

## First Year at Banff International Research Station

Contributed by Robert Moody, BIRS Scientific Director

I am sitting in the beautiful westward-facing dining room at the Banff Centre, watching the sun set gloriously into the Rocky Mountains. Even with it all in front of me, it is still hard to grasp the reality that the mathematicians of Western Canada have been so fortunate to have created a facility like BIRS in such a magnificent location. Not only is the setting exhilarating, but also BIRS itself has proved to be a congenial and well-ap-

pointed facility for the participants who have come here from every part of the world.

As you read this BIRS will have held on the order of 37 five-day workshops as well as some 20 research in teams, focused research groups, and two-day workshops involving well over 1200 participants. All reports indicate that participants and organizers love BIRS. Mountain air, a beautiful setting, modern and well-equipped facilities, good food, a friendly staff, all in a semi-isolated setting do wonders to put people into a creative and heightened frame of mind! To quote a few samples of comments that we have received:

*"This is BY FAR the most pleasant, satisfying and rewarding experience I have EVER had as an organizer of a conference, but most importantly I am very happy to*



Participants of the BIRS Creative Writing Workshop which ran from August 31–September 4, 2003.

*report that the meeting was a resounding scientific success! The participants spent a great deal of time in scientific discussions during the free times, and I saw people starting to collaborate on some of the open questions raised during the workshop. I am really thrilled about this ... I give BIRS an unqualified A+!" - NK*

*see First Year at BIRS on page 4*

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## New PIMS Director Appointed

The Pacific Institute for the Mathematical Sciences is pleased to announce the appointment of Professor Ivar Ekeland as Director for a period of 5 years starting September 1, 2003.

As Director of PIMS, Ivar Ekeland will also chair the Executive Committee of the Banff International Research Station (BIRS). He will also sit on the Steering Committee of the MITACS Network of Centres of Excellence, on the Scientific Board of the Atlantic Association for Research in the Mathematical Sci-

ences (AARMS), and on the Oversight Committee of the National Program on Complex Data Structures (NPCDS).

Professor Ivar Ekeland, who is presently Canada's Research Chair in Mathematical Economics at the University of British Columbia, brings to the institute a strong academic standing, excellent administrative and leadership experience, and a far-reaching international network of scientists.

*see New Director on page 5*

## Director's Notes

This newsletter serves as a reminder of everything the PIMS community has accomplished since Nassif Ghoussoub and the early PIMS supporters first conceived the idea of a distributed institute connecting science and industry through mathematics. There is now so much activity going on, and it has attracted many researchers to this part of the world. But there also remains so much to be done.

We must maintain the pioneer spirit that is so apparent in endeavours such as the *Industrial Problem Solving Workshop*, *Pi in the Sky*, and the *Banff International Research Station*. We have to ensure that the initial enthusiasm does not fade, and that the early expectations are fulfilled. We hope the Banff Station, for instance, will become an international reference so that the best mathematicians in the world will come to Western Canada on a regular basis, and our institute will become the hub

of a major international scientific network. We want the quality of our scientific meetings and periods of concentration to continually rise and thereby promote training of younger researchers and encourage creativity of older ones: there must be substantial challenges if we aim for high accomplishments. We want our connections with industry to expand, because this is where many of our graduates will find jobs, and because this is where many interesting mathematical problems come from. We want our educational programmes and *Pi in the Sky* to capitalize on their experience and continue to improve in quality, so that younger generations and the public at large will become more familiar with mathematics and more curious about science.

As more scientists are attracted to PIMS, the range of disciplines we cover will expand, and so will our industrial connections. The



*Ivar Ekeland, PIMS Director*

structure of PIMS will change to accommodate these needs. UBC has created a position of Associate Director of PIMS within the Mathematics Department which we expect to fill by the summer of 2004. Until then, Manfred Trummer has agreed to serve as Deputy Director. The Board of Directors of PIMS is being recomposed, and we hope that the University of Washington will seize the opportunity to strengthen its formal ties to the organization. The mathematicians at UW have submitted a VIGRE proposal to NSF in which PIMS is heavily involved. If successful, this proposal would provide a sound financial basis for the cross-border exchange of students and researchers.

In addition to the governing changes, building renovations will begin. UBC has agreed to provide more space to the PIMS Central Office. The new facility should be open by May 2004, in time for the 8<sup>th</sup> Industrial Problem Solving Workshop.

Visiting the various sites, I have noticed broadening interest in PIMS from previously unreached scientific communities, such as economics and health sciences. Here we find new opportunities as well, which we must be ready to seize. Doing all this while keeping up the momentum on earlier initiatives, such as the Postdoctoral Fellowships, the Banff Station, and the Collaborative Research Groups, is a huge challenge. PIMS will rise to it, as it has done in the past. It will be a proper and fitting tribute to the vision and dedication of Nassif and all those who have made PIMS what it is today.



*The staff from the PIMS sites together for a training meeting at UBC in November 2003*



# New PIMS Deputy Director and Site Directors

**Manfred Trummer** from Simon Fraser University is PIMS Deputy Director for a one-year term until August 31, 2004. The Deputy Director assists the PIMS Director with the development and implementation of scientific policies and the adjudication process of the scientific programmes, and supervises the organization of scientific activities and preparation of grant proposals. The position is the main scientific liaison between the institute and the PIMS community, and works closely with the coordinators and leaders of the Collaborative Research Groups. The Deputy Director also oversees the various aspects of communication of the institute, including traditional and electronic publishing.

Manfred has been PIMS site director at SFU for over two years. He has been involved in a number of PIMS events and activities such as the Pacific Northwest Numerical Analysis Seminar series, the Graduate Industrial Mathematics Modelling Camp, the IAM-CSC-PIMS Senior Undergraduate Math Modelling Workshop, and the PIMS-sponsored SciCADE 2001 conference.

Manfred Trummer has been Associate Professor of Mathematics and Computing Science at SFU since 1989. Manfred hails from Austria. He received his Ph.D. (Dr.sc.math.) from ETH Zürich. He has held academic appointments at the University of North Carolina,



*Manfred Trummer, PIMS Deputy Director and SFU Site Director*

University of British Columbia, Massachusetts Institute of Technology, Graz (Austria), Zürich (Switzerland), and Auckland (New Zealand). His area of research is numerical analysis and scientific computing; his current interests are adaptive and high-order methods for numerically solving differential equations.

**David Leeming** was appointed PIMS Site Director for the University of Victoria on July 1, 2003. Since 1998 he had been PIMS Site Deputy Director and Education Coordinator. He is acting for **Chris Bose** who will start his term on July 1, 2004.

David received his Masters degree from the University of Oregon and his PhD from the University of Alberta. Since 1969 he has been



*David Leeming, PIMS Site Director, U. Victoria*

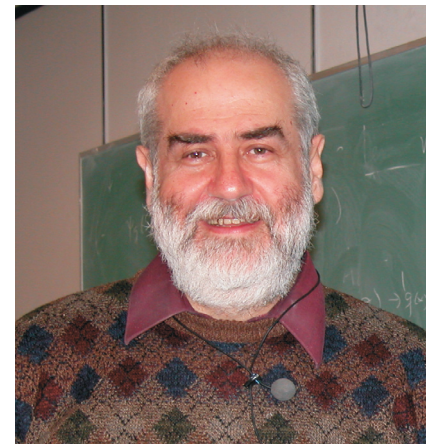
at the University of Victoria. He was promoted to Professor in 1986 and became Department Chair from 1989–94.

His research interests are in approximation theory, particularly in rational approximation. He has also published several papers on the Euler and Bernoulli polynomials.

David has a long interest in articulation and student transferability. Since 2000 he has been on the Council of the B.C. Council on Admissions and Transfer.

**Gunther Uhlmann** was recently appointed PIMS Site Director for the University of Washington.

Gunther Uhlmann received his PhD in 1976 at MIT under the direction of Victor Guillemin. He held postdoctoral positions at Harvard, the Courant Institute and MIT. In 1980 he became Assistant Professor at MIT and in 1985 he



*Gunther Uhlmann, PIMS Site Director, U. Washington*

moved to the University of Washington as an Associate Professor. He was promoted to Full Professor in 1987.

Uhlmann was awarded the Annual National Prize of Venezuela in Mathematics in 1982. He received the Alfred P. Sloan Research fellowship in 1984 and a John Simon Guggenheim fellowship in 2001. He was named Corresponding member of the Chilean Academy of Sciences in 2001. He has given numerous lectures throughout the world included an invited address at the Portland meeting of the AMS in 1991, the CBMS-NSF lectures on *Inverse Problems and Non-Destructive Evaluation* in 1995, an invited lecture at the International Congress of Mathematicians in Berlin in 1998 and the PIMS Distinguished Chair Lectures at UBC in 2002.

His current interest is inverse problems, in particular inverse boundary value problems and inverse scattering problems. In these problems one attempts to determine internal parameters of a medium by making measurements at the boundary of the medium or by remote observations.

The position of PIMS new Site Director at UBC will be taken over by the Deputy Director, once he/she has been appointed. Until then the duties have been taken over by the Director. Thank you **Dale Rolfsen** for fulfilling this role from 1997 to 2003 and contributing to the success of PIMS at UBC.

# First Year at BIRS

*continued from page 1*

*“Staff of BIRS and Banff were exceptionally friendly, extra competent and efficient and run a totally excellent facility. By far this is the most useful, pleasant and memorable meeting I have ever attended. Thanks!”* - EB

*“Loved the ambiance and setting of The Banff Centre — [it] made it possible to wholly concentrate on learning as much as I possibly could. I am very grateful to have had the opportunity to come here.”* - TR

*“The staff at BIRS was outstanding in helping the participants. This meeting was probably the best I have attended in my whole career!”* - EJ

*“All of the people with whom I spoke were truly very happy at BIRS and they enjoyed the workshop very much. The workshop was very active and productive. Also the weather was exceptionally nice, which contributed to the good spirits at the workshop. I want to thank you once again for having provided such outstanding conditions wherein fellow mathematicians may gather together from all over the world, and work together creating many new and stimulating mathematical ideas, leading to still more exciting research work. Certainly BIRS is helping mathematicians succeed in very important ways with their research ideas, goals and innovations.”* - JM

BIRS is first and foremost an international facility for five-day workshops. A glimpse at the activities for 2003 and 2004 on the BIRS website or on the two beautiful posters that Heather Jenkins at PIMS has made for us gives one an accurate indication of the depth and breadth of the workshops that organizers are creating at BIRS. Some of the most suc-

cessful workshops here have been those that have brought different disciplines or sub-disciplines together around some new developments in mathematics or science. Here the “work together, eat together, sleep together” idea on which BIRS is founded functions at its best. Scientists have the time to talk, to cross scientific language and conceptual barriers, and to form new collaborations. For example, the workshop on fuel cells brought biologists, chemists, engineers, workers from industry, as well as mathematicians together on the very real and difficult scientific and technical problems of hydrogen fuel cell design.

However it is interesting to see how BIRS can go beyond this and in doing so serves the Canadian and western Canadian mathematical community in unique ways. First, BIRS hosted the Canadian Mathematical Olympiad Team for about 10 days, just before their highly successful competition in Japan. It was wonderful to see these potential future mathematicians in full mathematical form, totally absorbed from morning until night, and beyond, in intense discussion and problem solving.

The five-day format at BIRS leaves two days per week open, and although it is hard on our ever ready staff, using these two day periods for short workshops has proven quite

popular. Examples:

1. A two-day workshop on mathematics fairs in Alberta. The workshop was organized to train mathematics teachers in Alberta how to run successful elementary school Math Fairs. We plan to repeat this workshop next year, expanding the boundaries to include British Columbia teachers as well.
2. A Northwest Functional Analysis Symposium. This was a two-day workshop which gathered together 40 of the analysts of the PIMS universities.
3. The annual Theoretical Physics meeting of the University of Alberta, which was the most successful ever. The group immediately applied to come back next year.
4. A two-day workshop on SARS. This was an example of how MITACS, PIMS, and Health Canada were able to move quickly on an important topical issue — the modeling and analysis of transmission dynamics of SARS — to bring in experts from Australia, Canada, China, the United Kingdom, and the USA for a very successful (and intense) two days.
5. The Mathematics Chairs of Canadian Universities 2003 annual meeting.

Yet there is another dimension to BIRS that we have only just begun to explore. BIRS is located in one of the world's premiere centres for creative arts. For over fifty years,

the Banff Centre has been a cross-roads and spiritual home for many thousands of artists from around the world — in music, stage, visual arts, writing and literature, multi-media, and mountain culture. Needless to say, our participants have been quick to take up the cultural opportunities afforded by concerts, displays, and even a new opera.

However BIRS and the Banff Centre clearly have the opportunity to do something far more creative here. We can look for ways to work together in bringing some reality to the often stated



*The participants of the BIRS Stochastic Partial Differential Equations Workshop which took place on September 28–October 3, 2003.*



dream of a renaissance of the arts/science dialogue.

As a start BIRS hosted a five day half workshop on Creative Scientific Writing. We had no idea how this would work out, but Chandler Davis (U. Toronto) and Marjorie Senechal (Smith College) made it into a marvellously stimulating and creative event. The next workshop in the series will be in April 2004, and we are hoping to create a real Banff Centre-BIRS combined workshop in 2005. Also in 2005 we plan to have a four day math-arts conference including one day set aside as a tribute to the late Donald Coxeter and his extensive interaction with the visual arts.

BIRS has also teamed up with the Banff Centre to produce a new public lecture series featuring distinguished scientists, writers, and academics addressing ideas, issues, and personalities in the world of the mathematical sciences as they relate to our times, culture, and society. The first lecture in the series, *The Wings of Madness*, was given by the author **Paul Hoffman**, who is well known for his biography of Paul Erdos: *The Man Who Loved Only Numbers*. The next lecture, on *The Art and Mathematics of Star Polygons*, was by **Reza Sarhangi**, the creator of the very successful *Bridges Conferences* that are designed around connections between mathematics and art, music, and science.

This type of activity will take time and imagination to build. There are no established formulae for creating meaningful arts/science interactions. But I sense a strong desire for it and much willingness from the artists and leaders at the Banff Centre to find ways to make it happen. Nurtured properly, the BIRS-Banff Centre connection has the potential to create in Banff a leading centre for this bridging of cultures.

As you know, BIRS is the result of the initiative of PIMS along with the MSRI and MITACS. Its funding comes from the two national science funding agencies in Canada and the United States, NSERC and the NSF, and especially from the Alberta Science and Research Authority, which not only contributes to the annual funding of BIRS, but also contributed extensively to renovating the physical facility in which BIRS operates in the Banff Centre. PIMS itself has made and continues to

make a large commitment to BIRS in terms of both equipping the facility and funding its day to day operations. This includes the on-site staff of Andrea Lundquist and her two part-time assistants, and Brent Kearney, our system administrator and technical support person. PIMS also provides all the amenities for the morning and afternoon tea/coffee breaks, the costs of breakfast supplements, the costs of the office equipment and supplies, and the travel expenses of the Scientific Director. It is a very significant contribution by our community.

As it stands, BIRS has funding until the end of 2005.

