

# Applied Mathematics Perspectives

July 12-17, 2011

## 2011 ICIAM Satellite Meetings Conference Manual



### Numerical Ricci Flow in Computer Science, Geometry, and Physics University of British Columbia



## Welcome to Applied Mathematics Perspectives, a sequence of workshops supporting the flagship ICIAM 2011 meeting of the Applied Mathematics community.

Although the historical roots of Applied Mathematics lie in applied analysis and the development of Newtonian physics, today Applied Mathematics is a far reaching set of interwoven disciplines and skills, which are applied to understanding every facet of nature, science, industry and modern life. This dynamic and ever-changing field includes classical mathematical areas (differential equations, numerical analysis, asymptotic and variational methods, applied probability), all pursued from the application perspective, as well as areas such as applied mechanics and fluid dynamics. Each of these areas advances apace, embracing in particular high speed computation, but newer areas are also intertwined into the applied mathematics fabric. The workshops of Applied Mathematics Perspectives 2011 reflect the diversity and strengths of applied mathematics today.

This sequence of workshops has been supported by PIMS, MITACS, BIRS, CAIMS/SCMAI. Substantial travel funds have been awarded to US participants by the NSF. Local support at UBC has come from PIMS, the UBC Department of Mathematics and the Institute of Applied Mathematics. We thank these organisations for their generosity and help in making this a successful scientific event. We also acknowledge with gratitude the contributions of members of the local organising committee and scientific committee.

Visit the official website: <http://www.mitacs.ca/goto/amp2011>

### Local Organising Committee:

I. Frigaard (University of British Columbia)  
T. Hillen (University of Alberta)  
B. Khouider (University of Victoria)  
M. Lamoureux (University of Calgary)  
R. LeVeque (University of Washington)  
N. Nigam (Simon Fraser University)  
R. Russell (Simon Fraser University)  
R. Spiteri (University of Saskatchewan)  
M. Ward (University of British Columbia)

### Scientific Committee

R. Craster (Imperial College, UK)  
M. Davidson (University of Western Ontario)  
I. Frigaard (University of British Columbia)  
G. Homsy (University of British Columbia)  
S. Howison (Oxford University, UK)  
H. Othmer (University of Minnesota, US)  
M. Overton (Courant Institute, US)  
O. Scherzer (Vienna, Austria)  
R. Spiteri (University of Saskatchewan)

## Numerical Ricci Flow in Computer Science, Geometry, and Physics

Over the past several years, the Ricci flow has come into its own. This geometric flow was first introduced by Richard Hamilton in order to study the geometry and topology of manifolds in low dimensions. With a number of important refinements, it provided the backbone of Perelman's famous proof of the Poincaré Conjecture. The flow itself has been implemented numerically, providing for the first time important information about the limiting metrics on spaces of interest to mathematicians, to physicists, and now even to computer scientists.

Recently, numerical algorithms based on Ricci flow have been developed for discrete surfaces and 3-manifolds. It has been demonstrated that Ricci flow is a powerful tool to design Riemannian metrics using curvatures, and extremely valuable for many fields in computer science. In computer graphics, Ricci flow has been applied for surface parameterization, texture mapping, vector field design; in computer vision, Ricci flow is used for matching surfaces with large deformations and shape analysis, geometric database retrieval; in medical imaging, it is widely used for brain cortex surface mapping, computer aided diagnosis, virtual colonoscopy; in geometric modeling, it offers a novel way to construct splines on general manifolds. In networking research, Ricci flow has been applied for designing novel routing protocols.

Ricci flow arises as the simplest renormalization group (RG) flow of the target space geometry in string theory. It is believed that RG flows approximate off-shell processes in string theory such as closed string tachyon condensation. There is thus enormous potential for using and generalizing the tools developed in the Ricci flow program in mathematics for addressing relevant questions in string theory.

