reverse direction when the AC frequency increases beyond a cross-over frequency. This reversal of polarization is associated with different electro-wetting direction of the contact line. Instead of blowing up algebraically at the contact line, the AC potential is shown to blow up in an oscillatory manner towards the singularity at the cross-over frequency, with algebraically growing wavelength and logarithmatically growing amplitude. These waves at the cross-over frequency are connected to a nanodrop ejection phenomenon that changes direction at the cross-over.

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[35]CONTACT: gordon.swaters@ualberta.ca
[36]CONTACT: 1-574-631-5697, hchang@nd.edu, http://www.nd.edu/~changlab

Session I6: Wednesday 1:30-3:30 in TL-B1

I6.1

Asymptotic Reductions of Water and Internal Waves and their Solitary Waves

Mark Ablowitz[37]
University of Colorado

Recently we have developed nonlocal 'spectral' reformulations of water and internal waves where the vertical coordinate is eliminated. These nonlocal spectral formulations are a convenient means to obtain asymptotic reductions. We use these systems to obtain high order asymptotic expansions describing one dimensional solitary waves with surface tension and various two-dimensional systems including the Benney-Luke (BL) equation, its intermediate long wave counterpart (ILW-BL), the reductions to the Kadomstsev-Petviashvili (KP) and ILW-KP equations and their two-dimensional lump-type solitary waves.

I6.2

Surface Waves

Chris Linton[38]
School of Mathematics, Loughborough University

Surface waves are a common phenomenon which exist in many different physical contexts. They are associated with the propagation of energy along a surface, or along an interface between two different media. Perhaps the most obvious example is a wave on the surface of water, but others include seismic waves in the earth's crust and, at a much smaller scale, surface acoustic waves on elastic substrates, which are used in filters found in mobile phones.

In this talk I will discuss a number of different types of surface wave, including some classical examples from water waves, acoustics, elasticity and electromganetsim, and then focus on array guided surface waves, which only exist when the surface or interface is periodic and which have only recently been studied theoretically.

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[37]CONTACT: mark.ablowitz@colorado.edu
[38]CONTACT: +44 1509 223484, c.m.linton@lboro.ac.uk, http://www-staff.lboro.ac.uk/~macml1

C7.1

Theory and Numerical Simulations of Two-Layer Bores

Laura K. Brandt[39]

Science Applications International Corporation

The problem of upstream propagating undular interfacial bores generated by two-layer flow over two-dimensional topography is reconsidered. Since existing theories of two-layer bores are not valid over the entire parameter range, we use a fully-nonlinear numerical model of two-layer stratified flow over two-dimensional topography in combination with known theories to obtain a semi-empirical formula for upstream propagating bores that is valid over a wider range of layer depth ratios and density differences.

C7.2

Gravity Currents in 2-Layer Stratified Media

Alan Tan[40] and M.R. Flynn

Dept. Mechanical Engineering, University of Alberta

An analytical and experimental investigation of boundary gravity currents propagating through a two-layer stratified ambient of vertical extent will be presented. The heavy gravity current fluid is assumed to span the entire channel depth, H, at the initial instant. The theoretical discussion considers slumping, supercritical gravity currents i.e. those that generate an interfacial disturbance whose speed of propagation matches the front speed and follows the classical analysis of Benjamin [J. Fluid Mech. 31, 209-248 (1968)]. In contrast to previous investigations, the interfacial disturbance is parameterized so that its amplitude can be determined solely from the layer depths. Measured front speeds show positive agreement with analogue model predictions which remain strictly single valued. Experimental observations show that the front speed remains independent of the interface thickness even in the case where the environment comprises of uniformly stratified ambient with no readily discernible upper or lower ambient layer.

Subcritical flows are anticipated in a relatively narrow band of parameter space. Experimental observations show that the presence of an interface significantly effects the propagation of a subcritical gravity current. Details of flow regime transition will be discussed.