This will be my last Newsletter report to you as the Scientific Director of BIRS: I will step down at the end of December 2003. A number of people have expressed surprise — after all, BIRS has only just started! Certainly in the sense of actually operating workshops this is true. However, the idea for creating BIRS began to really take shape in December 2000, so it has been three (rather exciting!) years that I have been involved with the project. In November 2003 we will make the workshop selections for the 2005 programme year, and that will complete the third successive year of overseeing the programme selection process.

BIRS is up and running, and it works. It has already become a leading international centre for workshops in the mathematical sciences. Its potential to become something beyond anything that currently exists in the world is very real. Of course there are things to improve and important decisions to be made: how much library access is required, what about expanding the programme year to 44 or 48 weeks, what are the appropriate activities for BIRS to pursue, how much can PIMS afford to put into the running of BIRS (every single event that runs at BIRS and every single person who stays at BIRS costs PIMS money), what about other organizations that want to partner with PIMS on BIRS (as MSRI and MITACS have), what is the appropriate amount of attention for the arts-BIRS connections?

Whatever the answers to these questions may be, I want to urge the PIMS community to treasure this new facility and put time and energy into its securing its future. We are incredibly fortunate to have BIRS.

## New Director

*continued from page 1*

Ivar Ekeland is an outstanding mathematical scientist, an internationally renowned mathematical economist, a dedicated educator, a prolific writer and disseminator of science, and a first class administrator. Ekeland has received prizes from the French Academy of Sciences, the French Mathematical Society, and the Belgian Academy of Sciences. He is a foreign member of the Norwegian Academy of Sciences and he holds honorary doctorates from UBC and from the University of Saint-Petersburg for Economics and Finance.

As a former President of Universite Paris-Dauphine, he has extensive experience in leading a large multi-faceted institution and in negotiating support from different levels of government. As former Department Chair, and as Director of the research centres CEREMADE and Institute Finance-Dauphine, he has provided scientific leadership for a large and diverse group of mathematical scientists.

Dr. Ekeland has supervised over 35 doctoral dissertations, has organized numerous symposia and has written numerous monographs on Convex analysis, Control theory, Game theory, Catastrophe theory, and Hamiltonian Mechanics. He is the founding editor of the “Annales de l’Institut Henri Poincaré-Analyse nonlineaire” and he sits on the editorial board of many other publications.

Ekeland has also written several books which are reflections on, or popularization of, mathematics. All of them have been translated into many languages. For these contributions, Dr Ekeland was awarded the “Prix Jean Rostand” by the Association des Ecrivains Scientifiques de France and the “Prix d’Alembert” by the Societe Mathematique de France. Ekeland is also a regular contributor to the journal “Nature” as well as to the magazine “Pour la Science”.

*Taken from a Press Release by Nassif Ghoussoub. Please see [www.pims.math.ca/whatsnew/ekelanddirector.html](http://www.pims.math.ca/whatsnew/ekelanddirector.html).*

# Nassif Ghoussoub's Farewell Speech at UBC

June 27, 2003

“Dear friends,

“As you can see all my family is here today, but you may not know why. They wanted to make sure that I am really quitting. Joseph is expecting me to play Lego with him. “This is Mathematics, Daddy!” Mireille is hoping that I can do something about her B in English, still not suspecting that her English is already better than mine. However, Michelle came here seeking assurances that once the new director is announced, the Board would make sure that he is not criticizing Harry Potter in the media. And the truth about Louise is that she is dreading it. Imagine all this energy now totally re-focused on gardening, dish-washing and answering the kids’ phone.

“Four years ago, I took my daughters to the first PIMS Elementary Math Contest held here in Buchanan. Klaus Hoechsmann was trying to shout instructions to the 300 plus kids (5–7 graders) and their parents. Then he asked, what is the organization behind this event? They all shouted PIMS. Then he asked: “What does PIMS stand for?” And my daughter Michelle looked at me and said, “Well, that’s easy. Even I know that!” Well Michelle, I have news for you. There is much more to know about PIMS.

“PIMS has a community spirit that arose in defiance to outdated beliefs and often flawed perceptions about mathematical research and education. PIMS is this dramatic change in attitude and this new state of mind of the Canadian mathematical community. PIMS is about thinking big, about thinking globally, about being a major partner in the country’s global R&D effort, and PIMS is about Canada playing a leading role on an international level.

“PIMS is about the early believers, Ed Perkins (Uncle Ed), Arvind Gupta, Claude Laflamme, Nicole Tomczak-Jaegermann, Reinhard Illner, Robert Miura ... the pioneers who gave selflessly when the light was not in sight and the dark tunnel was too long to bear.

“PIMS reflects the excellence of our world class researchers managing its scientific panels: David Boyd, Gordon Slade, Bob Moody, David Brydges, Ian Putnam, Bob Russell, and Hugh Williams, among many others.

“PIMS is about the dedication of Klaus Hoechsmann developing the poster campaigns for the Vancouver buses, the efforts of Joshua Keshet for the PIMS Elementary Math Contests, those of Wieslaw Krawcewicz and his colleagues on Pi in the Sky, those of Malgorzata Dubiel for the “Changing the Culture” conferences, those of David Leeming for the Math Mania, and those of Ted Lewis and Jim Muldowney for the Math Fairs.

“PIMS is the commitment of Anthony Peirce, Bernie Shizgal and Manfred Trummer to the Senior Undergraduate Modelling Camps. It is the volunteer work of Rachel Kuske, Chris Bose, Ian Frigaard, Randy LeVeque and so many others for the Industrial Problem Solving Workshops.

“The PIMS story has also been written by the organisers of thematic programmes who brought the scientific world to our door: Ed Perkins, Martin Barlow, Changfeng Gui, Gordon Semenoff, Arturo Pianzola, Bryant Moodie, Nicole Tomczak, Pavol Hell, Robert Miura, Leah Keshet, Gunther Uhlmann and so many others.

“PIMS is about the selfless efforts of Gary Margrave, Mark Lewis, Chen Greif, and Bruce Sutherland who developed the international summer schools not only for our graduate students but for the world’s students.

“PIMS is about the spirit of collaboration between our departments and institutes: Uli Haussmann, George Bluman, Brian Marcus and Rabab Ward at UBC, Sherm Riemenschneider and Akbar Rhemtulla at UA, Alistair Lachlan at SFU and John Phillips and Nigel Horspool at U. Victoria, among others.

“PIMS is also about breaking geographic barriers with the support of so many great friends and outstanding scientists from UW: Chris Burdzy, Doug Lind, Tatiana Toro, Gunther Uhlmann, Ron Irving, Selim Tuncel and of course Don Marshall who started it all and who is with us here today.

“PIMS is about the positive energy and generosity of Ken Foxcroft, Hugh Morris, and others in the private sector who honour us and stimulate us with their continuous support.

“PIMS lives by the efforts of its staff: a great and loyal bunch who are extremely committed to the vision. Sandy Rutherford, Andrea Hook, Heather Jenkins, Clarina Chan, Derek Bideshi, Caitlin Shepard, Shannon Perkins, Fanny Lui, Shervin Teymouri, Olga German, Dil Bains, Marian Miles and Shirley Mitchell. The BIRS staff: Andrea Lundquist, Amanda Kanuka, and Brent Kearney. The MITACS staff so ably represented by the great Olga Stachova. And we shall not forget the early support from the staff at Math: Mary-Margaret Daisley and Ann Artuso: Do you remember, Ann, the horrible and forgettable transcripts that Ed used to ask from you? I never did that to my staff.

“PIMS is about the leadership of Arvind Gupta, first as PIMS Deputy Director and now as leader of the MITACS Network. It is about the diligent work of Alan Mackworth on the MITACS Research Management Committee, about Leah Keshet, Uli Haussmann, Brian Wetton, Raymond Ng, Marty Puterman, Binay Bhattacharya, Mike Kouritzin, Jack Tuszynski, Gary Margrave, Michael Lamoureux and Richard Cleve



*Nassif Ghoussoub,  
PIMS Director 1996–2003*



who are aptly leading their challenging MITACS team projects.

“PIMS tells us a bit more about our universities and their leaders like Dick Peter, Mike Boorman, Martin Taylor, Indira Samarasekera, Colin Jones and Barry McBride, among others, who give senior administrators a good name and whose example we are determined to follow. Dick: Things have not been as fun since you stepped down from the Deanship at UA. And Barry: Let me confess that we are dreading the moment you will be leaving us because UBC will never be the same without your leadership.

“Let me conclude with a special tribute to three very special people who've had a tremendous impact on my own professional and personal life: Ed Perkins, Arvind Gupta, and Bob Moody. Without treasures like these, PIMS, MITACS and BIRS would never have become the great institutions they are today. Leadership, commitment, integrity and, above all, generosity. They gave their time and they gave their best. They gave me friendship and comfort. I couldn't have asked for a better deal from Providence.

“Let me also say that the last seven years should be remembered as a period where the community learned to look outward with confidence, to put forward bold visions and to lead. But allow me to give our young generation a healthy dose of paranoia. The hard-earned gains of the last few years need to be protected. So we need you, all of you, young colleagues to be vigilant, to step forward to positions of leadership, to stop those tempted to turn back the clock, and to keep the dream alive for our discipline, and for Canada's future.

“Thank you all. My life has been greatly enriched by your presence and by your friendship.”



*Clockwise from top left: Barry McBride, Bob Moody, Dick Peter, Dale Rolfsen and Hugh Morris*

## PIMS Postdoctoral Fellowships

### Call for Nominations

PIMS invites applications for Postdoctoral Fellowships from outstanding young researchers in the mathematical sciences for the year 2004-2005.

Applicants must be nominated by one or more scientists affiliated with PIMS or by a Department (or Departments) affiliated with PIMS. The fellowships are intended to supplement support made available through such a sponsor. The Institute expects to support up to 20 fellowships tenable at any of its Canadian member universities: Simon Fraser University, the University of Alberta, the University of British Columbia, the University of Calgary, and the University of Victoria, as well as the affiliated universities: the University of Northern British Columbia and the University of Lethbridge.

For the 2004–2005 competition, the amount of the award is \$20,000 and the sponsor(s) is (are) required to provide additional funds to finance a minimum stipend of \$40,000 (including benefits). Award decisions are made by the PIMS PDF Review Panel based on excellence of the candidate, potential for participation in PIMS programs and potential for involvement with PIMS partners. PIMS Postdoctoral Fellows will be expected to participate in all PIMS activities related to the Fellow's area of expertise and will be encouraged to spend time at other sites. To ensure that PIMS Postdoctoral Fellows are able to participate fully in Institute activities, they may not teach more than one single-term course per year.

Applicants must have a Ph.D. or equivalent (or expect to receive a Ph.D. by December 31 2004) and must have received their Ph.D. after January 1, 2001. The fellowship may be taken up at any time between April 1, 2004 and January 1, 2005. The fellowship is for one year and is renewable for a maximum of one additional year. Applications should include curriculum vitae, statement of research interests, three letters of reference (including one from a sponsoring scientist), and statement of anticipated support from the sponsor.

Nominations must be received by **February 6, 2004**.

Please see [www.pims.math.ca/opportunities/pdf.html](http://www.pims.math.ca/opportunities/pdf.html).

# A Glimpse at Upcoming Science Activities

## Seminar on Stochastic Processes 2004

University of British Columbia,  
May 20–22, 2004

There will be five invited speakers: **Rene Carmona** (Princeton), **Robert Dalang** (EPF Lausanne), **Alice Guionnet** (Ecole Normale Supérieure de Lyon), **Yves Le Jan** (Orsay) and **Balint Virag** (U. Toronto).

The local organizers are **Martin Barlow**, **Vlada Limic**, **Alexander Holroyd** and **Ed Perkins** (UBC).

For more information see <http://www.pims.math.ca/science/2004/ssp>.

## 10<sup>th</sup> International Workshop on Non-Monotonic Reasoning Delta Whistler Resort Hotel, Whistler BC, June 6–8, 2004

This is the 10<sup>th</sup> workshop in the NMR series. Its aim is to bring together active researchers in the broad area of nonmonotonic reason-



ing, including belief revision, reasoning about actions, planning, logic programming, causality, probabilistic and possibilistic approaches to KR, and other related topics. Workshop activities will include invited talks, tutorials, presentations of technical papers and special sessions.

NMR 2004 will be composed of six specialised subworkshops:

- *Foundations of Nonmonotonic Reasoning*
- *Computational Aspects of Nonmonotonic Reasoning*
- *Action and Causality*
- *Belief Change*
- *Uncertainty Frameworks*
- *Argument, Dialogue and Decision*

As well, this year there will be a session for

demonstrations of implemented NMR systems.

As part of the program of the 10th workshop, we will assess the status of the field after the 25 years since its inception, and discuss issues such as:

- significant recent achievements in the theory and automation of NMR
- critical short and long term goals for NMR
- emerging new research directions in NMR
- practical applications of NMR
- significance of NMR to knowledge representation and AI in general

NMR 2004 now welcomes the submission of papers broadly centred on issues and research in nonmonotonic reasoning. Papers of both a theoretical or practical nature are welcome. For more information please see [www.pims.math.ca/science/2004/NMR/](http://www.pims.math.ca/science/2004/NMR/).

## 16<sup>th</sup> Annual International Conference on Formal Power Series and Algebraic Combinatorics University of British Columbia June 28–July 2, 2004

*Contributed by Julian West, Malaspina and U. Victoria*

This conference will cover all aspects of combinatorics and their relations with other parts of mathematics, physics, computer science and biology.

The conference will include invited lectures, contributed presentations, poster session, problem session and software demonstrations. As usual there will be no parallel sessions. The invited speakers are:

- Norman Biggs** (London School of Economics)  
**Louis Billera** (Cornell U.)  
**Sara Billey** (U. Washington)  
**Takayuki Hibi** (Osaka U.)  
**Allen Knutson** (UC, Berkeley)



**Jean-Louis Loday** (CNRS, Paris)  
**Robin Pemantle** (Pennsylvania)  
**Anne Schilling** (UC, Davis)  
**Gordon Slade** (UBC)

For papers and posters, authors are invited to submit extended abstracts of at most twelve pages by November 22, 2003, via the conference web site [www.pims.math.ca/fpsac/](http://www.pims.math.ca/fpsac/).

For the first time at FPSAC, an award will be made for the best paper submitted by a graduate student. Students submitting extended abstracts are invited to indicate their eligibility for this award.

Limited funds are available for partial support of participants, in particular for students and junior scientists.

Before April 1, 2004 the registration fee is \$350 CAD (students \$175 CAD), and on or after April 1, 2004 it is \$450 CAD (students \$225 CAD).

This conference is being organized by **Julian West** (U. Victoria, chair), **Nantel Bergeron** (CRC at York), **Marni Mishna** (UQAM), **Tom Roby** (Cal State, Hayward), **Frank Ruskey** (U. Victoria) and **Stephanie van Willigenburg** (UBC). The chair of the scientific committee is **Jonathan Borwein** (SFU).

For more information please see: <http://www.pims.math.ca/fpsac/>.