### Organizers:

Charles Doran, University of Alberta ( doran@math.ualberta.ca )

David Gu, Stony Brook University ( gu@cs.stonybrook.edu )

Robert Gulliver, University of Minnesota ( gulliver@math.umn.edu )

Suneeta Vardarajan, IISER Pune ( suneeta@iiserpune.ac.in )

### Locations:

Workshops - Math 125

Lunches and coffee breaks PIMS office downstairs, WMAX110

## SCHEDULE :::

### Locations:

Workshops - Math 125

Lunches and coffee breaks PIMS office downstairs, WMAX110

### Wednesday, July 13

6:00pm Registration and reception at Mahoney's Irish Pub, see UBC map for location

### Thursday, July 14

9:00 - 10:00 Speaker: David Gu (SUNY Stony Brook )  
Title: Ricci Flow in Engineering Fields

10:00 - 11:00 Break

11:00 - noon Speaker: Huai-Dong Cao (Lehigh U.)  
Title: Singularity formation in the Ricci Flow

noon - 2:00 Lunch

2:00 - 3:00 Speaker: Mauro Carfora (U. of Pavia, Italy)  
Title: Ricci Flow: A Theoretical Physics Perspective

3:00 - 4:00 Break

4:00 - 5:00 Speaker: Adam Oberman (SFU)  
Title: Introduction to Numerical Methods for Nonlinear Elliptic Partial Differential Equations.

6:30pm Dinner cruise, see last page for details.

## Friday, July 15

9:00 - 10:00 Speaker: Dan Knopf (U. of Texas at Austin)  
Title: Asymptotics of degenerate Ricci flow neckpinches

10:00 - 11:00 Break

11:00 - noon Speaker: David Glickenstein (U. of Arizona)  
Title: Scalar curvature on piecewise flat manifolds and other topics

noon - 2:00 Lunch

2:00 - 3:00 Speaker: Ali Nassar (U. of Lethbridge)  
Title: Energy functionals for Calabi-Yau metrics

3:00 - 4:00 Break

4:00 - 5:00 Speaker: Guoyi Xu (U. of California at Irvine)  
Title: Short-time existence of the Ricci flow on noncompact Riemannian manifolds

5:00-6:00pm Speaker: Jack Gergenber (UNB)  
Title: Yang-Mills Flow and Uniformization

6:45pm Reception at SAGE BISTRO, see UBC map for location

## Saturday, July 16

9:00 - 10:00 Speaker: Ken Stephenson (U. of Tennessee)  
Title: Curvature Flow via Circle Packing

10:00 - 11:00 Break

11:00 - noon Speaker: Mauro Carfora (U. of Pavia, Italy)  
Title: Ricci flow conjugation and Initial data sets for Einstein Equations