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[39]CONTACT: +1 858 826-6271, laura.k.brandt@saic.com
[40]CONTACT: (780) 248-1313, awtan@ualberta.ca

C7.3

Intrusive Gravity Currents and the Solitary Wave Lifecycle in a Cylindrical Geometry

Justine McMillan[41] and Bruce R. Sutherland
Dept. Physics, University of Alberta

An "intrusive gravity current" or "intrusion" arises when a fluid of one density propagates at an intermediate depth within a stratified ambient. Numerous experimental and theoretical studies have examined the propagation of these currents in a rectilinear geometry, however, the dynamics of radially spreading axisymmetric intrusions is less well established. By way of full-depth lock release experiments and numerical simulations, we examine the propagation of vertically symmetric intrusions in a two-layer ambient in a cylindrical geometry. We show that the strong stratification at the interface supports the formation of a mode-2 solitary wave that surrounds the intrusion head and carries it outwards at a constant speed beyond 6 lock radii. The wave and intrusion propagate faster than a linear long wave; therefore, there is strong evidence to support that the wave is indeed nonlinear. By extending rectilinear KdV theory to allow the wave amplitude to decay as r^{-p} with $p \approx \frac{1}{2}$, we show that from a single measurement of wave amplitude, the theory can be used to accurately predict the amplitude, speed and spread of the wave during its nonlinear evolution phase.

C7.4

The Axisymmetric Collapse of a Mixed Patch in Uniformly Stratified Fluid

Amber Holdsworth[42] and Bruce R. Sutherland
Dept. Earth & Atmospheric Sciences, University of Alberta

Hurricanes are responsible for mixing localized patches of the upper ocean leaving cooler waters in their wakes. The region collapses into the stratified ambient forming an gravity current and generating internal waves beneath the mixed patch. When these waves break they form turbulent patches which mix the deep ocean. In an effort to understand the axisymmetric collapse of a mixed patch into uniformly stratified fluid laboratory experiments are performed and wave properties are determined using a non-intrusive technique called Synthetic Schlieren.

We find internal wave frequencies are set by the buoyancy frequency, ($\omega \approx 0.8N_0$) and that the horizontal wavelength is set by the radius of the cylinder so that $k_r \approx 2R_c$. Vertical displacement amplitudes scale with the depth of the mixed patch according to $|\xi|/H_m = .016 \pm .001$ and we find that about 2% of the available potential energy of the mixed region is extracted by vertically propagating internal waves.

The work presented here is a precursor to the more complicated rotating case which will more realistically simulate the oceanic example. Extrapolation of these results is certainly premature, but a conservative estimate of the energy extracted by internal waves through the process of mixed region collapse is on the order of $1\,\mathrm{GW}$. That is an estimated $2\,\mathrm{TW}$ of power over the generation time and is comparable to the power exerted by tides and winds over the ocean.

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[41]CONTACT: jmmcmill@ualberta.ca
[42]CONTACT: amberholdsworth@gmail.com

C8.1

Anomalously Strong Tides in the Gully, a Submarine Canyon on the Nova Scotia Shelf

Neil Swart[43] **Susan E. Allen**[44]
Dept Earth and Ocean Sciences, University of British Columbia
Blair Greenan
Ocean Sciences Division, Bedford Institute of Oceanography

Two major submarine canyons: Monterey Canyon off California and Gaoping Canyon off Taiwan, have strongly enhanced ocean tides. Tidal amplification theories include focusing, trapping and local generation of internal waves. A very similar canyon, The Gully off Nova Scotia is also observed to have these enhanced tides but with a significant difference. The enhanced tides in Monterey and Gaoping Canyons are the semi-diurnal tides whereas in The Gully the most strongly enhanced tides are diurnal and are thus sub-inertial. We will show observations of the tides and present two coupled wave theories that explain the enhanced sub-inertial tides in the Gully. Comparisons between theoretical properties of the amplified tides and observed properties support the applicability of the theories.