## 2<sup>nd</sup> International Conference on Permutation Patterns Malaspina University-College, Nanaimo BC July 5–9, 2004

The unifying theme of the conference is permutation patterns. The topics addressed will include enumeration questions, excluded pattern questions, study of the involvement order, algorithms for computing with permutation patterns, applications and generalisations of permutation patterns, and others.

The Invited Speaker is **Miklos Bona** (U. Florida)

The conference is being organized as a satellite conference of FPSAC 04, which takes place at UBC the preceding week. The orga-



nizing committee consists of **Michael Albert** (Otago, New Zealand), **Einar Steingrímsson** (Goteborg, Sweden), **Julian West** (Malaspina and U. Victoria) and **Zvezdelina Stankova** (Mills, California).

Papers may be submitted before March 1, 2004, and acceptances will be determined promptly thereafter. There will be a reduced fee for registrations before April 1, 2004.

Further information can be found at <http://web.mala.bc.ca/math/events.html> or write to [westj@mala.bc.ca](mailto:westj@mala.bc.ca).

### Third Pacific Rim Conference on Mathematics University of British Columbia, June 21–25, 2004

The Organizers are **Genghua Fan** (Academia Sinica, Beijing), **Nassif Ghoussoub** (PIMS and UBC), **Fon-Che Liu** (Academia Sinica, Taipei), **Tai-Ping Liu** (Academia Sinica, Taipei), **Gaven Martin** (U. Auckland),

**Masayasu Mimura** (Hiroshima U.), **Robert M. Miura** (NJ Inst. Tech.), **Colin Rogers** (U. New South Wales), **Dale Rolfsen** (UBC), **Neil Trudinger** (Australian National U.) and **Roderick Wong** (City U. Hong Kong).

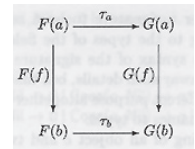
The topics covered in this conference will include, but are not limited to, the following: Algebra, Combinatorics, Asymptotics, Riemann Hilbert Problems, Holomorphic Dynamics, Low Dimensional Topology, Geometry, Nonlinear Analysis, PDEs, Phase Transitions, Vortex Dynamics, Nonlinear Evolution Equations including Dynamical Systems, and a special session on Mathematical Biology.

See [www.pims.math.ca/science/2004/prcm/](http://www.pims.math.ca/science/2004/prcm/).

### International Category Theory Conference (CT04) University of British Columbia, July 18–24, 2004

This conference will be held on the University of British Columbia campus. It will begin with a reception at 6pm on Sunday July 18, 2004,

and will end at 1pm on Saturday July 24, 2004. All those interested in category theory and its applications are welcome.



### PIMS PDF Meeting BIRS, April 15-17, 2004

This meeting will be for all PIMS PDFs to gather, meet one another, and talk about their research. It is being organized by Manfred Trummer (PIMS).

*Information about conferences associated with PIMS Collaborative Research Groups can be found on pages 13–22.*

For a full list of PIMS events please see [www.pims.math.ca](http://www.pims.math.ca).

## Recent Pacific Northwest Seminars

### PNW Geometry Seminar Spring Meeting PIMS-UBC, April 26–27, 2003

This meeting drew over 30 participants from other participating PNGS institutions, including Oregon State University, University of Oregon, and University of Washington. Parts of S.-T. Yau's lecture were filmed by a TV crew and may appear in an upcoming documentary about the Fields medallist!

The five speakers were:  
**Tobias Colding** (Courant Institute)  
**Jim Isenberg** (U. Oregon)  
**Michael Kapovich** (U. Utah)  
**Jun Li** (Stanford)  
**S.-T. Yau** (Harvard)

### West Coast Optimization Meeting University of Washington, May 9–10, 2003

*Contributed by James V. Burke, U. Washington*

The Spring West Coast Optimization meeting was a great success bringing together over 40 researchers in optimization and variational analysis. The Spring meeting was a particularly memorable occasion since it coincided with **R. Tyrrell Rockafellar's** retirement from the University of Washington. It was only fitting that the meeting was dedicated in honour of Professor Rockafellar's lifelong seminal contributions to the subject. Each of the invited speakers reminisced about the formative impact of Terry's research on their past and current research.

It was a truly joyous and inspiring event full of lively mathematical discussion as well as personal recollections of intellectual as well as wilderness explorations and adventures with Terry. Many of those in attendance recalled the thrill of collaborating with Terry while kayaking, x-country skiing, or trekking in the wilderness. Now that Terry is retired one can be sure that these mathematical back-country trips will be occurring with greater frequency. It must be something about intimately experiencing mountain passes and riptides that inspires a deeper insight into minimax theory and non-smooth analysis.

**Asen Dontchev** (U. Michigan, AMS) kicked off the morning session talks with a discussion of his joint work with Terry on metrically regular mappings and some recent work on selection theorems for such mappings.

*continued on next page*

## PNW Seminars

*continued from page 9*

Then **Mirjam Duer** (Darmstadt U. Technology) described a new probabilistic branch and bound method for solving non-convex optimization problems and its convergence theory.

**Lucien Polak** (UC, Berkeley) followed by describing the use of augmented Lagrangian techniques in the solution of generalized semi-infinite optimization problems.

**Jong-Shi Pang** (Johns Hopkins) began the afternoon session by discussing the connections between optimization and the theory and interpolation of continuous M-estimators.

**Roger Wets** followed by analyzing various notions of convergence for bi-variate functions giving examples from game theory.

**Boris Mordukhovich** (Wayne State U) then spoke on subdifferential and superdifferential optimality conditions in non-smooth optimization.

**Ivar Ekeland** (UBC, PIMS) concluded the programme by discussion the duality theory for functions of the determinant at one point producing his notes of a lecture given by Terry in 1968. The notes were still relevant to the topic at hand.

### Cascade Topology Seminar Portland State University, May 17–18, 2003

The speakers were:

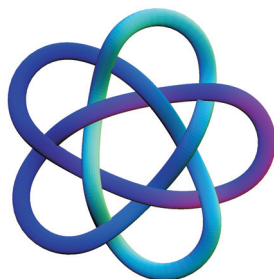
**Marta Asaeda** (U. Maryland, College Park)

**Dan Dugger** (U. Oregon): Motivic Cell decompositions

**Jens Harlander** (U. Frankfurt): Finiteness properties of groups

**Uwe Kaiser** (Boise State U.): Skein theory and the topology of mapping spaces

**Justin Roberts** (UC, San Diego)



### PDEfest PIMS-UBC, June 17, 2003

The speakers were:

**Jingyi Chen** (UBC): *Moving lagrangian submanifolds by mean curvature*

**Richard Froese** (UBC): *Some new examples of discrete Schrödinger operators with absolutely continuous spectrum*

**Nassif Ghoussoub** (PIMS & UBC): *The mother of many geometric inequalities*

**Changfeng Gui** (UBC): *Symmetry of mini-mizers*

**Tai-Peng Tsai** (UBC): *Asymptotic Stability and Completeness in Energy Space of Nonlinear Schrödinger Equations with Small Solitary Waves*

### Western Algebraic Geometry Seminar (WAGS) University of British Columbia, September 13–14, 2003

The WAGS speakers were:

**Dan Abramovich** (Brown U): *Valuative criteria for stable complexes*

**Gavril Farkas** (U. Michigan): *The Mori cones of moduli spaces of pointed curves*

**Sándor Kovács** (U. Washington): *Recent advances in the Minimal Model Program, after Shokurov, I*

**James McKernan** (UC, Santa Barbara): *Recent advances in the Minimal Model Program, after Shokurov, II*

**Alexander Polishchuk** (Boston University): *A-infinity homogeneous coordinate rings*

**Bernd Sturmfels** (UC, Berkeley): *Tropical Algebraic Geometry*

**Ravi Vakil** (Stanford U): *A geometric Littlewood-Richardson rule*

### West Coast Optimization Meeting Simon Fraser University, October 3–4, 2003

The speakers were:

**Jonathan Borwein** (SFU): *CoLab One function variational principles*

**Ian Coope** (U. Canterbury, NZ): *NZ Grids and Frames in Computational Optimization*

**John Dennis** (Rice University): *Mesh Adaptive Direct Search Algorithms*

**Rafal Goebel** (UC, Santa Barbara): *Optimality, stability, and duality of value functions for convex control problems*

**Chris Hamilton** (SFU CoLab): *Symbolic (Computational) Convex Analysis*

**Adrian Lewis** (CECM, SFU): *The structured distance to ill-posedness for conic systems*

**Terry Rockafellar** (U. Washington): *Regularity and Conditioning of Solution Mappings in Variational Analysis*

### Alberta IO Conference University of Alberta, October 24–25

The speakers were:

**John Boyce**: *Learning to Play Nash: The Case of Unstable Equilibria (with Rob Oxoby)*

**Sungchul Choi**: *Unpredictability and Mixed-Strategy Pricing: An Experimental Study of Competitive Price Promotional Strategies (with Paul R. Messinger)*

**Greg Dow**: *Appropriation, Information, and the Structure of the Firm*

**Curtis Eaton**: *Reciprocal Altruism and the Theory of the Family Firm*

**Loretta Fung**: *Large Real Exchange Rate Movements, Firm Dynamics, and Productivity Growth*

### PNW Geometry Fall Seminar Portland State University, October 25–26, 2003

The speakers were:

**Michael Anderson** (SUNY, Stony Brook): *Surgery construction of Einstein metrics*

**Christine Escher** (Oregon State): *The topology of manifolds with nonnegative sectional curvature*

**Victor Guillemin** (MIT): *Signature quantization*  
**Matthew Gursky** (U. Notre Dame): *A notion of maximal volume in conformal geometry and some applications*

**Rafe Mazzeo** (Stanford): *Positive Scalar Curvature and Poincaré-Einstein Fillings*

For reports on PNW seminars and activities of PIMS CRGs please see pages 13–22.



# Seminar on Stochastic Processes

University of Washington, March 27–29, 2003

Contributed by Krzysztof Burdzy, University of Washington

This conference attracted about 100 participants, mainly from the United States, but also from countries as diverse as Algeria, Colombia, Germany and Hungary. About one half of the participants were junior people: recent graduates and even advanced graduate students. The conference followed the format of the previous conferences in the same series (see <http://www.math.yorku.ca/Probability/ssparch.html> for the history of the conference). Only five one-hour invited talks were given by the top experts in probability theory. All other sessions were devoted to short informal talks. Every participant had a chance to present his or her most favourite recent result or an interesting open problem. Both senior mathematicians and junior researchers actively participated in the informal sessions. Ample time was left for discussions in small groups.



Wenbo Li  
(U. Delaware)

The main speakers and their lecture titles were:

**Ioannis Karatzas** (Columbia U.): *Some Stochastic Optimization Problems in Mathematical Finance*

**Wenbo Li** (U. Delaware): *Large Deviations for Intersection Local Times*

**Russ Lyons** (Indiana U.

and Georgia Tech): *Stationary Determinantal Processes (Fermionic Lattice Gases)*

**Carl Mueller** (U. Rochester): *Some Wave Equations with Noise*

**Balint Toth** (Budapest U. of Technology & Economics): *Between Equilibrium Fluctuations and Eulerian Scaling*



Balint Toth  
(Budapest University of Technology & Economics)

Some more information related to the event, including a few photographs, can be found at <http://www.math.washington.edu/~burdzy/SSP2003/index.shtml>. The conference was sponsored by the Institute of Mathematical Statistics and was financially supported by PIMS, the Milliman Fund at the Department of Mathematics, at the University of Washington, a VIGRE grant at the University of Washington, the National Science Foundation, the University of Washington, College of Arts and Sciences at the University of Washington and the Graduate School at the University of Washington. A big portion of the conference budget was spent on travel grants for junior researchers, women, minorities, and participants with no grants.

# Graph Theory of Brian Alspach

Simon Fraser University, May 25–29, 2003

Contributed by Joy Morris, University of Lethbridge

This special event was held in honour of the 65<sup>th</sup> birthday of **Brian Alspach**. There were twenty invited plenary talks spread over the five days, in addition to daily sessions of contributed talks. Many of the talks focused on areas of research in which Brian's own work has been particularly influential, including graph decomposition problems, hamilton cycles, Cayley graphs and vertex-transitive graphs.

The organisers were **Pavol Hell** (SFU, chair), **Gena Hahn** (U. Montréal, cochair), **C-Q Zhang** (West Virginia U, cochair), **Luis Goddyn** (SFU), **Wolf Holzmann** (U. Lethbridge), **Hadi Kharaghani** (U. Lethbridge), **Jiping Liu** (U. Lethbridge) and **Joy Morris** (U. Lethbridge).

The invited speakers ranged from some of the best-known, most respected mathematicians in graph theory, to mathematicians still establishing their careers; but they all clearly shared a deep admiration and respect for Brian's work, and many anecdotes throughout the week de-

scribed his impact on the lives and research of everyone present. In keeping with Brian's tradition of encouraging and helping promising young mathematicians, graduate students were welcomed to the conference, and most talks were pitched at a suitable level for their comprehension. There were 103 registered participants at the conference, 21 of whom were graduate students. Mathematicians came from more than 10 countries around the world, including France, England, Germany, Slovenia, New Zealand, Puerto Rico, Spain, Japan, India and the United States, demonstrating the widespread influence that Brian has had on graph theory. The invited talks generally provided some history and an overview of work on a particular problem, leading up to some of the speaker's recent research. In this way, current cutting-edge research and open problems were presented without an excess of technical detail. The background provided by these presentations often proved useful in some of the

shorter, more technical contributed talks. The focus of the conference, on Brian's work, gathered a group of mathematicians who are not often brought together. This provided an exciting, dynamic environment where many ideas were exchanged, acquaintances renewed, and new collaborative relationships were formed that may last for many years. Despite the demanding schedule, participants enjoyed themselves, and left with a much better understanding of many research problems.