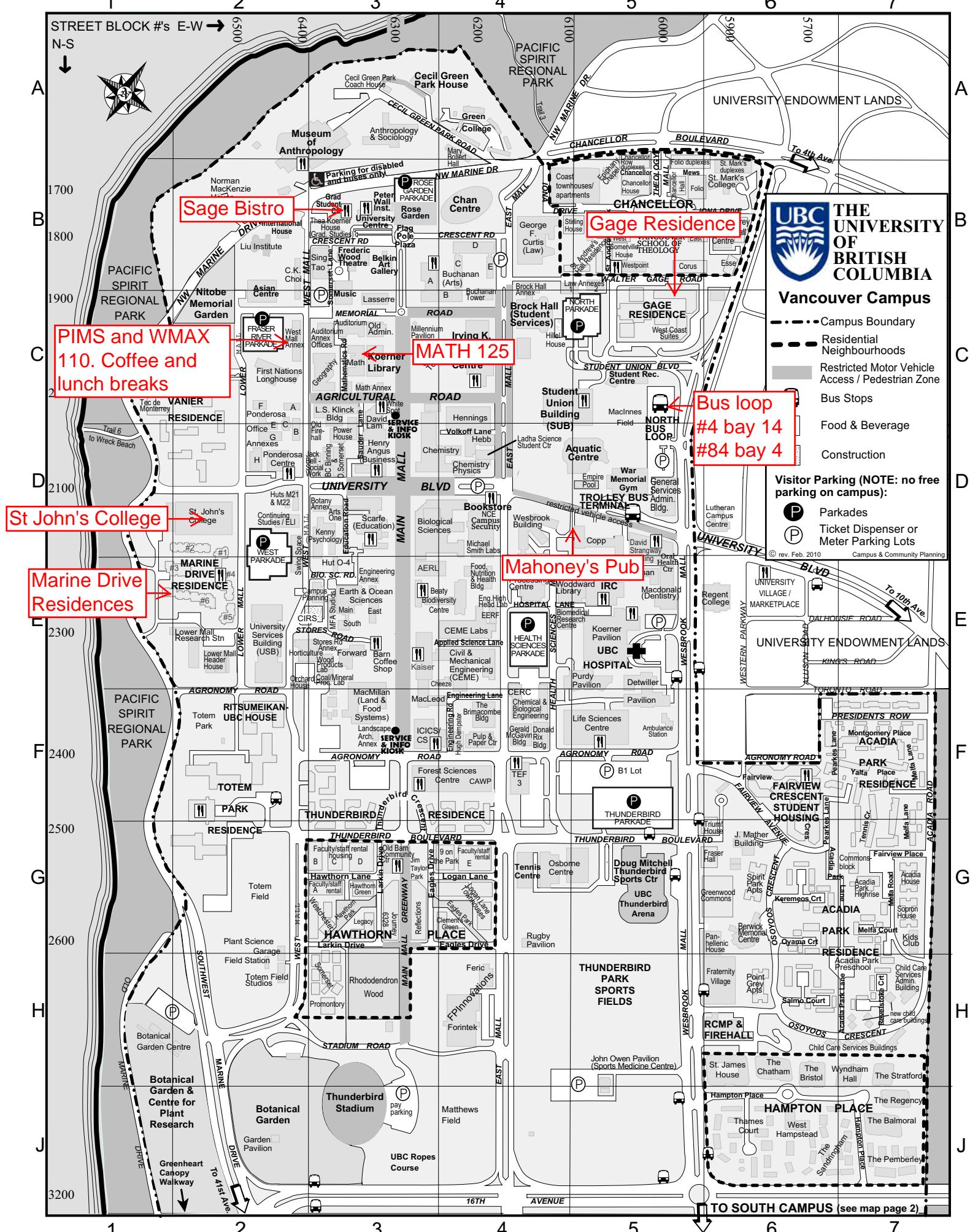
noon - 2:00 Lunch

2:00 - 3:00 Speaker Albert Chau (UBC)  
Title: The complex parabolic Monge Ampere equation and applications

3:00 - 4:00 Break + poster session

4:00 - 5:00 Speaker: Huai-Dong Cao (Lehigh U.)  
Title: Geometry of gradient Ricci solutions

5:00 - 6:00pm Speaker: Warner Miller (Florida Atlantic U.)



## Abstracts :: (alphabetical)

**Mauro Carfora (two seminars): (U. of Pavia, Italy)**

**Expository seminar Title: Ricci Flow: A Theoretical Physics Perspective**

Abstract: I will touch upon some aspects of the connection between Ricci flow and theoretical physics.

**Research seminar title: Ricci flow conjugation and Initial data sets for Einstein Equations**

Abstract: We discuss a natural form of Ricci flow conjugation between two distinct general relativistic data sets given on a compact manifold.

**Huai-Dong Cao (Lehigh University)**

**Expository Talk: Singularity formations in the Ricci flow**

Abstract: This is an expository talk on singularity formation of the Ricci flow on 3-manifolds.

**Research talk: Geometry of gradient Ricci solutions**

Abstract: Ricci solutions are natural extensions of Einstein metrics.

There are special solutions to Hamilton's Ricci flow and arise as possible singularity models. In this talk we shall present some recent progress on classifications of gradient steady and shrinking Ricci solitons.

**Albert Chau (University of British Columbia)**

**Title. The complex parabolic Monge Ampere equation and applications**

Abstract. In this talk I will introduce a class of complex parabolic Monge Ampere equations, then discuss applications to the Kahler Ricci flow on non compact Kahler manifolds. A review of some results for Kahler Ricci in this context will also be given.

**Jack Gergen (University of New Brunswick)**

**Yang-Mills Flow and Riemannian Geometry**

Abstract: I will explore the idea of using the Yang-Mills flow to examine uniformization of 2D manifolds. We build the gauge theory connection from the topology of the manifold, and identify the components of the gauge potential with a frame-field and a linear connection on the manifold. I will describe both analytic and numerical solutions to the flow, and discuss its consistency with the uniformization theorem for closed compact 2D manifolds. I will speculate on extending this to higher dimensions.

