Quantum computation with “classical” states

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The question of what resources, states, evolutions, etc. are useful for quantum computational speed-up is central to the study of quantum computation. This work – a collaboration with Matty Hoban, Joel Wallman, Hussain Anwar Naïri Usher, Robert Raussendorf and Hugo Cable – considers the quantum computations where “classical states”, mixed states with no entanglement or quantum discord, play a central role.

Consider an n-qubit state ρ. We shall say ρ is **classical** when it can be written:

$$\rho = U_L \sum_x p_x |x\rangle\langle x| U_L^\dagger$$

where the sum is over all n-bit strings x, $|x\rangle = |x_1\rangle|x_2\rangle \cdots |x_n\rangle$ is a tensor product of computational basis states determined by bitstring x and U_L is a local unitary. Defined in this way, such classical states have (trivially) no entanglement and no quantum discord with respect to any bi-partitions.

In my talk, I will report two recent and contrasting results considering the utility of such states for non-classical computation. First [1] I will report strong evidence that classical states (and thus probability distributions) exist which, in spite of their classical nature, can be resources for non-classical computation. I will present two such resource models, an oracle model, and a measurement-based quantum computation, which use such states to perform computations which can be described we simple families of quantum circuits, and show how prior theorems and methods by Aaronson [2] and Bremner, Shepherd and Josza [3] can be exploited to show that such computations are unlikely to be exactly realisable (via sampling) on a classical device.

If time permits, I will then present recent work [4] considering the computational power of unitary circuit model computations, where, after each gate in the computation, the qubit register is in a classical state. Eastin showed that for a sub-class of such computations [5] (where initial state is in a product distribution, and all gates act on at most two qubits) efficient classical sampling simulation of the circuit could be achieved. We provide a simpler and (with caveats) more general simulation algorithm, which generalises (with caveats) Eastin's result to a broader family of circuits.

These complementary results show that the resource requirements for quantum speed up can depend strongly on the model of computation.

[1] Matty J. Hoban, Joel J. Wallman, Hussain Anwar, Naïri Usher, Robert Raussendorf, Dan E. Browne, arXiv:1304.2667.
[2] S. Aaronson, *Proc. R. Soc. A* **461**, 3473 (2005).
[3] M. J. Bremner, R. Jozsa, and D. J. Shepherd, *Proc. R. Soc. A* **467**, 459 (2011).
[4] H. Cable and D. E. Browne, in preparation.
[5] B. Eastin, arXiv:1006.4402.