Brian Alspach (SFU)

continued on the next page

The Invited Speakers were:

**J. Adrian Bondy** (U. Lyon 1 and U. Paris 6): *The Erdos-Posa property for long circuits*

**Ted Dobson** (Mississippi State U): *On groups of odd prime-power degree that contain a full cycle*

**Luis Goddyn** (SFU): *Explaining Youngs' bimodality phenomenon for embedded graphs*

**Chris Godsil** (U. Waterloo): *Colouring interesting graphs*

**Gena Hahn** (U. Montréal): *And now for something somewhat different (what you may not know about Brian Alspach)*

**Pavol Hell** (SFU): *Homomorphisms of graphs with bounded degrees*

**Bill Jackson** (Queen Mary, U. London): *Connected rigidity matroids and unique realizations of graphs*

**H.A. Jung** (TU Berlin): *Degree estimates for the circumference of graphs*

**Curt Lindner** (Auburn U): *A brief history of embedding partial 4-cycle systems*

**Jim Liu** (U. Lethbridge): *A characterization of pancyclic complements of line graphs*

**Dragan Marusic** (U. Ljubljana): *Symmetry in graphs - some open problems*

**Joy Morris** (U. Lethbridge and Oklahoma State U.): *Hamiltonian paths and cycles in vertex-transitive graphs and digraphs, On automorphisms of circulant graphs*

**Brooks Reid** (California State U, San Marcos): *Tournaments in which every arc is in a Hamiltonian path*

**Alex Rosa** (McMaster U):

**Moshe Rosenfeld** (U. Washington, Tacoma): *Variations on Hamiltonian themes*

**Gert Sabidussi** (U. Montréal): *Subholomorphic Cayley graphs*

**Mateja Sajna** (U. Ottawa): *Cycle decompositions: the past, present, and future*

**Wal Wallis** (Southern Illinois U): *Totally magic graphs*

**Cun-Quan Zhang** (West Virginia U): *Chords of longest circuits in 3-connected graphs*

## Workshop on Structural Graph Theory PIMS-UBC, July 9–19, 2003

*Contributed by Bruce Reed, McGill U. and Luis Goddyn, SFU*

In July 2003, a small group of researchers gathered together for 10 days at the PIMS UBC facility to work together on a number of problems in structural graph theory. The main focus was on parity minors and generalizations to Matroid theory. The collaboration was fruitful and at least five papers will result from work carried out during the meeting. PIMS was a perfect location for such a meeting. The staff arranged the housing, computers accounts, rooms in which to work, and coffee breaks. We enjoyed the stunning landscape on and around the campus and the glorious weather. The participants initially focused on the *odd path packing* problem: "How many odd-length disjoint paths having endpoints in a prespecified set of vertices, exist in a given undirected graph?" They solved a generalization of this problem concerning "non-zero" paths in graphs whose arcs have group-valued weights. For example, one can substitute "noncontractible" for "odd-length" in the case of embedded graphs. Additionally, one can substitute "nonmonochromatic" for "odd-length" in the case of coloured graphs. Progress was also made on problems related to perfect graphs, matroids, and the structure theory of signed graphs. For more information please see [www.pims.math.ca/science/2003/structural/](http://www.pims.math.ca/science/2003/structural/).

## Banach Algebras and their Applications, Banach Algebras 2003 University of Alberta, July 27–Aug. 9, 2003

*Contributed by Volker Runde, University of Alberta*

This conference was organized by **Anthony To-Ming Lau** and **Volker Runde** (University of Alberta). It was the sixteenth in a series of conferences on Banach algebras that started 1974 in Los Angeles. The organizers of the first meeting, **William G. Bade** of Berkeley and **Philip C. Curtis, Jr.**, of UCLA, were among the 136 participants from all continents (except Antarctica) at the Edmonton conference.

In addition to the regular conference programme, there were five workshops on the following topics, each of which was chaired by an internationally recognized specialist in the respective area:

**Joachim Cuntz** (U. Münster): *K-theory for Banach and locally convex algebras*

**Alexander Ya. Helemskii** (Moscow State U, and PIMS Distinguished Chair): *Topological homology*

**Anthony To-Ming Lau** (U. Alberta): *Banach algebras in abstract harmonic analysis*

**Michael M. Neumann** (U. Saarbrücken): *Banach algebras in operator theory*

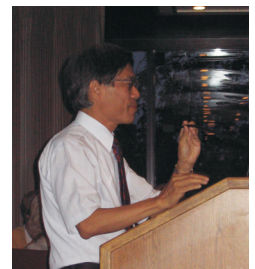
**Z.-J. Ruan** (U. Illinois, Urbana-Champaign): *Banach algebras and operator spaces*

The workshop on harmonic analysis was held in the honour of **Eberhard Kaniuth of Paderborn**, Germany, who retired this year.

The mathematical highlights included a solution — by **Viktor Losert** of Vienna, Austria — of the so-called inner derivation problem of the late B. E. Johnson: "Every derivation on the group algebra of a locally compact group is an inner derivation implemented by a measure." This problem had been open for more than three decades.

There will be conference proceedings published by the AMS in their Contemporary Mathematics series.

Please see <http://www.math.ualberta.ca/~ba03/>.



*Anthony To-Ming Lau at the conference banquet*



*Viktor Losert explaining his resolution of the inner derivation problem*



*Excursion to Elk Island*



# Periods of Concentration for PIMS Collaborative Research Groups

The Periods of Concentration (POC) are designed to promote and support longer term, multi-event, multi-site coordinated activities of competitively selected Collaborative Research Groups (CRGs), in tandem with their national and international collaborators and visitors. Every year, the PIMS Scientific Review Panel will select, on a competitive basis, proposed POCs. The selected areas will be the focus of much of the institute's programme over a 1-2 year period of concentrated activities that will be delivered through the selected CRGs. At any given time, it is expected that 5-8 CRGs will lead the PIMS scientific enterprise. Proposals can vary greatly according to the needs of the particular group and may combine a number of existing PIMS activities. During its period of concentration, a CRG can expect to receive priority for:

- PIMS postdoctoral fellowships
- 5-day workshops and other activities at BIRS
- Intensive graduate courses
- Graduate students exchanges
- Industrial training camps
- Pacific Northwest seminar series
- Workshops and conferences at PIMS sites
- Distinguished chairs and long term visitors
- Graduate and senior undergraduate schools
- International collaborations

With this support, a CRG can plan to gather a significant portion of the world's experts in its scientific area of research for periods of intense collaboration. The fruits of such intensity can be expected to persist for many years and to be exponentially greater than the results of more normal activity levels.

Each CRG must have significant participation and leadership from at least two Canadian PIMS universities. Each CRG will designate a coordinator for its period of concentration. This coordinator must be based at a Canadian PIMS university; she/he will co-ordinate the various CRG activities, sign off on all CRG events, and will be the CRG's liaison with the PIMS scientific and administrative personnel.

Each CRG designs its activities according to its specific needs within the guidelines provided by PIMS. Facilitating the training of highly qualified personnel has been identified as a priority for PIMS. Hence, CRGs are encouraged to take full advantage of the opportunities provided through the PIMS Postdoctoral Fellowship Programme, and to take a leadership role in the training of graduate and senior undergraduate students. PIMS encourages CRGs to develop innovative programmes in consultation with the Deputy Director. The proposed POC programme is evaluated by the PIMS Scientific Review Panel.

CRGs have priority access to the PIMS PDF programme. Individual PDF applications have to be submitted to the local PIMS site as part of the regular PIMS PDF competition. Please see [www.pims.math.ca/opportunities/pdf.html](http://www.pims.math.ca/opportunities/pdf.html) and page 7 of this magazine.

Proposals for POCs should be developed by the CRG leaders in consultation with local site directors and the PIMS deputy director. Proposals should be sent to [proposal@pims.math.ca](mailto:proposal@pims.math.ca) by the October deadline for review by the PIMS Scientific Review Panel. Proposals for POCs starting in 2005 or later must be submitted by October 15, 2004. Since a POC is an ambitious programme, researchers are encouraged to prepare and submit applications well in advance of their period of concentration; for example, the best time to apply for a POC starting in April 2006 is October 2004.

## Period of Concentration on Number Theory



### 7th Annual PNW Number Theory Conference

U. Washington, April 5-6, 2003

This conference was organized by **Ralph Greenberg** and **Joe Buhler** with support from the University of Washington Milliman Fund, the Number Theory Foundation, Reed College, and PIMS. The speakers were:

**Michael Bennett** (UBC): *Perfect Powers from Progressions*

**Joe Buhler** (Reed College): *The Probability that a  $p$ -adic Polynomial Splits*

**Cheewhye Chin** (UC, Berkeley): *Lafforgue's Work on the Langland's Correspondence over Function Fields*

**Stephen Choi** (SFU): *Small Prime Solutions for Quadratic Equations with Five Variables*

**Henry Cohn** (Microsoft): *Horosphere Packings*

**Karl Rubin** (Stanford): *Kolyvagin Systems*

About 40 people attended the conference, including graduate students and faculty members from universities in the Pacific Northwest and as far as the University of Chicago. The lectures covered a diverse set of topics in number theory.

### Mahler Measure of Polynomials

Simon Fraser University, June 2-29, 2003

*Contributed by Stephen Choi, SFU*

The PIMS summer program on *Mahler Measure* was funded by PIMS with support from Simon Fraser University. There were a total of 50 faculty members, postdoctoral fellows, graduate and undergraduate students from various universities and countries including Canada, US, France, Italy, Australia, Austria and Greece. The keynote speaker for this program was Professor **Jeffrey Vaaler** from the

University of Texas at Austin. Prof. Vaaler was also PIMS Distinguished Chair visiting Simon Fraser University in 2003. During the program, Prof. Vaaler gave four very stimulating talks in his distinguished lectures series. Before his lectures series, there was a short graduate course, taught by **Stephen Choi** (SFU) and **Mike Mossinghoff** (Davidson College), serving as a preparation for the lecture series and later talks in the program. Prof. **David Boyd** (UBC) gave two talks on *Multivariable Mahler measure*. The program ended with a workshop in the last week of June. In the workshop 18 speakers gave talks on different aspects of Mahler measure. A list of the speakers is given below. A student session was also organized throughout the whole program to encourage graduate students to present current work to the participants. Almost all the participating students gave talks in this student session. Most of the slides and notes of the talks from this

program are on the PIMS website <http://www.pims.math.ca/science/2003/mahler/>. Prof. Vaaler's lecture series has been videotaped and will be put on the PIMS website in the near future. Moreover, Prof. Vaaler also wrote an article based on the notes from his lecture series in this newsletter.

The Invited Speakers were:

**Iskander Aliev** (Technische U. Wien): *Decompositions of Integer Vectors and Some Related Problems*

**Arthur Baragar** (UNLV): *Vector Heights on Surfaces*

**Jason Bell** (U. Michigan): *Cohen-Macaulay Rings and Polynomials with Real Zeros*

**Marie José Bertin** (U. Paris, Pierre et Marie Curie): *Mahler's Measures of Calabi-Yau's Varieties: Examples*

**David Boyd** (UBC): *Explicit Formulas for Multivariable Mahler's Measure; Mahler's Measure and Hyperbolic Manifolds*

**Edward Dobrowolski** (The College of New Caledonia)

**Tamas Erdelyi** (Texas A & M)

**Lenny Fukshansky** (U. Texas, Austin): *Small Zeros of Quadratic Forms with Linear Conditions*

**Kevin Hare** (UC, Berkeley): *Gaps in the Spectra of Pisot Numbers*

**Angel Kumchev** (U. Texas, Austin): *On Waring-Goldbach Problem*

**Matilde Lalin** (U. Texas, Austin): *Examples of Mahler Measures as Special Values of the Riemann Zeta Function and L-Series*

**Friedrich Littmann** (UIUC, UBC, SFU): *Entire Extremal Majorants*

**Michael Mossinghoff** (Davidson College): *Computational Aspects of Problems on Mahler's Measure; Mahler's Measure of Polynomials with Odd Coefficients*

**Nathan Ng** (U. Montréal): *Mean Values of L-functions*

**Clay Petsche** (U. Texas, Austin): *The Height of Algebraic Units in Local Fields, and Lehmer's Problem; The Quantitative Distribution of Galois Orbits of Small Height*

**Chris Pinner** (Kansas State U.): *Some Bounds for Complete Exponential Sums*

**Igor E. Pritsker** (Oklahoma State U.): *Gelfond-Schnirelman Method in Prime Number Theory*

**Georges Rhin** (U. Metz): *Integer Transfinite Diameter and Polynomials of Small Mahler Measure*

**Chris Sinclair** (U. Texas, Austin): *Heights of Polynomials, Asymptotic Estimates and the Mellin Transform; The Distribution of Mahler's Measures of Reciprocal Polynomials*

**Jeff Vaaler** (U. Texas, Austin): *Mahler's Measure and the Number of Irreducible Factors of a Polynomial, Mahler's Measure and the ABC Inequality, The Distribution of Values of Mahler's Measure, Estimates for the Number of Algebraic Numbers of Fixed Degree and Bounded Height*

**Carlo Viola** (U. Pisa): *Birational Transformations and the Arithmetic of Euler's Integrals*

**Qiang Wu** (U. Metz): *An Effective Algorithm to Compute the Integer Transfinite Diameter and Some Applications*

### PIMS Number Theory Day SFU Harbour Centre, December 5, 2003

This meeting is being organized by Michael Bennett (UBC), Peter Borwein (SFU), David Boyd (UBC), Imin Chen (SFU) and Stephen Choi (SFU).

The Invited Speakers are:

- Valentin Blomer** (U. Toronto)
- Alina Cojocaru** (Princeton U.)
- Benjamin Green** (PIMS)
- Friedrich Littmann** (PIMS)
- Nathan Ng** (U. Montreal)
- Robert Osburn** (Queen's U.)
- Christopher Rowe** (PIMS)

### PIMS PNWNT Conference Corvallis, Oregon, April 17–18, 2004

**Sergei Konyagin** (Moscow State University) will deliver a lecture series in March 2004 at UBC. Winner of the Salem Prize in 1990, Prof. Konyagin has made numerous significant contributions in number theory, approximation theory and harmonic analysis.

### BIRS Workshops

The Many Aspects of Mahler's Measure, 5-Day Workshop, April 26–May 01, 2003



Participants of the The Many Aspects of Mahler's Measure Workshop

Current Trends in Arithmetic Geometry and Number Theory, 5-Day Workshop, August 16–21, 2003

Explicit Methods in Number Theory, 5-Day Workshop, November 13–18, 2004

Diophantine Approximation and Analytic Number Theory, 5-Day Workshop, November 20–25, 2004

#### PIMS PDFs of the CRG:

- SFU: Ron Ferguson, William Galway, Alexa van der Waall
- UBC: Ben Green, Friedrich Littman, Christopher Rowe

#### Faculty of the CRG:

- Group Leaders: Peter Borwein (SFU), David Boyd (UBC)
- SFU: I. Chen, S. Choi, P. Lisonek
- U. Alberta: J. D. Lewis
- U. Calgary: R. Guy, J. P. Jones, R. Mollin, R. Scheidler, H. Williams
- UBC: M. Bennett, W. Casselman, R. Gupta, I. Laba, G. Martin, N. Vatsal
- U. Washington: R. Greenberg, A. Iovita, N. Koblitz, B. Solomyak
- Other institutions: A. Akbary (U. Lethbridge), E. Dobrowolski (College of New Caledonia), M. Klassen (DigiPen Inst. Tech.), K. Lauter (Microsoft)

The Number Theory webpage is at [www.pims.math.ca/CRG/number/](http://www.pims.math.ca/CRG/number/). Further information and a preliminary list of activities for 2004–05 may be found there.



# Period of Concentration on String Theory



## Frontiers of Mathematical Physics Summer School on Strings, Gravity and Cosmology

PIMS-UBC, July 14-25, 2003

Contributed by Moshe Rozali, UBC

*Frontiers in Mathematical Physics* was organized as a summer school this year. The school was concentrated on the interface between gravity, cosmology and string theory. This is an active research area in string theory, and the school was designed to prepare the students for active research in this field. By choosing a particular section of string theory, the school was fairly comprehensive in the topics covered, providing both elementary introduction to the subject matter and ending with reviews of recent literature. The organizers were **Taejin Lee** (APCTP), **John Ng** (TRIUMF, UBC), **Moshe Rozali** (UBC), **Alexander Rutherford** (PIMS) and **Gordon W. Semenoff** (UBC). The first week of the school was dedicated to introducing the basic tools of string theory. Those include the basic technical tools of (perturbative and non-perturbative) string theory and quantum field theory, and some of their applications. Basics of cosmology were covered in preparation for the second week. The second week of the school covered more advanced topics, by and large developments of the last year or two (or in some cases the last week or two). These included recent developments in matrix models, Liouville theory, cosmic singularities and much more. Among those were the excellent lectures of **Ashoke Sen**,

PIMS distinguished speaker, on *Tachyon Dynamics in Open String Theory*.