**David Glickenstein: (U. of Arizona)**

**Scalar curvature on piecewise flat manifolds and other topics**

Abstract: One route to a numerical Ricci flow is to construct spaces that admit nice finite elements, such as triangulated piecewise flat manifolds. Such spaces admit natural scalar curvature measures, proven to converge to scalar curvature measure on a Riemannian manifold by Cheeger, Muller, and Schrader (as conjectured by T. Regge). Using variation of geometric functionals, these scalar curvature measures lead to discrete Yamabe flows (and hence Ricci flows in 2D). We will discuss some of the relationships between these scalar curvatures, conformal variations, circle packings, graph Laplacians, and the Einstein equation. Time permitting, we will also discuss some additional topics relevant to the conference: visualizing abstract piecewise flat manifolds and embedding abstract geometric flows.

## Abstracts :::

**David Gu: (SUNY Stony Brook )**

**Title: Ricci Flow in Engineering Fields**

**Abstract:** Ricci flow is a powerful tool for designing Riemannian metrics by prescribing curvatures, which plays an fundamental role in many engineering fields. In this talk, we summarize computational algorithms for surface Ricci flow and its main applications, which include surface registration in medical imaging, surface parameterization in computer graphics, 3D face recognition and shape retrieval in computer vision, spline construction and fitting in geometric modeling and design, efficient routing in wireless sensor network and so on.

**Dan Knopf : (U. of Texas at Austin)**

**Title: Asymptotics of degenerate Ricci flow neckpinches**

**Abstract:** Formation of degenerate (Type-II) Ricci flow neckpinches was studied numerically by Garfinkle and Isenberg in 2005--8. An existence proof for the formation of such singularities from nongeneric rotationally symmetric initial data was supplied by Gu and Zhu in 2008. I will report on recent joint work with Angenent and Isenberg that provides a precise asymptotic profile of these singularities.

**Warner Miller (Florida Atlantic U.)**

[[Organizers' description: This talk will be on constructing a discrete simplicial Ricci tensor in higher dimensions for applications to DRF.]]

**Ali Nassar: (U. of Lethbridge)**

**Energy functionals for Calabi-Yau metrics**

**Abstract:** We identify a set of “energy” functionals on the space of metrics in a given Kaehler class on a Calabi-Yau manifold, which are bounded below and minimized uniquely on the Ricci-flat metric in that class. Using these functionals, we recast the problem of numerically solving the Einstein equation as an optimization problem. We apply this strategy, using the “algebraic” metrics (metrics for which the Kaehler potential is given in terms of a polynomial in the projective coordinates), to the Fermat quartic and to a one-parameter family of quintics that includes the Fermat and conifold quintics. We show that this method yields approximations to the Ricci-flat metric that are exponentially accurate in the degree of the polynomial (except at the conifold point, where the convergence is polynomial), and therefore orders of magnitude more accurate than the balanced metrics, previously studied as approximations to the Ricci-flat metric. The method is relatively fast and easy to implement. On the theoretical side, we also show that the functionals can be used to give a heuristic proof of Yau’s theorem.

**Adam Oberman (Simon Fraser University)**

**Introduction to Numerical Methods for Nonlinear Elliptic Partial Differential Equations.**

**Abstract:** Nonlinear elliptic and parabolic partial differential equations (PDEs) appear in geometric flows, image processing, and mathematical finance. The theory of viscosity solutions has been enormously successful in addressing the problems of existence, uniqueness, and stability for a wide class of such equations.

A problem which has not been addressed with as much success is the construction of solutions. In some cases, exact solutions formulas exist, but for the most part, solutions must be found numerically. Viscosity solutions are the correct class of weak solutions for these types of PDE. Correspondingly,

## Abstracts :::

monotone methods are the correct class of numerical schemes.

We introduce a framework for building monotone schemes which converge to the viscosity solution. This framework allows explicit nonlinear finite difference schemes to be built. In particular we address how to handle: obstacle problems, degeneracy, and singularities in the solutions. We will discuss a number of geometric PDEs which can be solved using Wide Stencil finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, Mean Curvature, the Porous Medium Equation, and others.