C8.2

Long Nonlinear Surface Waves Propagating above a Current Varying with Depth

Jan Feys[45] and Sherwin Maslowe
Dept. Mathematics, McGill University

For the class of waves we consider, Benney (1966) showed that the amplitude evolves according to the KdV equation. It was proved earlier by Burns that the propagation speed of such waves is such that there is no critical layer (i.e., the phase speed lies outside the range of $U(y)$, the velocity profile). R. S. Johnson (1986 JFM), however, recognized that if the critical layer is nonlinear, the result of Burns may not apply and he determined streamline patterns for such flows. He did not, however, solve the associated eigenvalue problem.

In the present work, we investigate the eigenvalue problem for specific velocity profiles, assuming that there is no phase change across the critical layer. Such solutions often do not exist, the case of the asymptotic suction profile $U = 1 - e^{-y}$ being a notable example. We find, however, that singular solutions can be obtained for Falkner-Skan boundary layer profiles for both favorable and adverse pressure gradients. This and other examples will be discussed both for the Rayleigh equation and its long wave limit, as required to be consistent with the KdV as the amplitude equation.

C8.3

Interactions of Equatorially Trapped Waves with a Barotropic Background Zonal Shear

Maryam Namazi[46]
Mathematics and Statistics, University of Victoria

We are interested in the dynamics of equatorially trapped waves evolving in a sheared environment, consisting of a barotropic background zonal shear mimicking the equatorial trade winds. Our set up consists of the equatorial beta-plane shallow water equations with advective interactions with a prescribed barotropic zonal mean shear. We solve this system by using a second-order central scheme in the zonal-direction combined with a Galerkin projection in the meridional direction, on the few first parabolic cylinder functions. As they evolve, the equatorial waves exchange energy with the background mean flow and ultimately experience important deformations of their meridional structures and propagation properties.

More importantly, other equatorial waves are generated/excited at later time such as Kelvin and westward-inertial gravity waves as well as MJO-like eastward-moving intra-seasonal disturbances characterized by a quadropole vortex, when the model is evolved from an initial solution consisting of one specific equatorially trapped wave although the model is linear and direct linear resonances are not possible through the time-independent mean flow. The shear effect becomes more significant for larger wavenumbers (of the initial-seed wave) and stronger shear gradients. We emphasize the cases of Kelvin waves that are observed in organized tropical convective systems, and Rossby waves that are believed to play a primary role in tropical-extratropical tele-connections.

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[46]CONTACT: 1 (250) 472-5315, maryam@math.uvic.ca

C9.1

Film Flow Over Heated Wavy Inclined Surfaces

Serge D'Alessio [47] and J.P. Pascal[48]
Dept. Applied Mathematics, University of Waterloo

The two-dimensional problem of gravity-driven laminar flow of a thin layer of fluid down a heated wavy inclined surface will be discussed. The coupled effect of bottom topography, variable surface tension and heating has been investigated both analytically and numerically. A stability analysis is conducted while nonlinear simulations are used to validate the stability predictions and also to study thermocapillary effects. The governing equations are non-hydrostatic approximations to the Navier-Stokes equations which exploit the assumed relative shallowness of the fluid layer with the cross-stream dependence eliminated by means of a weighted residual technique. New interesting results regarding the combined effect of bottom topography and surface tension on the stability of the flow will be presented along with the influence resulting from heating.