The full list of speakers is: **Vijay Balasubramanian** (UPenn): *Time and String Theory*

**Micha Berkooz** (Weizmann Institute): *Cosmic Singularities in General Relativity and String Theory*

**Robert Brandenberger** (Brown): *Basics of Cosmology for String Theorists*

**Michael Dine** (UC, Santa Cruz): *String Phenomenology*

**Simeon Hellerman** (Stanford): *Supersymmetric Gauge Theories*

**David Kutasov** (U. Chicago): *Little String Theory*

**Yuri Makeenko** (ITEP, Moscow): *Large N Gauge Theories*

**Volker Schomerus** (Saclay): *Strings in Exact Non-Compact Backgrounds*

**Ashoke Sen** (Harish-Chandra Institute): *Tachyon Dynamics in Open String Theory*

**Matthew Strassler** (U. Washington): *Confinement and String Theory: The Duality Cascade and its Applications*



Vijay Balasubramanian (U. Pennsylvania)



David Kutasov (U. Chicago)



Leonard Susskind (Stanford)

**Leonard Susskind** (Stanford): *deSitter Space*  
**Richard Szabo** (Heriot-Watt): *Perturbative String Theory*

**Mark Van Raamsdonk** (Stanford/UBC): *Introduction to AdS/CFT*

**Don Witt** (UBC): *QFT in Curved Space*

**Piljin Yi** (KIAS): *Low Energy Dynamics of Unstable D-Branes*

We were fortunate to have a large group of very enthusiastic and dedicated students, 88 in all. Student seminars were given at the end of the day, and were well-attended. The lectures were very interactive, resulting in interesting discussions during the breaks. It is our hope the students are now able to start making their own contributions to our field.

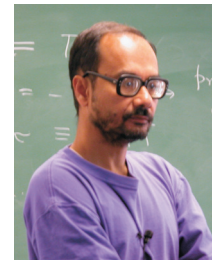
The next Frontiers in Mathematical Physics Summer School will be on Particles, Fields and Strings, and it will be held in early August 2004.

## Ashoke Sen, PIMS Distinguished Lecturer

Contributed by Moshe Rozali, UBC

As part of our summer programme in string theory, we were fortunate to host Ashoke Sen as PIMS distinguished speaker. Professor Sen is one of the most influential string theorists in the world, leading the recent effort aimed at understanding tachyon dynamics in open string theory, one of the most active research areas in string theory. This was the topic of his 5 lectures in Vancouver.

Professor Sen was educated in India, and Obtained his PhD from State University of New York at Stony Brook. He completed postdoctoral appointments in SLAC (Stanford) and Fermilab (Chicago). He joined the faculty of the Tata institute in Mumbai in 1988, and the Harish-Chandra research institute in Allahabad in 1995.



Ashoke Sen (Harish-Chandra Institute)



Frontiers in Mathematical Physics Summer School Participants

Pacific Northwest String Seminar  
PIMS-UBC, November 15–16, 2003

The meeting is cosponsored by PIMS and the Pacific Institute for Theoretical Physics.

The speakers were:

- Dominic Brecher** (UBC)
- Michael Gutperle** (UCLA)
- Shamit Kachru** (Stanford University)
- Andreas Karch** (University of Washington)
- Albion Lawrence** (Brandeis University)
- Hiroshi Ooguri** (Caltech)
- Stephen Shenker** (Stanford University)
- Matthew Strassler** (U. Washington)

PNW String Seminar on Mathematical Aspects of Open-Closed String Dualities  
U. Washington, December 4–5, 2003

The organizers are Mina Aganagic, Charles Doran, Andreas Karch and Matthew Strassler (U. Washington).

This seminar is supported by PIMS, the Milliman Fund at the UW Department of Mathematics, the U.S. Department of Energy, and the Dean of the UW College of Arts and Sciences.

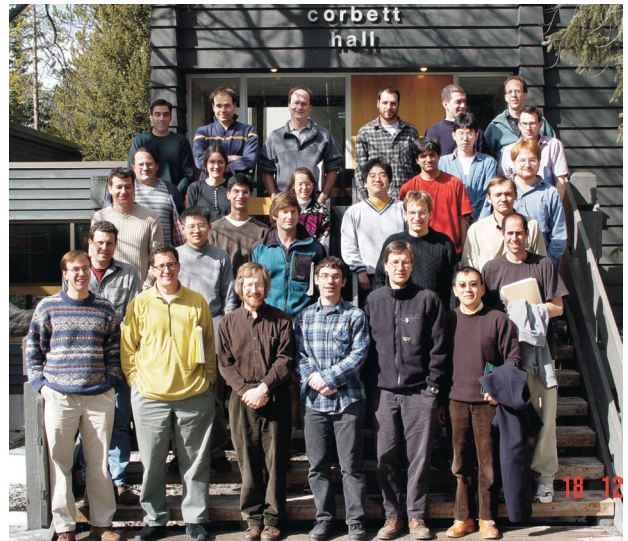
The external speakers include **Adrian Clinger** (Stanford), **Ezra Getzler** (Northwestern), **Sergei Gukov** (Clay/Harvard), **Kentaro Hori** (Toronto), **Ken Intriligator** (UCSD), **Amer Iqbal** (Harvard) and **Melissa Liu** (Harvard).

BIRS Workshops

BIRS Workshop on Recent Developments in Superstring Theory, 5-Day Workshop, March 15–20, 2003

New Horizons in String Cosmology, 5-Day Workshop June 12–17, 2004

String Field Theory Camp, Focused Research Group, July 10–25, 2004



Participants of the Workshop on Recent Developments in Superstring Theory

PIMS PDFs of the CRG

UBC: Dominic Brecher, Kazuyuki Furuuchi and Ehud Schreiber

Faculty of the CRG

Leaders: Gordon Semenoff (UBC), Eric Woolgar (U. Alberta)

SFU: K. Viswanathan

U. Alberta: B. Campbell, V. Frolov, T. Gannon, D. Page

UBC: K. Behrend, J. Bryan, M. Choptuik, M. Van Raamsdonk, M. Rozali, K. Schleich, W. Unruh, D. Witt

U. Lethbridge: M. Walton

UofT: A. Peet

U. Washington: A. Karch

Perimeter Institute: R. Myers, L. Smolin

APCTP: T. Lee

The String Theory CRG webpage is at [www.pims.math.ca/CRG/string/](http://www.pims.math.ca/CRG/string/). Further information and a preliminary list of activities for 2004–05 may be found there.

# Period of Concentration on Scientific Computing



Workshop on Numerical Linear Algebra and Applications

PIMS-UBC, August 4–8, 2003

This workshop was organized by Chen Greif (UBC). It featured two short courses, and a mixture of survey talks and advanced talks.

The short courses were accessible to non-experts, and slides from these courses are available on the web page.

The short course speakers were:

**Alison Ramage** (Strathclyde University): *An*

*Introduction to Iterative Solvers and Preconditioning Techniques*

**Eldad Haber** (Emory U):

*PDEs and Optimization*

The other speakers were:

**Uri Ascher** (UBC): *On the*

*modified conjugate gradient*

*method in cloth simulation*

**Xiao-Wen Chang** (McGill

U.): *Numerical linear algebra*

*in the Global Positioning*

*System*



Alison Ramage  
(Strathclyde)

**Edmond Chow** (Lawrence Livermore National Lab): *A survey of incomplete factorization preconditioners*

**Iain Duff** (RAL and CERFACS): *The symbiosis of direct and iterative methods for solving large sparse systems & Solving large industrial problems in electromagnetics at CERFACS*



Eldad Haber  
(Emory)



**Gene Golub** (Stanford): *Numerical Solution for Solving Least Squares Problems with Constraints*

**Anne Greenbaum** (U. Washington): *Polynomial Numerical Hulls of Jordan Blocks and Related Matrices*

**Chen Greif** (UBC): *Techniques for solving indefinite linear systems*

**Misha Kilmer** (Tufts U.): *Numerical Methods for Ill-Posed Problems*

**Scott MacLachlan** (U. Colorado): *Solving PDEs with Multigrid Methods*

**Carl Meyer** (North Carolina State U.): *Updating Markov Chains*

**Peyman Milanfar** (UC, Santa Cruz): *Applications of Numerical Linear Algebra to Imaging Inverse Problems*

**Esmond Ng** (Lawrence Berkeley National Lab): *Computational Challenges in Electron Microscopy of Macromolecules*

**Michael Overton** (NYU): *Optimizing Matrix Stability*

**David Watkins** (Washington State U.): *Hamiltonian and Symplectic Lanczos Processes*

Please see <http://www.pims.math.ca/science/2003/numerical/>.

### Pacific Northwest Numerical Analysis Seminar

U. Washington, October 4, 2003

This year's speakers were:

**Marsha Berger** (NYU)

**William Ferng** (Boeing)

**Jonathan Goodman** (NYU)

**Boualem Khouider** (UVic)

**Ian Mitchell** (UBC)

**Michel Pettigrew** (UW)

**Yun-Qiu Shen** (WWU)

### IAM-CSC-PIMS Senior Undergraduate Math Modelling Camp

UBC and SFU,

February 14–15, 2004

### PIMS Distinguished Series in Scientific Computing at UBC

This is a new distinguished speaker series, comprising six talks by world leading experts, and spanning a wide variety of topics in scientific computing. It is organized by **Uri Ascher** and **Chen Greif** (Computer Science, UBC).



Participants of the Workshop on Numerical Linear Algebra and Applications

The series is taking place at UBC in September 2003–January 2004. Additional support is being provided by NSERC grants of the organizers and by the Institute of Applied Mathematics at UBC.

The lectures are:

**Heinz W. Engl** (Industrial Mathematics Institute, Johannes Kepler Universität, Austria): *Iterative Regularization of Non-linear Inverse Problems*

**Pat Hanrahan** (Computer Science, Stanford U.): *Why is Graphics Hardware so Fast? Implications for Scientific Computing*



Andy Wathen (Oxford)

**Tom Hou** (Applied Mathematics, CalTech): *Multiscale Modelling and Computation of Flow in Heterogeneous Media*

**Jorge Nocedal** (Electrical & Computer Engineering, Northwestern U.): *The New Faces of Nonlinear Optimization*

**Christoph Schwab** (Applied Mathematics, ETH, Switzerland): *Wavelet algorithms for asset pricing under Levy Processes*

**Andy Wathen** (Oxford U. Computing Lab): *Preconditioning in Scientific Computation*

Please see [www.pims.math.ca/science/2003/scicomp/](http://www.pims.math.ca/science/2003/scicomp/).

Otmar Scherzer (U. Innsbruck, Austria) will be a PIMS Distinguished Chair at UBC during the summer of 2004.



Participants of the Computational Techniques for Moving Interfaces BIRS Workshop

## BIRS Workshops

Computational Fuel Cell Dynamics, 5-Day Workshop, April 19–24, 2003

Computational Techniques for Moving Interfaces, 5-Day Workshop, August 23–28, 2003

Mathematical Foundations of Scientific Visualization, Computer Graphics and Massive Data Exploration, 5-Day Workshop, May 22–27, 2004

### PIMS PDFs of the CRG:

SFU: Jian-Jun Xu

UBC: Jianying Zhang

### Faculty of the CRG:

Coordinator: Steve Ruuth (SFU)

CRG Leaders: Elana Braverman (U. Calgary), Chen Greif (UBC), Randy Leveque (U. Washington), Yanping Lin (U. Alberta), Steve Ruuth (SFU), Manfred Trummer (SFU)

SFU: R. Choksi, M.C. Kropinski, T. Möller, D. Muraki, K. Promislow, B. Russell, S. Ruuth, L. Trajkovic, M. Trummer, J. Verner, R. Zahar. U. Alberta: Y. Lin, J. Macki, P. Mineev, Y.S. Wong

UBC: U. Ascher, O. Dorn, S. Dunbar, I. Frigaard, A. Peirce, B. Seymour, B. Shizgal, J. Varah, M. Ward, B. Wetton, M. Yedlin

U. Calgary: T. Ware, R. Westbrook

U. Victoria: P. van den Driessche, D. Olesky

U. Washington: L. Adams, C. Bretherton, J.

Burke, D. Durran, A. Greenbaum, G. Hakim, N. Kutz, R. LeVeque, R. O'Malley, P. Schmid  
Ballard Corp: R. Bradean, J. Kenna  
Boeing Corp: M. Epton, S. Filipowski, J. Lewis  
Quadrus Financial Technologies: S. Reddy

The Scientific Computing CRG webpage is at [www.pims.math.ca/CRG/scientific/](http://www.pims.math.ca/CRG/scientific/). Further information and a preliminary list of activities for 2004–05 may be found there.

# Period of Concentration on Mathematical Ecology



## 2<sup>nd</sup> Annual PIMS Mathematical Biology Summer Workshop

University of Alberta, April 30–May 9, 2003

*Contributed by Kym Schreiner, U. Alberta*

The Centre for Mathematical Biology (CMB), University of Alberta, hosted its 2<sup>nd</sup> Annual PIMS Mathematical Biology Summer Workshop entitled *Mathematics of Biological Systems*. The aim of this 10-day workshop was to introduce undergraduate mathematics students to mathematical modeling and analysis applied

to real biological systems.

The instructors were **Gerda de Vries**, **Thomas Hillen**, **Mark Lewis**, **Frithjof Lutscher** (all from the University of Alberta), and guest instructor **Pauline van den Driessche** (University of Victoria). There was further assistance provided by volunteer graduate students, postdoctoral fellows, and staff.

Twenty-two students came to the workshop from 14 different universities across Canada and the United States, many on their own funding. More than half of the attendees were women.

The workshop was 10 days in length and

consisted of a combination of classroom instruction, computer lab sessions, pen and paper exercises, guided group project work, and project presentations. Communication between and among workshop participants and instructors was promoted

through scheduled breaks and social events. For the second consecutive year, the PIMS Mathematical Biology Summer Workshop received extremely positive feedback. Instructors are planning the 3<sup>rd</sup> Annual PIMS Workshop to be held May 4–14, 2004. The present structure of the workshop is very effective and requires little alteration. Instructors are also revising a book for publication that was used as the text for the present workshop and will be used for future workshops. This text is to be published by the Society for Industrial and Applied Mathematics.



Participants of the 2<sup>nd</sup> Annual PIMS Mathematical Biology Workshop

consisted of a combination of classroom instruction, computer lab sessions, pen and paper exercises, guided group project work, and project presentations. Communication between and among workshop participants and instructors was promoted

**Bryan Grenfell** of the University of Cambridge was the PIMS Distinguished Chair in Mathematical Ecology in September 2003. He gave five lectures with two at the University of Alberta, one overview lecture at UBC, and two more lectures at the University of Calgary. These lectures were videotaped and will be made available on the web. Notes from his talks will also be made available.

