Generalizing the uniqueness of equilibrium states in a conditional setting

Jisang Yoo

Bowen and Sinai introduced many tools which enabled them to investigate properties of equilibrium states on systems beyond full shifts: Anosov systems and Axiom A systems. Since then people have been trying to generalize the theory in many directions such as certain classes of Z^d-actions or a relative setting. In a relative setting, we are concerned with a dynamical system (X, T) equipped with a distinguished factor map $\pi : (X, T) \to (Y, S)$ and we ask which invariant measures $\mu \in M_T(X)$ maximize $h_\mu(T) + \int V \, d\mu$ among the measures whose projection on (Y, S) is fixed to be $\nu \in M_S(Y)$, where $V \in C(X)$ is usually assumed to be Hölder-continuous.

Such measures, called relative equilibrium states, are not necessarily unique even when both (X, T) and (Y, S) are full shifts. Nonetheless, Petersen, Quas, and Shin showed that there are only finitely many such ergodic measures if $V = 0$ and (X, T) and (Y, S) are transitive subshifts of finite type [3]. Recent progress in this area is as follows. Allahbakhshi and Quas strengthened the finiteness result by giving a specific bound c_π related to the structure of π [2]. Allahbakhshi, Antonioli, and Yoo generalized it by dropping the condition $V = 0$ [1]. Yoo proved that there are exactly c_π such measures, counting multiplicities [4].

References

[1] M. Allahbakhshi, J. Antonioli, and J. Yoo, *Relative equilibrium states and class degree*, Ergodic Theory and Dynamical Systems (2017), 1–24.

[2] M. Allahbakhshi and A. Quas, *Class degree and relative maximal entropy*, Trans. Amer. Math. Soc. **365** (2013), no. 3, 1347–1368. MR 3003267

[3] K. Petersen, A. Quas, and S. Shin, *Measures of maximal relative entropy*, Ergodic Theory Dynam. Systems **23** (2003), no. 1, 207–223.

[4] J. Yoo, *Multiplicity structure of preimages of invariant measures under finite-to-one factor maps*, Trans. Amer. Math. Soc. ((to appear) 2017).