C9.2

The Nonlinear Schrödinger Equation as a Model of Bose-Einstein Condensate Formation

Hayder Salman[49]
School of Mathematics, University of East Anglia

Nonequilibrium phenomena in condensates form an important and growing area of research in low temperature systems. Yet, a systematic theoretical treatment of this class of problems is hindered by the lack of useful approximations that one can introduce to analyse such systems. In this work we consider one particular example of a nonequilibrium system, namely the process of Bose-Einstein condensate (BEC) formation from a strongly nonequilibrium initial condition in a finite-temperature atomic gas. We begin by reviewing recent results obtained within the framework of the classical fields approximation for the idealized scenario of a homogeneous Bose Gas. Using weak turbulence theory, we are able to derive a set of kinetic equations from the classical field equations that provide a complete nonequilibrium description of the condensation process. Our formulation effectively corresponds to a micro-canonical ensemble. Therefore, it follows that the temperature and the condensate mass fractions are fully determined by the total number of particles and the initial total energy. The results are extended to the more relevant configuration of a trapped BEC. This is achieved by invoking a scale separation argument. A novel feature of the work presented is the derivation of new analytical results of the equilibrium properties of the system.

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[47]CONTACT: (519) 888-4567 x35014, sdalessio@uwaterloo.ca, www.math.uwaterloo.ca/~sdalessi/
[48]Ryerson University
[49]CONTACT: +44(0)1603 591666, H.Salman@uea.ac.uk

C9.3

Resonant Oscillations Between Two Concentric Spheres

David E. Amundsen[50], M.P. Mortell[51] and B.R. Seymour[52]
Carleton University

We consider the propagation of acoustic waves in a resonantly forced spherical shell. Governing equations are derived assuming an isentropic gas, and arbitrary underlying density profile. The resulting evolution is characterized by the existence or non-existence of shock solutions. These are studied in detail in terms of the underlying physical characteristics of the system. In particular the transition between these two regimes is analyzed in the limit that the forcing is weak. Numerical simulations are then presented to provide comparison and further insight into the nature of the interplay between the geometric and nonlinear effects.

[50]CONTACT: dave@math.carleton.ca
[51]University College Cork
[52]University of British Columbia

Session I7: Thursday 9:30-12:00 in TL-B1

I7.1

Evolution of Solitary Waves in a Two-Pycnocline System

Patrick Weidman[53]

Dept. Mechanical Engineering, University of Colorado at Boulder

Three decades ago some numerical studies and laboratory experiments identified the phenomenon of leapfrogging internal solitary waves propagating on separate pycnoclines. The long-time behavior of this system based on the Liu, Kubota and Ko (1980) model for inviscid interactions is revisited using modern spectral methods. The physical system is comprised of separated pycnoclines in a three-layer fluid bounded by rigid horizontal top and bottom walls. We first show that mode-two solitary waves exhibit classical soliton behavior and identify the Lax interactions for overtaking waves. This is followed by a study to determine the nature of solutions on input parameters: the three fluid densities and two interface thicknesses, for fixed initial conditions describing isolated mode-two disturbances on each pycnocline. Generally, the initial disturbances immediately separate and evolve into two distinct solitary waves. In a narrow region of parameter space, however, the waves pair up and oscillate in leapfrog fashion. The motion is only quasi-periodic as each wave sheds energy into its dispersive tail, which causes the oscillation magnitude and period to increase until the waves eventually separate. The separation time, oscillation period and magnitude, and final amplitudes and celerities of the separated waves are determined. A simple asymptotic mode is developed to aid in the interpretation of numerical results.

I7.2

Ermakov Structure in 2+1-dimensional Magneto-Gasdynamics

Colin Rogers[54]

Dept. Applied Mathematics, The Hong Kong Polytechnic University

The 2+1-dimensional equations of rotating homentropic gasdynamics with a parabolic gas law are shown to admit a broad class of elliptic vortex type solutions determined by a Ermakov-Ray-Reid system. A novel invariant associated with the latter is used in cojunction with underlying Hamiltonian structure to construct a class of "accelerated' motions in terms of an elliptic integral representation. A complementary class is shown to lead to pulsrodons.