His talks were entitled:

- *Comparative dynamics of childhood microparasitic infections and the impact of vaccination*
- *Waves, sparks and wavelets: measles in space*



and time

- *Childhood infections in space and time*
- *Dynamics and evolution of pathogens*
- *Dynamics and control of foot and mouth disease*



Bryan Grenfell  
(U. Cambridge)

For more information please see [www.pims.math.ca/science/2003/distchair/grenfell/](http://www.pims.math.ca/science/2003/distchair/grenfell/).

From November 20 until December 13 **Brian Sleeman** (U. Leeds) will visit the University of Alberta as an Endowment Fund for the Future Distinguished Visitor. Brian is a world leading expert in applied mathematics, in particular the mathematical modelling of angiogenesis in tumor growth. He gives a series of four lectures in Edmonton, one talk in Calgary, and one in Saskatoon.

In November 2003 **Pauline van den Driessche** (U. Victoria) is visiting the Centre for Math Biology at U. Alberta. She will be doing research with PIMS PDF **Joanna Renclawowicz**.

**Roger Nisbet** (UC, Santa Barbara) will be a PIMS Distinguished Chair in the fall of 2004.

### International Conference on Differential Equations and Applications in Mathematical Biology

Nanaimo, BC, July 18-23, 2004

### BIRS Workshops

Mathematical Biology: From molecules to ecosystems; The legacy of Lee Segel, 5-day Workshop, July 5–10, 2003

Mathematical Models for Plant Dispersal, Focused Research Group, September 20–October 2, 2003

Retreat on Mathematical Ecology and Evolution, 2-day Workshop, March 18–20, 2004

Mathematical Models for Biological Invasions, 5-day Workshop, November 27–December 2, 2004

#### PIMS PDFs of the CRG:

U. Alberta & U. Calgary: Frithjof Lutschern  
U. Alberta & U. Victoria: Joanna Renclawowicz

#### Faculty of the CRG:

Coordinator: Thomas Hillen (U. Alberta)

Co-organizers: Michael Doebeli (UBC), Mark Lewis (U. Alberta), Edward McCauley (U. Calgary)

SFU: E. Palsson, B. Roitberg

U. Alberta: M.

Boyce, H. Freedman, S. Lele, M. Li, J. Roland, J. So

UBC: F. Brauer, L. Keshet, D. Schluter  
U. Calgary: S. Richards

U. Victoria: P. van den Driessche

U. Washington: J. Anderson, C. Bergstrom, D. Grunbaum, R. Hilborne, M. Kot



Thomas Hillen  
(U. Alberta)



Participants of The legacy of Lee Segel Workshop at BIRS

The Mathematical Ecology CRG webpage is at [www.pims.math.ca/CRG/ecology/](http://www.pims.math.ca/CRG/ecology/). Further information and a preliminary list of activities for 2004–05 may be found there.

## Period of Concentration on Dynamics & Related Topics $\pi$

### Aperiodic Order, Dynamical Systems, Operator Algebras and Topology University of Victoria, August 4–8, 2002

This workshop was devoted to recent developments in the area of aperiodic tilings and quasicrystals. In particular, the participants discussed contributions from diverse fields such as operator algebras, topology, K-theory and foliated spaces, as well as ergodic theory and diffraction. The number of talks was limited

so as to allow informal discussion.

It was organized by Jean Bellissard (U. Toulouse), Johannes Kellendonk (U. Cardiff) and Ian Putnam (U. Victoria).

The speakers were:

**Michael Baake** (U. Greifswald): *Mathematical Diffraction Theory and Model Sets*

**Marcy Barge** (Montana State U.): *The Topology of One-dimensional Tiling Spaces*

**Laurent Bartholdi** (UC, Berkeley): *Tilings and Groupoids Acting on Rooted Trees*

**Bob Burton** (Oregon State U.): *Dynamic Systems with Evenly Spread Out Orbits*

**Claire Anantharaman-Delaroche** (U. Orléans): *Amenable Groupoids. Examples and Applications*

**Thierry Fack** (U. Lyons I): *Introduction to Cyclic Cohomology*

**Franz Gähler** (ITAP U. Stuttgart): *Cohomology of Quasiperiodic Tilings*

**Franz Gähler** (ITAP U. Stuttgart): *Modelling Aperiodic Solids: Concepts and Properties of*

*Tiling and their Physical Interpretation*  
**Jean-Marc Gambaudo** (U. Bourgogne, Dijon): *Delone Sets, Tilings and Solenoids: from Finitetranslation Type to Finite Isometry Type*

**Thierry Giordano** (U. Ottawa): *Affable Equivalence Relations and Orbit Structure of Cantor Minimal Systems*

**Chaim Goodman-Strauss** (U. Arkansas): *Triangle Tilings and Regular Productions*

**John Hunton** (U. Leicester): *New Models and Methods for Tiling Spaces*

**Jerry Kaminker** (IUPUI): *Index Theory on Foliated Spaces and Applications*

**Alex Kumjian** (U. Nevada, Reno): *Actions of  $Z^k$  Associated to Higher Rank Graphs*

**Daniel Lanz** (TU-Chemnitz): *Uniform Ergodic Theorems on Delone Dynamical Systems and Applications*

**Jeong-Yup Lee** (U. Alberta): *Consequences of Pure Point Diffraction Spectra for Discrete Point Sets*

**N. Christopher Phillips** (U. Oregon): *The Structure of the  $C^*$ -algebras of Free Minimal Actions of  $Z^d$  on the Cantor Set*

**Charles Radin** (U. Texas, Austin): *Aperiodicity: Lessons from Various Generalizations*

**Lorenzo Sadun** (U. Texas, Austin): *When Size Matters: the Effect of Geometry on 1-D Tiling Dynamics*

**Klaus Schmidt** (U. Vienna): *Shifts of Finite*

*Type, Wang Tilings and Fundamental Cocycles*  
**Claude Schochet** (Wayne State U.): *Life After K-theory*

**Robert Williams** (U. Texas, Austin): *Tiling Spaces as Cantor Set Fiber Bundles*

### Alexander Helemskii, PIMS Distinguished Chair

*Contributed by Volker Runde, U. Alberta*

The 2003 holder of the PIMS Distinguished Chair in Dynamics at the University of Alberta was Alexander Ya Helemskii of Moscow State University. He arrived in Edmonton on July 11 and stayed till August 13. Besides giving a series of four lectures as PIMS Distinguished Chair, he participated in the 16th international conference on Banach algebra (Banach Algebras and their Applications, July 27 to August 9 — another PIMS sponsored event), where he chaired a workshop on his speciality: topological homology.

His lecture notes may be downloaded from [www.pims.math.ca/publications/distchair/](http://www.pims.math.ca/publications/distchair/).

A Summer School in Aperiodic Order at the University of Victoria is planned for the Summer 2005.

### BIRS Workshops

Northwest Functional Analysis Symposium, 2-Day Workshop, March 27–29, 2003

Coordinate Methods in Nonselfadjoint Operator Algebras, 5-Day Workshop, December 13–18, 2003

Aperiodic Order: Dynamical Systems, Combinatorics, and Operators, 5-Day Workshop, May 29–June 3, 2004

#### Faculty of the CRG:

Group Leaders: Douglas Lind (U. Washington), Ian Putnam (U. Victoria)

U. Alberta: A. Lau, R. Moody, V. Runde, A. Weiss  
U. Calgary: B. Brenken, M. Lamoureux, I. Nikolaev

U. Victoria: C. Bose, R. Edwards, M. Laca, J. Phillips

U. Washington: M. Einsiedler, C. Hoffman, D. Lind, S. Rohde, B. Solomyak, S. Tuncel

Visitors and other contributors: M. Boyle (Maryland), C. Denninger (Muenster), W. Parry (Warwick), D. Rudolph (Maryland), K. Schmidt (Vienna)

The Dynamics & Related Topics webpage is at [www.pims.math.ca/CRG/dynamics/](http://www.pims.math.ca/CRG/dynamics/). Further information and a preliminary list of activities for 2004–05 may be found there.

## Period of Concentration on Probability & Statistical Mechanics

There will be a period of concentration in Probability and Statistical Mechanics at PIMS in the period April 2004–August 2006.

### Summer School in Probability

UBC, May 25–June 25, 2004

*Contributed by Martin T. Barlow, UBC*

This summer school will consist of two advanced graduate courses, given by **Martin T. Barlow** (UBC) and **Gregory F. Lawler** (Duke University). The intention is that these will be official courses at UBC. Therefore graduate students at

universities in Western Canada can receive credit for them through the Western Deans Agreement.

Those interested in attending these courses (graduate students, PDF's, faculty members) are encouraged to sign up using the online registration form.

There will be some financial support available for graduate students and postdoctoral fellows who would like to attend. The deadline for applications for financial support is December 1, 2003. For more information please see [www.pims.math.ca/science/2004/ssprob](http://www.pims.math.ca/science/2004/ssprob)

The two courses are:

*Random walks and geometry of graphs*

Instructor: Martin T. Barlow (UBC)

In the late 1950s and early 1960s de Giorgi, Moser and Nash obtained regularity estimates for divergence form PDEs. Their ideas have been very fruitful, and have been developed in many directions, but it is only fairly recently that they have been extended to the context of random walks on graphs.

This course will survey these ideas, the interactions between geometric properties of graphs, and the behaviour of random walks, transition densities, and harmonic functions.

Prerequisites will be a knowledge of first-year graduate probability at roughly the level of Durrett, Probability: Theory and Examples.

*The Schramm-Loewner evolution and other*



*conformally invariant processes in the plane*

Instructor: Gregory F. Lawler (Duke University)

A number of lattice systems in statistical physics "at criticality" are conjectured to have scaling limits that are in some sense conformally invariant.

This course will focus on the mathematics of these limits, including the Schramm-Loewner evolution (SLE) and restriction measures (which are related to 2-d Brownian motions), as well as some discussion of discrete models.

Prerequisites will be first-year graduate courses in probability and complex variables (at the level of Durrett, Probability: Theory and Examples and Ahlfors, Complex Analysis, respectively).

**Seminar on Stochastic Processes**  
UBC, May 20–22, 2004

This meeting will host 80-100 participants. In addition to the five plenary lectures there will be parallel informal sessions in a number of topics stressing current projects and open problems.

The invited speakers are:

**Rene Carmona** (Princeton)

**Robert Dalang** (EPF Lausanne)

**Alice Guionnet** (Ecole Normale Supérieure de Lyon)

**Yves Le Jan** (Orsay)

**Balint Virag** (University of Toronto)

The meeting is being organized by M. Barlow, A. Holroyd, V. Limic, E. Perkins (UBC)

**Analysis, Probability, and Logic: a Conference in Honor of Edward Nelson**  
UBC, June 17–18, 2004

**Edward Nelson** (Princeton University) has done beautiful and influential work in probability, functional analysis, mathematical physics, non-standard analysis, stochastic mechanics, and logic. This conference will be an occasion where his students, colleagues, and friends can meet for an overall look at his work and its influence on current research. The experience is intended to be serious, illuminating, and enjoyable.

The programme will include review talks on Nelson's research and a few invited talks on current directions in the areas of his interest. It will also allocate time for enthusiasts of relevant research areas to have less formal encounters. Some people may wish to stay the following weekend for hiking or sight-seeing or for further discussions.

The organizing committee consists of David Brydges, Eric Carlen, Bill Faris, and Greg Lawler.

The plenary speakers are **Sam Buss** (UCSD), **Eric Carlen** (Georgia Tech.), **Len Gross** (Cornell), **Greg Lawler** (Cornell), **Barry Simon** (Cal Tech) and **Cedric Villani** (ENS de Lyon).

In the fall of 2004 there will be a Pacific Northwest Probability Seminar at the University of Washington.

**Summer School in Probability**  
June 6–July 1, 2005 at UBC

**BIRS Workshops**

BIRS Workshop on Statistical Mechanics of Polymer Models, May 10–15, 2003 at BIRS

BIRS Workshop on Stochastic Partial Differential Equations, September 27–October 2, 2003 at BIRS.

BIRS Workshop on Analytic and Geometric Aspects of Stochastic Processes, April 10–15, 2004 at BIRS.

Stability and Computations for Stochastic Differential Delay Equations, Research in

Teams, July 24–August 7, 2004, BIRS  
Participants: **Salah Mohammed** (Southern Illinois University), **Evelyn Buckwar** (Humboldt University), **Tony Shardlow** (Manchester), **Rachel Kuske** (UBC)

Competing Species and Predator-Prey Stochastic Models, Research in Teams, August 2004, BIRS  
Participants: **Rick Durrett** (Cornell U.), **Leonid Mytnik** (Technion), **Ed Perkins** (UBC)

**Faculty of the CRG:**

Group Leaders: David Brydges (UBC), Chris Burdzy (U. Washington), Ed Perkins (UBC), Byron Schmuland (U. Alberta)

U. Alberta: M. Kouritzin

UBC: M. Barlow, J. Feldman, A. Holroyd, V. Limic, G. Slade, J. Walsh

U. Saskatchewan: C. Soteros, R. Srinivasan

U. Washington: Z.-Q. Chen, B. Erickson, C. Hoffman, L. Korf, S. Rohde

Microsoft Research: C. Borgs, J. Chayes, O. Schramm, D. Wilson

Other Institutions: D. Dawson (McGill), R. van der Hofstad (Eindhoven)

The Probability & Statistical Mechanics CRG webpage is at [www.pims.math.ca/CRG/prob-ability/](http://www.pims.math.ca/CRG/prob-ability/). Further information and a preliminary list of activities for 2004–05 may be found there.



Participants of the BIRS Workshop on Statistical Mechanics of Polymer Models

# Period of Concentration on Topology & Knot Theory $\pi$

This Period of Concentration will start in 2004.

The PIMS community has an active group of researchers in topology and related fields. Their research may be roughly divided into two major themes: geometric and algebraic. Among the geometric issues being studied by PIMS topologists are the classification of manifolds (particularly in dimension 3 and 4), group actions on Riemann surfaces, knot theory and its applications, and relating 3-manifold topology to relativity theory. A sample of the contributions in algebraic topology are: application of algebraic topology to robotics, developing equivariant minimal models in homotopy theory, applying subtle algebraic properties of projective spaces and bundles to solve classical problems in quadratic forms and combinatorics. Because of their geographic separation and diversity of interests, this community of scientists is particularly well-served by forming a collaborative research group.

## MSRI-PIMS Summer Graduate Programme: Knots and 3-Manifolds UBC, July 7-20, 2004

This programme is cosponsored by the Mathematical Sciences Research Institute (MSRI) and PIMS. It is open only to graduate students nominated by MSRI's Academic Sponsors.

The mathematical theory of knots has become one of the most active areas of mathematics in the last few decades. Two important reasons for this

are that many fields of mathematics (and physics) converge in the study of knots, and secondly there are applications to the study of manifolds as well as fields such as stereochemistry and molecular biology.

