Changing the Culture 2007

Workshop 3 – Report/Summary
Rethinking Precalculus Mathematics

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Abstract: *Do our Precalculus courses (at universities, colleges and high schools) prepare students for Calculus? What are concepts and skills that are essential for students' understanding of Calculus, and how can we test whether students do have them? How can we deal with students' lack of these skills in our Calculus courses? How can we design better Precalculus courses?*

A large number of students just don't seem prepared for the challenges of Calculus; this includes the ones who have just completed a Precalculus course. The goal of this workshop was to investigate the questions in the abstract and have participants provide possible answers. In order to facilitate discussions the participants were split up into small groups and asked the two following questions.

1. What are the most important mathematical concepts students should bring to your course (Math12, Precalculus, Calculus)?
2. What are the greatest organizational impediments to students' success in mathematics when they move from secondary to post-secondary studies?

After having an hour to discuss these questions all groups then came together and presented their findings to all participants. The following is a summary of what was presented. Also included are additional comments we received from participants after the conference.

Question 1: What are the most important mathematical concepts students should bring to your course (Math12, Precalculus, Calculus)?

- Arithmetic and Algebra:
    - The ability to work with and manipulate fractions without calculators.
    - Understanding basic arithmetic operations at both a practical and conceptual level.
    - Zero product: ab=0 implies a=0 or b=0.
    - Factoring – integers, polynomials
    - Solving equations:
        - Knowing what it means to solving an equation (or systems of equations), and being able to find such solutions.
        - Being able to solve systems of two equations in two unknowns; knowing that the solution is the intersection of the lines.
        - Knowing that a linear equation in 3 variables represents a plane; knowing that the solution of a systems of such equations is the intersection of planes.
        - Knowing when no / one / infinitely many solutions exist geometrically.

        - Knowing that non-linear equations could have more than one solution.
        - Absolute value equations and inequalities: for example, being able to solve $|x-2|<|3x-4|$.
- Functions:
    - A very clear understanding of what a function is.
    - Function notation (don't always use f and x – change it up a bit!)
    - Representations: verbal, formula, table, graph – ability to go between different representations.
    - Arithmetic combinations of functions, composition, inverse.
    - A good mental picture of a function such as a "function machine" is useful.
    - Ability to graph (without a calculator) – in particular, graphing polynomials and rational functions.
    - Knowledge of basic functions (the elementary functions) used in simple modeling, and that other esoteric functions (Bessel, etc) exist.
- Modeling:
    - given a problem can they represent algebraically by a function or equation (language of math).
- Proofs & Logic:
    - Understanding why theorems are true.
    - Understanding what it means to "prove" a statement is true.
    - Math 11 – this year they cut logic/proof sections, Venn diagrams, indirect proof, logic statements – these are important for understanding "why".
- Trigonometry:
    - Unit circle definition of sine and cosine as Cartesian coordinates of points on circumference.
    - Understanding the difference between right-angle and unit-circle definitions of trigonometric functions.
    - Understanding of basic trigonometric identities (i.e. Pythogorus)
- Problem Solving:
    - The ability to read and understand a problem. Furthermore, the ability to write a solution so that full understanding is shown. The ability to show work and justify results. The ability to reflect on the answer and determine if it is reasonable.
    - More highly developed checking/verification skills and strategies.
    - Students tend to give up to soon and rely on others to show them.
    - Lack of real word applications
    - The ability to generalize or specialize results.
- Writing Skills:
    - More emphasis on mathematical notation. (mathematical language skills)
    - Proper use of notation - students should not abuse the equal sign.
    - Writing should be neat and kept within margins.
- Study Skills (Learning for Understanding):
    - Better approach to studying/learning (students approach is based on memorizing).
    - The ability to reflect on ones work.
    - Understanding of connections between concepts:
        - One definition per topic – some concepts have multiple definitions, i.e. right-angle vs. unit circle definition of trigonometric functions.

        - Understanding vs. memorization vs. application
        - They think each section represents a different concept – students don't think there is a relationship between topics.
- Math Textbook Resources:
    - A math textbook should not be treated as an "exercise book".
    - Students need to improve their ability to read the textbook. They need to have a pencil and paper handy to fill in details and work through examples.
    - Emphasis on "reading for understanding". Reading a math textbook is much different than reading a novel: it takes much longer to read one page of a math textbook.
    - Leading questions – not really problem solving.
- Student Misconceptions:
    - "People who are good at math see the solution quickly." – teachers don't model the process of approaching a problem.
    - "It's ok to be bad at math." – this is not acceptable
    - "To be good at math you need to memorize formulas." – completely untrue, understanding is much more important than memorization
    - Students don't realize that Mathematics, by its very nature, is cumulative. Any difficulties they have with the material will not go away unless they take the time to get help.
- Calculators promote lack of numeration.
- Notion of infinity.

Question 2: What are the greatest organizational impediments to students' success in mathematics when they move from secondary to post-secondary studies?

- Assessment:
    - In post secondary they have only two midterms and one exam – insufficient feedback on their knowledge, students need to learn how to evaluate themselves.
    - Provincial exams – there is an issue with multiple choice test taking skills
    - Process is more important than answer – students have a hard time grasping this when in high school the final answer is the only thing marked.
    - More assessment with problems that students have not already seen.
- Student Responsibility:
    - Self assessment of student knowledge – Are teachers holding their hands too much and they leave high school unable to cope with too much independence?
    - Seek help, ask "why" – responsibility is on student not teacher
    - Students lack the ability to reflect on homework assignments. They need to be taught how to effectively reflect on their work.
    - Lack of questions and reflections in lectures.
    - Independent learning
    - Higher level courses require more effort – homework completion is essential
- Teacher Responsibility:
    - Teachers have to redesign the presentation of the curriculum to overcome its deficiencies.
    - Control pace by choosing how much can be taught in the allotted time.

- Suggestions:
  - Calculus should focus less on going over prerequisite material and more on new material.
  - Online homework questions will allow for more questions to be marked and provide students with immediate feedback. Could also be used to encourage students to review material of previous lecture by getting them to answer a question online before the next lecture.
  - At the end of the lecture students submit index cards stating concepts they struggled with.
  - Encourage students to reflect on their work by having them submit corrections to their homework assignments.