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[53]CONTACT: (303) 492-4684, weidman@colorado.edu
[54]CONTACT: (852) 2766-4592, macolinr@inet.polyu.edu.hk

I8.1

Numerical modelling of internal tides in the ocean

Stephen D. Griffiths[55] and W.R. Peltier[56]
Dept. Applied Mathematics, University of Leeds

Topographically generated internal waves are ubiquitous in both the atmosphere and ocean. Here we focus on internal tides in the ocean, which are generated as the barotropic (surface) tide flows over submarine topography. At sufficiently low latitudes, the internal tide propagates away from the generation region, leading to an important energy transfer from the barotropic tide to the ocean interior. In the coastal oceans, this phenomenon often leads to intense wave activity and breaking, and the generation of upstream propagating solitary waves.

On a global scale, internal tide generation accounts for a substantial fraction of the dissipation of the barotropic tide (the rest being associated with bottom friction in shallow seas). As such, internal tides are heavily implicated in the evolution of the lunar orbit, a phenomenon of planetary scale relevance. High-resolution numerical modelling, using a simple yet flexible formulation for three-dimensional free-surface stratified flow over arbitrary topography, is used to understand internal tide generation and propagation on a global scale. In conjunction with a coupled numerical model of the global barotropic tide, a corresponding analysis is used to estimate internal tide generation during the ice-ages of the late Quaternary, during which the tidal regime shifted due to changes in the geometry of the ocean basins.

I8.2

Settling in Stratified Fluids: A Tortoise-and-Hare Experimental and Mathematical Tale

Roberto Camassa[57]
Dept. Mathematics, University of North Carolina at Chapel Hill

The interplay between forces generated by rigid bodies moving through fluids can be significantly affected by the presence of density variations in the fluid. This talk will present a theoretical and experimental study of what can be viewed as one of the simplest set-ups where density effects can be studied in detail in this context. We consider spheres falling under gravity through miscible, sharply stratified fluids under viscous dominated dynamics. A first-principles, numerically-assisted mathematical model of this system will be presented. Analysis of our theory identifies parametric trends, which are also partially explored in the experiments, further confirming the predictive capability of the theoretical model. Even for such simple systems, some new, and perhaps non-intuitive, physical phenomena emerge, as illustrated by the example of two identical spheres racing to the bottom of homogeneous vs. stratified fluids. Mathematically, when viscosity dominates, fluid flow solutions acquire paradoxical properties in approximate limits. An example of how stratification may influence the resolution of the Stokes (or Whitehead) paradox will be discussed, as well as the mathematical implications for analysis of the equations of motion in exterior domains. A conjecture about reversal of the sphere's motion in the presence of internal waves when viscous domination is sufficiently weakened will be illustrated.

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[55]CONTACT: S.D.Griffiths@leeds.ac.uk
[56]Dept. Physics, University of Toronto
[57]CONTACT: camassa@amath.unc.edu

C10.1

Numerical Investigation of Internal Wave-Vortex Interactions

Tyler D. Blackhurst[58] and Julie C. Vanderhoff

Brigham Young University

A numerical investigation of internal waves interacting with a pancake vortex dipole is presented and is validated by the experimental study of Godoy-Diana, Chomaz and Donnadieu (2006). Two general wave-vortex interactions are simulated using three-dimensional linear ray theory. The first interaction involves wave propagation initially in the same direction as the dipole translation. In this co-propagating case, a spreading of wave energy, termed defocusing, is observed as rays interact with the dipole and then diverge in the spanwise direction. Waves near the center of the dipole can approach critical levels where the wave energy is absorbed by the dipole or where the waves are overturned and possibly break. As wave breaking cannot be simulated with this linear model, an analysis of changes in wave steepness aids in estimating the onset of breaking. The second interaction involves waves counter-propagating with the dipole. A concentration of wave energy, termed focusing, can be seen as symmetric, off-center rays converge upon the center of the dipole.