This course will begin with an introduction to the subject, covering classical subjects such as the knot group, Seifert surfaces, Dehn surgery, branched coverings, Alexander polynomial as well as more recent work such as knot polynomials, skein theory, etc. The second week will be devoted to more specialized subjects such as hyperbolic geometry in knot theory, quantum invariants, Vassiliev theory, representation theory and the Casson invariant.

The lecturers include Colin Adams, Steve Boyer, Roger Fenn, Louis Kauffman, Dale Rolfsen, and Susan Williams.

It is being organized by Steven Boyer (UQAM), Roger Fenn (Sussex), Dale Rolfsen (Chair, UBC), Denis Sjerve (UBC).

For more information please see [www.pims.math.ca/science/2004/KT3Mgrad/](http://www.pims.math.ca/science/2004/KT3Mgrad/).

This programme will be followed by:

## Knots at PIMS: Workshop in Knot Theory and 3-Manifolds, UBC, July 21-30, 2004

For more information please see [www.pims.math.ca/science/2004/KT3Mwksp/](http://www.pims.math.ca/science/2004/KT3Mwksp/).

## BIRS Workshops

Topology of Manifolds and Homotopy Theory, 5-day Workshop, March 19–24, 2004

Knots and their Manifold Stories, 5-day Workshop, May 7–12, 2004

Braid Groups and Applications, 5-day Workshop, October 15–20, 2004

Cascade Topology Seminar, 2-day Workshop, 2004

## Faculty of the CRG:

Group Leaders: George Peschke (U. Alberta), Dale Rolfsen (UBC), Laura Scull (UBC), Peter Zvengrowski (U. Calgary)

U. Alberta: J. Timourian  
UBC: J. Bryan, K. Lam, D. Sjerve

U. Calgary: K. Dale Rolfsen (UBC) Varadarajan

U. Washington: E. Babson, E. Devinatz, M. Freedman, S. Mitchell, J. Palmieri, J. Segal

The Topology & Knot Theory CRG webpage is at [www.pims.math.ca/CRG/topology/](http://www.pims.math.ca/CRG/topology/).



Dale Rolfsen (UBC)

## Yuri Gurevich, PIMS Distinguished Chair at SFU

Yuri Gurevich was a PIMS Distinguished Chair at Simon Fraser University in 2003. In April he gave one lecture on *Executable Specifications: The Abstract State Machine Approach* and in July he spoke twice about *The Theory of Abstract State Machines*.

Yuri Gurevich is Senior Researcher at

**András Hajnal** (Rutgers University) will be a PIMS Distinguished Chair at the University of Calgary in fall 2004.

Microsoft Research in Redmond, WA. He heads the group on Foundations of Software Engineering. Dr. Gurevich started his career as an algebraist. Later he became a logician. Then he moved to computer science, where his main projects have been Abstract State Machines, Average Case Computational Complexity, and Finite Model Theory. Dr. Gurevich has been honored as Dr. Honoris Causa of the University of Limburg, Belgium (1998), a Fellow of the Association for Computing Machinery (1996), as well as a Fellow of the John Simon

Guggenheim Memorial Foundation (1995).

These lectures were organized by Alistair Lachlan, Uwe Glässer and Evgenia Ternovska (SFU).

For more information or to watch his lectures please see [www.pims.math.ca/science/2003/distchair/gurevich/](http://www.pims.math.ca/science/2003/distchair/gurevich/).



Yuri Gurevich  
(Microsoft Research)



# PIMS Thematic Programme on Inverse Problems and Applications

Contributed by *Gunther Uhlmann (U. Washington)*, *Gary Margrave (U. Calgary)* and *Robert McCann (U. Toronto)*

Inverse problems are problems in which the goal is to find material or biological properties of objects or information about the objects' surrounding environment which cannot be measured directly or it is not desirable to do so. These problems arise in many areas of applications including geophysics, medical imaging, remote sensing and non-destructive evaluation of materials. During the last twenty years or so there have been remarkable developments in the mathematical theory of inverse problems. These developments together with the enormous increase in computing power and new powerful numerical methods have made it possible to make significant progress on increasingly more realistic and difficult inverse problems. The purpose of the Thematic Programme was to bring together mathematicians and practitioners to work on these problems. A series of workshops on inverse problems were held at different locations, during the year 2003, emphasizing these different applications. Several of these activities were sponsored jointly by PIMS and the NSF. **Gunther Uhlmann** was the coordinator of the PIMS thematic year.

Please see [www.pims.math.ca/inverse/](http://www.pims.math.ca/inverse/) for more information.

## PASI on PDE, Inverse Problems and Non-Linear Analysis Santiago, Chile, Jan 6–18, 2003

The Pan-American Advanced Studies Institute (NSF) are funded by the US NSF and DOE. The PASI is modelled on the NATO Advanced Studies Institute. Canadian participation in the PASI in Chile was supported by PIMS. The PASI was organized by Rafael Benguria (Universidad Católica de Chile), Carlos Conca (U. Chile), Nassif Ghoussoub (PIMS and UBC), Raul Manasevich (U. Chile), Wei-Ming Ni (U. Minnesota), Gunther Uhlmann (U. Washington) and Michael Vogelius (Rutgers).

The PASI on Partial Differential Equations (PDE), Inverse Problems (IP) and Non-Linear Analysis (NLA) was held at the Centro de Modelamiento Matemático (CMM), Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile. The interaction between PDE, IP and NLA has produced remarkable developments in the last couple of decades or so. One of the main objectives of the PASI was to bring many of these developments to advanced graduate students, postdocs and other scientists in the Americas interested in these fields and their applications. Another important objective of the PASI was to foster international cooperation throughout the Americas by bringing different areas of expertise in PDEs, IP and NLA in one event. More than 160 participants mainly from several countries of the Americas participated in this unique workshop.

During the first week of the PASI there were a series of minicourses given by **Jean-Bernard Baillon** (U. Chile), **Jerome Busca** (U. Paris IX), **Luis Caffarelli** (UT, Austin), **F. Alberto Grunbaum** (Berkeley), **Maarten de Hoop** (Colorado School of Mines), **Peter Kuchment** (Texas A&M), **Yan Yan Li** (Rutgers) and **Michael Vogelius** (Rutgers). During the second week the workshop focused on recent developments in the interaction between IP, NLA and PDE.

## BIRS Workshop on Scattering and Inverse Scattering BIRS, March 22–27, 2003

This workshop was organized by **Richard Froese** (UBC) and **Gunther Uhlmann** (U. Washington).

In the fields of scattering and inverse scattering theory techniques of microlocal analysis, including the use of eikonal equations and of complex geometrical optics solutions to Schrödinger and other equations, have led to substantial progress in recent years.

Despite close mathematical connections between the fields of scattering and inverse scattering there has not always been a strong interaction between these fields. Part of the rationale of this workshop was to bring together workers who might not ordinarily interact, but could benefit from sharing ideas. The 40 participants took full advantage of this opportunity.

Scattering theory seeks an understanding of spectral phenomena for noncompact manifolds. There has been a recent focus in this subject on what is now termed geometric scattering, which amounts to the study of scattering on classes of noncompact complete manifolds with regular structures at infinity. Several of the lectures were concerned with questions about the smooth parametrization of the continuous spectrum by functions on some ideal boundary, the structure of the scattering matrix as an operator on this ideal boundary, and the study of resonances, which are poles of the meromorphic continuation of the resolvent. There are many subtle connections between these objects and the geometry of the underlying manifold.

Inverse scattering is how we obtain a large part of our information about the world. An everyday example is human vision: from the measurements of scattered light that reaches our retinas, our brains construct a detailed three-dimensional map of the world around us. Dolphins and bats are able to do the same thing from listening to scattered sound waves.

The inverse scattering problems discussed in the

workshop included the determination of the interior structure of the Earth by measuring the travel times of seismic waves, and the inverse quantum scattering problem which attempts to determine the inner structure of the atom and its constituents from studying the scattering when materials are bombarded with particles. Another important inverse scattering problem discussed in the workshop is reflection seismology which uses the reflection of seismic waves to locate oil deposits. Inverse obstacle scattering is used in radar and sonar and several of the lecturers dealt with this topic.

## Seismic Wave Simulation and Seismic Imaging: A PIMS Summer School U. Calgary, July 14–18, 2003

The first part of this summer school examined real seismic data, and several mathematical models for the forward problem (the simulation of seismic waves) were presented. Wave equations and their solutions were explored. Essential mathematical techniques such as Greens functions, Kirchhoff diffraction theory, and ray theory were developed and examined.

In the second part, these concepts and tools were applied to develop the prototypical approaches to the seismic imaging problem. The Born and Kirchhoff approximations were shown to lead to direct schemes for the estimation of subsurface reflectivity that are the basis for modern imaging techniques. The strengths and weaknesses of these techniques were examined and a survey of more advanced, emerging methods was presented. Emphasis was placed on understanding the assumptions and limitations of each technique.

The instructors at this summer school were:

**Len P. Bos** (Math & Statistics, U. Calgary)  
**Robert J. Ferguson** (Jackson School of Geosciences, Geological Sciences, U. Texas, Austin)  
**Gary F. Margrave** (Geology & Geophysics, U. Calgary)

## PIMS Geophysical Inversion Workshop U. Calgary, July 20–25, 2003

The University of Calgary campus was the setting for more than eighty mathematicians, physicists, and geophysicists from around the world, who gathered to discuss geophysical inversion in theory and practise.

The workshop was organized by **Gary Margrave** (U. Calgary), **Martijn de Hoop** (Colorado State of Mines), and **William (Bill) Symes** (Rice U.).

There were ten invited speakers and twenty-three contributed papers which made for a very full agenda. In addition, there were more than forty non-presenting registered attendees and many more people dropped in for the occasional presentation. The strong linkage between the subject matter and the business interests of the Calgary oil and gas exploration community bolstered workshop attendance. Though there were more geophysicists than mathematicians in attendance, there were some very well known people in the latter category, while many of the former were effectively applied mathematicians. Many graduate students were also present and some of these had stayed over from the Seismic Imaging Summer School of the previous week.

There were many presentations on the mathematics of wave propagation and the implied inverse problem, which is often called seismic imaging. Seismic imaging is a central theme in geophysical inversion and is a rapidly moving field. Fourier integral operators and pseudodifferential operators were found in numerous presentations suggesting that many re-

searchers are finding these operators useful in seismic inversion. Another common theme in seismic inversion is the recognition that the inversion operation can also be formulated as a generalized Radon transform. In addition to papers on the seismic reflection problem, there were also a variety of papers dealing with the inverse travelttime problem and other aspects of kinematics. Finally, there were several more general presentations on inverse theory that were not limited to seismological data.

The general atmosphere of the workshop was informal and congenial. Many presentations elicited a lively exchange of ideas. The two social events, an opening barbeque and a dinner in the latter part of the week, were well attended. In the end, there were many expressions of gratitude to PIMS for funding the event and a general desire to have another meeting in a few years.

The invited speakers were:

**Norman Bleistein** (Colorado School of Mines): *Seismic Modelling, Migration and Inversion: From String Construction to Theory to Computer Implementation*

**Michael Bostock** (UBC): *Inverse scattering of teleseismic wavefields*

**Hans Duistermaat** (U. Utrecht): *Fourier integral operators as a tool to take care of the high frequency part of waves*

**Louis Fishman** (MDF International previously at NRL): *One-Way Wave Equation Modeling in Seismo-Acoustic Wave Propagation and Inversion*

**Rob van der Hilst** (Massachusetts)

**Larry Lines** (U. Calgary): *Geophysical Inversion - Provehito in Altum!*

**Doug Oldenburg** (UBC): *Inversion for applied geophysics: Applications to mineral exploration*

**Mauricio Sacchi** (U. Alberta): *Regularized Least-squares Migration/Inversion*

**Christiaan Stolk** (Ecole Polytechnique): *The determination of medium discontinuities by migration: results from microlocal analysis*

**Gunther Uhlmann** (U. Washington): *The inverse kinematic problem*

## Workshop on Inverse Problems and Medical Imaging PIMS-UBC, August 4–8, 2003

This workshop was organized by **John Schotland** (Chair, Biomedical Engineering, U. Pennsylvania), **Richard Albanese** (Armstrong Research Lab, Brooks AFB), **Tom Budinger** (Biomedical Engineering, Berkeley), **David Isaacson** (Rensselaer Polytechnical Institute), **Amir Gandjbakhche** (National Institute of Health) and **Gunther Uhlmann** (U. Washington).



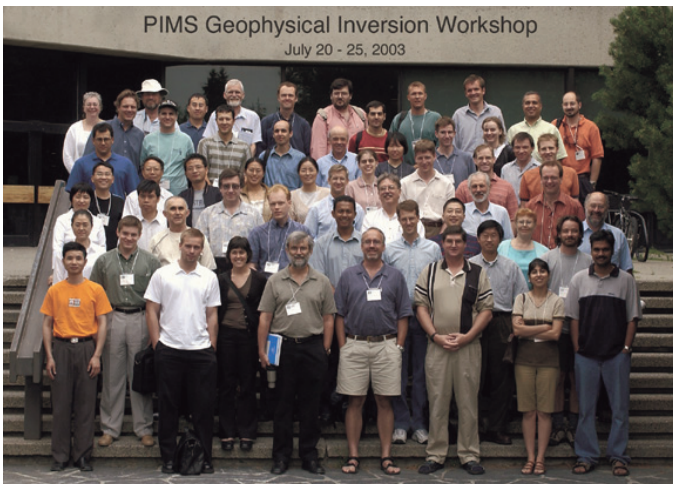
John Schotland (U. Penn) and Gunther Uhlmann (U. Washington)

It was sponsored in collaboration with the US National Science Foundation.

The workshop focused on recent developments in medical imaging, particularly the advances in mathematics which have allowed for significant enhancement of widely used imaging techniques such as x-ray tomography, magnetic resonance imaging, and ultrasonic imaging. Mathematical developments in emerging medical imaging modalities were also surveyed. The more than 70 participants included physicians, physicists and mathematicians directly involved in the mathematical and practical aspects of medical imaging.

Several of the lectures described relatively recent medical modalities which might improve on the earlier detection and diagnosis of breast cancer. These new imaging methods are non-invasive and they all provide for more detailed information than given by standard x-ray mammography about different properties of breast tissue. For instance, electrical impedance tomography images the electrical conductivity of tissue by making voltage and current measurements at the boundary of the body. Breast tumours have a much larger conductivity than surrounding tissue. Transient elastography uses information given by shear ultrasound waves

*continued on the next page*







Participants of the workshop on *Onverse Problems and Medical Imaging*

to measure the breast tissue hardness or elasticity. In optical tomography one sends infrared light through breast tissue to locate and quantify regions of oxygenated and deoxygenated hemoglobin. This might help detect early tumour growth and characterize the stage of a tumour by learning about its vascular makeup.