In this introductory lecture, I'll show how these finite difference schemes can be constructed and implemented.

**Ken Stephenson: (U. of Tennessee)**

**Curvature Flow via Circle Packing**

**Abstract:** In “circle packing”, it helps to view the circles as imposing a geometry on combinatorial situations. Typically the combinatorics are triangulations and the geometry of “packed” circles is conformal in nature. But circle packing is very maleable:

- \* General combinatorics can be modified to triangulations -even, perhaps, preserving some initial geometry.
- \* Circles need not be “packed”: generic radii impose cone structures.
- \* We all know and understand circles!
- \* Software “CirclePack” permits sophisticated and open-ended experimentation, manipulation, and visualization of circle configurations.

The computation of circle packing radii itself has been modelled as a discrete Ricci flow by Bennett Chow and Feng Luo. Through serendipity, experiments have uncovered a more mysterious “curvature flow”. The talk is an invitation to consider this approach in studying surface parameterizations.

**Guoyi Xu: (U. of California at Irvine)**

**Short-time existence of the Ricci flow on noncompact Riemannian manifolds**

**Abstract:** In this talk, using the local Ricci flow, we prove the short-time existence of the Ricci flow on noncompact manifolds, whose Ricci curvature has global lower bound and sectional curvature has only local average integral bound. The short-time existence of the Ricci flow on noncompact manifolds was studied by Wan-Xiong Shi in 1990s, who required a point-wise bound of curvature tensors. As a corollary of our main theorem, we get the short-time existence part of Shi's theorem in this more general context.

## Workshop Participants :::

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# Vancouver :: UBC On Campus

## WEBSITES

- :: <http://www.ubc.ca/vancouver/about/services.html>
- :: <http://www.ubc.ca/about/maps.html>

WIRELESS INTERNET - Free wireless internet is available at the PIMS office (WMAX bldg) during breaks. Password required, signs will be posted in the office. Please abide by UBC internet use policy <http://www.universitycounsel.ubc.ca/files/2010/08/policy104df>

## FOOD

- :: in the Student Union Building - 6138 Student Union Boulevard
  - : The Gallery (Montreal-style bagels)
  - : The Pit Pub (quarter pound patties and burger specials everyday)
  - : Pie R Squared (pizza)
- :: in the University Village - corner of University Boulevard & Western Pkwy
  - : One More Sushi (located on the 2nd floor, this is a sushi restaurant)
  - : Curry Point (in the underground foodcourt you'll find many foods including curry)
  - : Pita Pit (healthy and filling wraps)

## RECREATIONAL ACTIVITIES, VISIT [HTTP://WWW.REC.UBC.CA/](http://WWW.REC.UBC.CA/)

- :: The Birdcoop Fitness Centre (fitness gym \$9 daily drop-in)
  - : 6000 Student Union Blvd. - in the Student Recreation Centre
- :: Thunderbird Arena (skating arena)
  - : 6066 Thunderbird Blvd.
- :: UBC Aquatic Centre (pool)
  - : 6121 University Blvd.
- :: Pacific Spirit Regional Park (hiking and biking trails)
  - : parking available at 4915 W. 16th ave.
- :: UBC Tennis Centre (indoor courts \$16/hr and outdoor courts \$5/hr)
  - : 6160 Thunderbird Blvd.
- :: UBC Botanical Garden (Canada's oldest university garden \$12 entrance fee)
  - : 6804 SW Marine Drive
- :: Norm Theatre (check the schedule <http://ubcfilmsociety.com/> \$5 admission)

## EMERGENCY/FIRE/SAFETY

- \*\*\* in an emergency dial 911 for Police, Fire, or Ambulance
- :: UBC Campus Security
  - : Emergency: 604-822-2222
  - : Non-Emergency: 604-822-8609

## HEALTH

- :: University Village Medical & Dental Clinic (open Mon-Fri 8am-6pm)
  - : 604-222-2273
  - : 228-2155 Allison Road