C10.2

Internal Wavefield Generated by a Towed Sphere: Theory/Experiment Comparison

Alan Brandt[59]

Applied Physics Laboratory, Johns Hopkins University

Bodies moving in a stratified fluid generate internal waves by two primary mechanisms: body displacement (i.e. lee wave forcing), and wake turbulence. The physical mechanisms underlying the former are well known, however the mechanisms coupling wake turbulence to the internal wavefield are not well understood. The present study provides experimental data that demonstrate the complexity of body generated internal wavefields and provides a dataset for model validation covering a wide range of Froude numbers, $0.1 < F < 10$, including regimes where each internal wave generation mechanisms dominates. The integrated potential energy obtained from these experimental data at low Froude numbers compares favorably with a body generated (lee-wave) analytical model wherein a depth dependent resonance is present. Comparisons of the kinematical properties, e.g. wavefield opening angle and vertical structure, are more problematical. In the high Froude number regime the turbulent wake forces the internal wavefield causing random internal waves that by nature vary in each experimental realization. Models of this process are ad-hoc and can at best predict the mean of this random component. The adequacy of models put forth to date will be discussed.

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[58]CONTACT: +1 801 422-2320, tyblackhurst@byu.net
[59]CONTACT: alan.brandt@jhuapl.edu

C10.3

First and Second Harmonic Internal Waves from a Horizontally Oscillating Sphere

Evgeny Ermanyuk[60], Jan-Bert Flór[61] and **Bruno Voisin**[62]
LEGI, CNRS and Université de Grenoble

A horizontally oscillating sphere in a density-stratified fluid is studied experimentally and theoretically, as a paradigm of the generation of three-dimensional internal tides by supercritical topography. The experiments implement a novel technique for the measurement of the spatial structure of internal wave fields, based on horizontal fluorescent dye planes and a mobile vertical laser sheet; they are compared with an original linear theory. Spectral analysis reveals the presence of two harmonics, namely a first harmonics at the fundamental frequency and a second harmonics at twice this frequency. The first harmonics has a dipolar structure, an amplitude varying linearly with the amplitude of oscillation, and is quantitatively described by the theory. The second harmonics is present at amplitudes of oscillation higher than one tenth of the sphere radius and has a quadrupolar structure. Its amplitude varies quadratically with the amplitude of oscillation, and may exceed the amplitude of the first harmonics. At frequencies smaller than half the buoyancy frequency, the second harmonics is evanescent and confined to the vicinity of the sphere; at frequencies larger than half the buoyancy frequency, it propagates away.

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[60]Siberian Branch of the Russian Academy of Sciences
[61]University of Grenoble
[62]CONTACT: +33 4 76 82 50 45, bruno.voisin@legi.grenoble-inp.fr, http://www.legi.grenoble-inp.fr/web/?lang=en

C11.1

Pre-saturation Evolution of Kelvin-Helmholtz Instabilities

Mona Rahmani[63] , G. A. Lawrence and B. R. Seymour
Dept. Civil Engineering, University of British Columbia
Dept. Mathematics, University of British Columbia

The two-dimensional evolution of Kelvin-Helmholtz (KH) instabilities is investigated using direct numerical simulation (DNS). The primary two-dimensional instability starts by rolling up the interface and forming a series of periodic, two-dimensional, core vortices. These are connected by thin braids containing the remnants of the core vorticity with hyperbolic stagnation points in the middle of the braids. The semi-analytical model of Corcos and Sherman (1976) describes the evolution process up to saturation by modeling the depletion of braid vorticity from stagnation points towards the cores. The model benefits from its simplicity and computational efficiency as only the braid region is examined. The DNS results are used to test the predictions of the model for an extended range of Reynolds and Prandtl numbers.

The similarity solution of Corcos and Sherman (1976) is also used to predict the minimum length scale and braid centerline vorticity before saturation. The significance of the minimum length scale prediction to DNS is discussed.

