PIMS CRG Workshop on Geometric Analysis

Abstracts

- Arunima Bhattacharya, University of Oregon

  Interior Schauder Estimates for the Fourth Order Hamiltonian Stationary Equation in Two Dimensions (This is on a joint work with Micah Warren.)

  We study the regularity of the Lagrangian Hamiltonian stationary equation, which is a fourth order nonlinear PDE. Consider the function $u : B_1 \to \mathbb{R}$ where B_1 is the unit ball in $\mathbb{R}^2$. The gradient graph of u, given by $\{(x, Du(x)) | x \in B_1\}$ is a Lagrangian submanifold of the complex Euclidean space. The function θ is called the Lagrangian phase for the gradient graph and is defined by

  $$\theta = F(D^2u) = Im \log \det(I + iD^2u).$$

  The non homogeneous special Lagrangian equation is given by the following second order non-linear equation

  $$F(D^2u) = f(x). \tag{0.1}$$

  The Hamiltonian stationary equation is given by the following fourth order nonlinear PDE

  $$\Delta_g \theta = 0 \tag{0.2}$$

  where g is the induced Riemannian metric from the Euclidean metric on $\mathbb{R}^4$, which can be written as

  $$g = I + (D^2u)^2.$$

  We consider the Hamiltonian stationary equation for all phases in dimension two and show that solutions that are $C^{1,1}$ will be smooth and we also derive a $C^{2,\alpha}$ estimate for it.

- Mikhail Karpukhin, UC Irvine

  Index of minimal surfaces in spheres and eigenvalues of the Laplacian.

  The Laplacian is a canonical second order elliptic operator defined on any Riemannian manifold. The study of upper bounds for its eigenvalues is a classical problem of spectral geometry going back to J. Hersch, P. Li and S.-T. Yau. It turns out that the optimal isoperimetric inequalities for Laplace eigenvalues are closely related to minimal surfaces in spheres. At the same time, the index of a minimal surface is defined as a number of negative eigenvalues of a different second order elliptic operator. It measures the instability of the surface as a critical point of the area functional. In the present talk we will discuss the interplay between index and Laplace eigenvalues, and present some recent applications, including a new bound on the index of minimal spheres as well as the optimal isoperimetric inequality for Laplace eigenvalues on the projective plane.

- Daniel Ketover, Rutgers University and IAS

  TBA

- Man Shun Ma, Rutgers University

  Uniqueness Theorem for non-compact Mean Curvature Flow with possibly unbounded curvatures

  We discuss uniqueness for mean curvature flow of non-compact manifolds. We use an energy argument to prove a uniqueness theorem for mean curvature flow with possibly unbounded curvatures. These generalize the results in Chen and Yin (CAG, 07). This is a joint work with Man-Chun Lee.

- Peter McGrath, University of Pennsylvania

  Area Bounds for Free Boundary Minimal Submanifolds

  Fraser-Schoen and Brendle proved that the area of a k-dimensional free boundary submanifold of the unit n-ball is bounded from below by the area of the k-dimensional unit ball. I will discuss (joint work with Brian Freidin) some recent generalizations of these works in positively curved ambient manifolds.

- Robin Neumayer, IAS

  TBA

- Daniel Stern, Princeton University

  Variational methods for the Ginzburg-Landau equations and minimal varieties of codimension two

  We will discuss recent developments in the variational theory and asymptotic analysis of the complex Ginzburg-Landau functionals in Riemannian manifolds, emphasizing connections between the space of solutions to the Ginzburg-Landau equations and the space of weak minimal submanifolds of codimension two.

- Celso Viana, IAS

  Index one minimal surfaces in spherical space forms

  Minimal surfaces are critical points of the area functional. In this talk I will discuss classification results for minimal surfaces with index one in 3-manifolds with positive Ricci curvature and outline the proof that in spherical space forms with large fundamental group the genus of such surfaces is at most two.