# Vancouver::: Off Campus

## TRANSPORTATION

- :: Vancouver Taxi (604-871-1111) or Yellow Cab (604-681-1111)
- :: Public Transit
  - : a single fare is valid for 90 minutes (hop on and off as you please)
  - : one zone - \$2.50, two zone - \$3.75, three zone - \$5.00
  - : [www.translink.ca](http://www.translink.ca)
  - : UBC Bus Loop is located at 1950 Wesbrook Mall

## TOURISM VANCOUVER

- :: visit [www.tourismvancouver.com](http://www.tourismvancouver.com) or call 1-800-663-6000

- :: Granville Island - bustling marketplaces with food and crafts
  - : <http://www.granvilleisland.bc.ca>



Photo: Tourism Vancouver

- :: Telus World of Science - omnimax theatre and lots to discover
  - : <http://www.scienceworld.bc.ca>



Photo: Tourism Vancouver

- :: Grouse Mountain - challenge yourself to a hike & enjoy the view
  - : <http://www.grousemountain.com/summer/>

- :: Chinatown - visit the weekend Night Market
  - : <http://www.vancouverchinatown.ca>

- :: Gastown - experience the birthplace of Vancouver
  - : <http://www.gastown.org>

- :: Robson Street - shop 'til you drop
  - : <http://www.robsonstreet.ca>



Photo: Tourism Vancouver

- :: Metropolis at Metrotown - shop, eat, and play
  - : <http://www.metropolis.shopping.ca>

- :: English Bay - visit the beach
  - : [http://en.wikipedia.org/wiki/English\\_Bay\\_\(Vancouver\)](http://en.wikipedia.org/wiki/English_Bay_(Vancouver))

- :: Vancouver Aquarium - over 70,000 amazing animals
  - : <http://www.vanaqua.org>

- :: Vancouver Public Library - pick up a book for some quiet time
  - : <http://www.vpl.ca>

# Organized activities

Wednesday July 13: 6pm

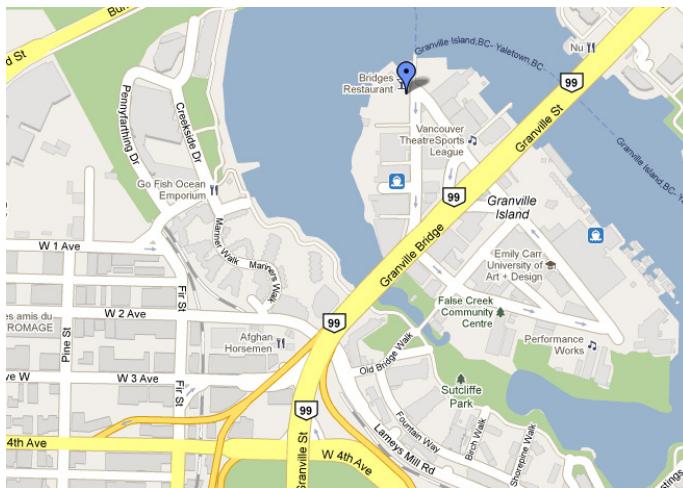
Registration and dinner reception in Mahony's Irish Pub, see UBC campus map for location.

Thursday, July 14: Dinner Cruise, boarding at 6:30pm

Our workshop banquet will take place on the M.V. Burrard Queen as we cruise around Vancouver Harbour. Boarding will begin at 6.30pm and the ship departs at 7.15pm (return around 10.30pm). Please don't be late, the ship doesn't wait!

The dock is located on the West side of Granville Island, to the left of Bridges' Restaurant. Granville Island is accessible by bus. Your workshop organiser will provide you with bus tickets. From UBC take either #84 (bay 14) from the North bus loop or #4 (bay 14) from the Trolley bus terminal. Ask the bus driver for the nearest stop to get off (4th avenue at Fir St). See UBC map for bus loop location.

Link to map: <http://maps.google.ca/maps/ms?ie=UTF8&hl=en&msa=0&ll=49.271949,-123.134723&spn=0.020805,0.038581&z=15&msid=200222965460381506379.0004a538d61370ed2d31d>



Allow 45 minutes for the bus + walk to the boat. Granville Island is a pleasant tourist area with indoor market, boutique shops, restaurants etc., to entertain you.

Friday July, 15: 6.30pm

Formal reception at the Sage Bistro, see UBC map for location.