C11.2

Shear Instabilities in Stratified Parallel flows and Implications for Mixing Efficiency

Ali Mashayek[64] and W.R. Peltier
Dept. Physics, University of Toronto

We investigate the transition process through which a two dimensional Kelvin-Helmhlotz (KH) instability becomes turbulent. KH billows are known to undergo merging processes. The braid region of the primary KH wave is also susceptible to a secondary shear instability which can happen before, during, or after the merging process. The KH billows are also known to be susceptible to three-dimensional convective instabilities occurring in the outer regions of their billows in which isopycnals overturn which provides a fast route to turbulent collapse. Occurrence of the latter instability may eliminate the possibility of the merging and secondary shear instabilities by quickly destroying the laminar structure of the two dimensional bilow dominated flow. We investigate the possibility of occurrence of these three instabilities in the Reynolds and Prandtl (Re-Pr) number space using a theoretical approach. A map is provided which determines the dominant instability in different zones of Re-Pr space which identifies the regions of possible coexistence of multiple instabilities. The map is developed on a theoretical basis and is tested against high-resolution two and three dimensional direct numerical simulations (DNS). As each of the instabilities have their specific implications on the mixing efficiency, the map allows identification of the appropriate value for the mixing efficiency based on the ambient physical properties of the flow. It also enables a prediction to be made on a priori grounds of the structures that will characterize the turbulent flow once transition has occurred.

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[63]CONTACT: (604) 339-5272, mrahmani@civil.ubc.ca
[64]CONTACT: 416 946 3019, amashaye@atmosp.physics.utoronto.ca, http://sites.google.com/site/alirezamashayek/

C11.3

Instability of Inclined Film Flow over Wavy Permeable Surfaces

S.J.D. D'Alessio[65] and J.P. Pascal[66]
University of Waterloo

This talk reports on a theoretical study of the two-dimensional flow of a Newtonian fluid film along an inclined permeable surface exhibiting sinusoidal undulations. The goal of our investigation is to analyze the combined effect of surface tension and bottom topography and permeability on the hydrodynamic instability of the flow. A depth-integrated model is derived by applying a weighted residual method to a second-order approximation of the Navier-Stokes equations for shallow flow. The effect of substrate permeability is incorporated into the model by imposing a slip condition suited for the nonplanar geometry of the fluid-porous medium interface. A linear stability analysis is employed to determine the critical conditions for instability. Nonlinear numerical simulations are also carried out and used to verify the predictions of the linear theory and to calculate the evolution of unstable flows.

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[65]CONTACT: sdalessi@math.uwaterloo.ca
[66]Ryerson University

Session C12: Thursday 4:00-4:40 in TB-81

C12.1

Stable and Unstable Large Amplitude Internal Solitary Waves

Stuart King[67], Magda Carr and David Dritschel
School of Mathematics and Statistics, University of St. Andrews

A new approach to modelling large amplitude internal solitary waves is considered in a Boussinesq fluid. Subject to weak smoothness conditions on the background stratification, a fast spectral method may be used to obtain the steady state form of such waves. This method has also been extended to deal in a novel way with closed streamlines when they occur in the domain. The steady states thus found may be stable or unstable, and a newly-developed contour-based numerical method allows one to accurately resolve their evolution, including breaking.

C12.2

Nonlinear Rossby Wave Patterns in Polar Areas

Oleg Derzho[68]
Dept. Physical Oceanography, Memorial University of Newfoundland

Nonlinear barotropic vortical patterns on a polar gamma-plane are investigated analytically. The solutions describe large scale Rossby waves rotating anticyclonically with zero circulation and a current with a cyclonic circulation.The Rossby waves are predicted to rotate with a specific angular velocity. The stream function-vorticity relation is assumed to be nonlinear, which may lead to a pronounced asymmetry within the pattern. The similarity between the simulated patterns and the Antarctic Circumpolar Wave is highlighted. We are able to explain why the Antarctic Circumpolar Wave is a quadrupole and why the observed sea ice manifestations at Antarctic polar stations have periods of 2, 4 and 8 years.

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[67]CONTACT: +44 (0)1334 463723, stuart@mcs.st-and.ac.uk
[68]CONTACT: oderzho@mun.ca