The invited speakers were:

**Simon Arridge** (UCL): *Reconstruction Methods in Optical Tomography and Applications to Brain Imaging*

**Yoram Bresler** (UIUC): *Fast Hierarchical Algorithms for Tomography*

**Thomas Budinger** (UC Berkeley): *How Medical Science will Benefit from Mathematics of Inverse Problems*

**Emmanuel Candes** (Cal Tech): *New Multiscale Thoughts on Limited-Angle Tomography*

**Scott Carney** (UIUC): *Computed Imaging for Near-Field Microscopy*

**Anna Celler** (UBC): *Inverse Problems and Nuclear Medicine*

**Rolf Clackdoyle** (U. Utah): *Reconstruction from Truncated Fanbeam and Parallel-Beam Projections*

**Mathias Fink** (ESPCI): *Transient Elastography and Supersonic Shear Imaging*

**Amir Gandjbakhche** (NIH, USA): *Effects of Target Non-localization on the Contrast of Optical Images: Lessons for Inverse Reconstruction*

**Frederick Greensite** (UC Irvine): *Multivariate Inverse Problems*

**David Isaacson** (RPI): *Progress and Problems in Electrical Impedance Imaging*

**Matti Lassas** (Rolf Nevalinna Institute, Finland): *Inverse scattering problem with a random potential*

**Brian Litt** (UPenn): *Predicting Epileptic Seizures From Intracranial EEG*

**Joyce McLaughlin** (RPI): *Interior Elastodynamics Inverse Problems: Recovery of Shear Wavespeed in Transient Elastography*

**Michael Miller** (Johns Hopkins University): *Image Analysis Models in Computational Anatomy*

**Frank Natterer** (U. Muenster): *3D Emission*

*Tomography Via Plane Integrals*

**Joseph O'Sullivan** (Washington U., St. Louis): *Information Geometry, Alternating Minimizations, and Transmission Tomography*

**George Papanicolaou** (Stanford): *Imaging in Clutter*

**Sarah Patch** (GE Medical Systems): *Thermoacoustic Tomography - An Inherently 3D Generalized Radon Inversion Problem*

**Todd Quinto** (Tufts): *Limited Data Tomography in Science and Industry*

**Yoram Rudy** (Case Western Reserve U.): *ECGI: A Noninvasive Imaging Modality for Cardiac Electrophysiology and Arrhythmias*

**John Schotland** (UPenn): *Tomography and Inverse Scattering with Diffuse Light*

**Meir Shinnar** (UMDNJ): *Inversion of the Bloch Equation*

Some of the talks were videotaped and may be watched in RealVideo format from the web page.

## Optimal Transportation and Nonlinear Dynamics Workshop PIMS-UBC, August 11–15, 2003

This workshop was organized by **Michael Cullen** (UK Meteorological Office), **Lawrence C Evans** (UC, Berkeley), and **Wilfrid Gangbo** (Georgia Tech) and cosponsored by their NSF Focused Research Group.



Wilfrid Gangbo

The conference commenced with an survey lecture by **Robert McCann** (U. Toronto) on *Optimal Transportation: The Lay of the Land*, and featured special sessions organized by **Felix Otto** (U. Bonn) about *Analyzing Gradient Descent Models in Physical Systems* and **Michael Cullen** on *Atmospheric Dynamics and Scientific Computation* problems from optimal transport theory.

A minicourse on *Hessian and Curvature Equations* was given by **John Urbas** (Australian National University). These second-order elliptic PDEs arise in geometric and optimization problems, but are fiendishly difficult to analyse due to nonlinearities which result from replacing the Laplacian operator in Poisson's equation with the Hessian determinant or (in 3 dimensions and higher) other elementary symmetric functions of the Hessian's eigenvalues.

The conference was a successful sequel to the *Transport me to Toronto!* workshop held at the Fields Institute in August 2001.

The invited lecturers were:

**Jean-David Benamou** (INRIA): *Numerical Resolution of an "Unbalanced" Mass Transport Problem*

**Guy Bouchitte** (U. Toulon et du Var): *Asymptotic of Optimal Location Problems*

**Yann Brenier** (U. Nice Sophia-Antipolis): *Displacement of 2D Vector Fields and 3D Electromagnetism*

**Peter Constantin** (U. Chicago): *Diffusive Lagrangian Transformations*

**Mike Cullen** (U. Reading): *Fast Algorithm for the Monge-Kantorovich Problem (joint work with Purser)*

**Jean Dolbeault** (U. Paris IX - Dauphine): *Direct Entropy Methods*

**Mikhail Feldman** (U. Wisconsin): *Lagrangian Solutions of 3D Semigeostrophic Equations in Physical Space*

**Jim Feng** (U. Massachusetts Amherst): *Large Deviations, Hamilton-Jacobi and Mass Transport*

**Nassif Ghoussoub** (UBC, PIMS): *A Unifying Framework for Geometric Inequalities*

**Lorenzo Giacomelli** (U. di Roma "La Sapienza"): *A Variational Approach to Lubrication Approximation*

**Bernd Kawohl** (U. Köln): *Symmetries in Anisotropic Media*

**David Kinderlehrer** (Carnegie Mellon U.): *Diffusion Mediated Transport: Can We Understand Motion in Small Systems?*

**Robert McCann** (U. Toronto): *Optimal Transportation: The Lay of the Land*

**John Norbury** (Oxford)

**Adam Oberman** (U. Texas): *Exact Solution to the Semigeostrophic Equations in Elliptical Domains*

**Vladimir Olliker** (Emory U.): *Fermat's Principle and Mass Transport Problem*

**Felix Otto** (U. Bonn): *A Transportation Problem in Type I Superconductors*

**Max von Renesse** (U. Bonn): *Robust Lower Ricci Bounds by Optimal Transportation*

**Ian Roulstone** (U. Reading): *Semigeostrophic Theory on The Sphere*

**Giuseppe Savaré** (U. degli Studi Pavia): *Gradient Flows in Wasserstein Spaces*

**Andrei Sobolevskii** (Observatoire de la Cote D'Azur): *Reconstruction of the Early Universe by a Monge-Ampere-Kantorovich Mass Transportation Method*

**John Urbas** (Australian National U.): *Minicourse on Regularity and Estimates for Hessian and Curvature Equations*

**Xu-Jia Wang** (Australian National U.): *Light Reflection and Optimal Transportation*

Some lectures were videotaped and may be watched in RealVideo format from the web page.

# Recent Progress on De Giorgi's Conjecture

by Changfeng Gui, University of Connecticut



*Changfeng Gui was the winner of the 2002 PIMS Research Prize.*

## 1 Introduction

Symmetry is a common natural phenomenon which appears in many different areas of science. Many important partial differential equations (PDEs) have symmetries, i.e., invariance under certain transformations (or simply put, changes of variable). Researchers are very interested in special solutions which preserve all or part of the symmetry of the equations. These special solutions are the obvious solutions to study first, but more importantly they display the important symmetry properties of the underlying objects. To mathematicians however, it often seems a more interesting question whether or not all physical solutions to a differential equation have a certain symmetry. Ultimately “PDEers” would hope that all solutions could be classified according to certain symmetries. The study of symmetry of solutions to certain PDEs has attracted much attention over the last two decades. Among numerous problems and papers in this direction, a conjecture of De Giorgi stands out as a classical problem. Not only it is physically important but it is also mathematically challenging and of rich structure. In this article I will describe De Giorgi's Conjecture, and its connection to phase transition and differential geometry. I will also give a brief survey of recent developments and methods used in the investigation.

## 2 The Conjecture and its Relation to Phase Transition and Minimal Surfaces

J. W. Cahn is a world renowned material scientist at the National Institute of Standard and Technology (NIST). In 1956 Cahn and J. E. Hilliard suggested a mathematical model to study antiphase transition. The two pure phase states are modelled by constant functions  $u \equiv 1$  and  $u \equiv -1$ . A general state then should be represented by a function with values between  $-1$  and  $1$ , depending on the relative density of each of the phases in the mixture. Cahn and Hilliard used an energy functional to gauge such physical states,

$$(1) \quad E_\epsilon(u) = \int_{\Omega} \left( \frac{\epsilon^2}{2} |\nabla u|^2 + F(u) \right) dx,$$

where  $\epsilon > 0$  is the diffusion rate, which is usually a small positive constant, and  $\Omega$  is the physical region of the antiphase material. The potential function  $F(u) = (1 - u^2)^2/4$  is a so-called double-well potential with equal depths.

The first term in the energy functional measures the tension-related transitional energy and the second term is the Van der Waals free energy. The equilibria of interest are the minimizers of the energy functional under various constraints. If one starts with a non-equilibrium state, then the state evolves according to the gradient flows under suitable norms. For

example, under the usual  $L^2$  norm the gradient flow is described by the Allen-Cahn equation

$$(2) \quad u_t = \epsilon^2 \Delta u - F'(u), \quad (x, t) \in \Omega \times \mathbf{R}^+;$$

while under the  $H^{-1}$  norm the gradient flow becomes the Cahn-Hilliard flow

$$(3) \quad u_t = -\Delta(\epsilon^2 \Delta u - F'(u)), \quad (x, t) \in \Omega \times \mathbf{R}^+.$$

The latter preserves the total mass of each of the phases in the process. The study of the equilibria or the flows of the Cahn-Hilliard energy has been a very active and interesting area of research, particularly when it is concerned with the limiting process as  $\epsilon$  goes to 0. For the flow cases, a central question is how the sharp interfaces in the limiting process form and evolve. One remarkable phenomenon is that an interface moves by its mean curvature (see e.g. [11] and references therein).

Here, however, I would like to discuss in some detail the equilibria of (1). The standard functional analysis methods (variational methods) imply that the minimizers of (1) exist under appropriate constraints. Note that the minimizers without constraint are only the pure states  $u \equiv -1$  and  $u \equiv 1$ . Let us focus on the case where the total mass of each phase is fixed, i.e.  $\int u dx = m$  for some  $m \in (-1, 1)$ . Denote such a minimizer by  $u_\epsilon$ . It is easy to see that  $u_\epsilon$  must tend to 1 or  $-1$  almost everywhere except on a set of measure zero (after possibly taking a subsequence). One could argue that such a convergence is indeed in the sense of the so-called  $\Gamma$  convergence (see [19] for the exact meaning of  $\Gamma$  convergence). We can simply say that  $u_\epsilon \rightarrow \chi_{\Omega_1} - \chi_{\Omega_{-1}}$  locally in  $L^1$  where  $\{\Omega_1, \Omega_{-1}\}$  is a partition of  $\Omega$  and  $\chi_\Omega$  is the characteristic function on  $\Omega$ . It is obvious that such a partition with fixed volume cannot be arbitrary. A remarkable result of Modica [17] basically says that the partition must minimize the area of interfaces between  $\Omega_1$  and  $\Omega_{-1}$  among all possible partitions with fixed volume. In this sense, the limit of the minimizers is pretty well understood globally, or in the physical terminology, the macrostructure of antiphases is pretty clear. Nevertheless, one may want to study further the microstructure near the phase boundary. We may choose a point on the interface and zoom in on the neighbourhood of this point in order to see the detailed transition behaviour. Mathematically, we set the origin at this point and scale the spatial variable by  $\epsilon$ , i.e., we look at  $U_\epsilon(y) = u(\epsilon y)$ ,  $x = \epsilon y \in \Omega$ . It can be shown that  $U_\epsilon$  converges nicely (say,  $C_{local}^{2,\alpha}(\mathbf{R}^n)$ ) to an entire solution

$$(4) \quad \Delta u - F'(u) = 0, \quad |u| \leq 1, \quad x \in \mathbf{R}^n.$$

De Giorgi was an influential Italian mathematician in the 20th century. He became famous for his work on the regularity of solutions to elliptic and parabolic equations, and his joint work with Bombieri and Giusti on the construction of nonplanary minimal graphs in  $\mathbf{R}^n$  with  $n \geq 9$ . In his pursuit of using nonlinear PDEs to study minimal surfaces and, in general, sharp interfaces, he proposed the following problem which later became known as **De Giorgi's Conjecture**:



**Conjecture 1** Suppose that  $u$  is an entire solution of equation (4) satisfying

$$(5) \quad \frac{\partial u}{\partial x_n} > 0, \quad x \in \mathbf{R}^n.$$

Then, at least for  $n \leq 8$ , the level sets of  $u$  must be hyperplanes.

The conjecture may be considered together with the following natural condition:

$$(6) \quad \lim_{x_n \rightarrow \pm\infty} u(x', x_n) = \pm 1.$$

From the viewpoint of phase transition, the solutions in De Giorgi's conjecture describe the microstructure of antiphase transition near the antiphase boundary. Condition (6) simply means the outer regions are close to the two pure phase states. If the conjecture is true, the solutions will depend on one variable, i.e.,  $u(x) = g(a \cdot x + b)$  for some  $a \in (S^{n-1})^+$ ,  $b \in \mathbf{R}^n$ , where  $g(s)$  is the solution of a simple ordinary differential equation (ODE)

$$(7) \quad g''(s) + g(s) - g(s)^3 = 0, \quad g'(s) > 0, \quad s \in \mathbf{R}.$$

It is well-known that the only solution of this ODE (up to translation) is  $g(s) = \tanh(s/\sqrt{2})$ . In this sense, the conjecture essentially states that the basic local configuration near the interface of antiphases should be unique and should depend solely on the distance to the interface. A similar discussion may also be done for the gradient flows of the Cahn-Hilliard energy functional to reveal the microstructure near the moving interfaces.



Ennio De Giorgi

The conjecture concerns all dimensions, in particular dimensions less than or equal to 8. One might wonder why dimensions higher than 3 are considered and why dimension 8 is special. The reason here is more related to geometry than to phase transition, unless one goes further and worries about the more fundamental question of the dimension of our universe. In differential geometry, one major area of interest is the study of minimal surfaces (see a recent article in the Notices of the AMS [9]).

A traditional theorem of Bernstein says that any two-dimensional complete minimal graph in  $\mathbf{R}^3$  must be planar. A Bernstein type theorem was proven for  $\mathbf{R}^n$  when  $n$  is less than or equal to 8; while for dimensions higher than 8, there are many non-planary minimal graphs. The first such graph was constructed by Bombieri, Giusti and De Giorgi in 1969. The study of the Bernstein problem is closely related to the understanding of regular and singular sets of minimal hypersurfaces and other similar objects. It seems that De Giorgi wanted to find another viewpoint to understand minimal surfaces and the related geometric objects. In De Giorgi's conjecture, the level sets of the solutions are complete graphs and are expected to be similar to minimal surfaces. Indeed, we could scale the solutions of (4) back to any fixed bounded domain, which is the reverse process of the zoom-in process described above. The global structure of the solution can then be revealed. The level sets of solutions to (4) would become the boundaries of minimal cones. In this sense, the level sets of solutions are asymptotically like minimal graphs at infinity. Therefore it is believed that De Giorgi's conjecture should be true for at least  $n \leq 8$ . (See [2] for a detailed discussion.)

In both the phase transition model and De Giorgi's conjecture, the potential  $F$  can be replaced by a more general double-well potential, i.e.

$$(8) \quad \begin{cases} F(u) > 0, & u \in (-1, 1), & F(-1) = F(1) = 0 \\ F'(-1) = F'(1) = 0, & F''(-1) > 0, & F''(1) > 0. \end{cases}$$

### 3 Liouville Property, Local Minimality and Energy Estimates

Equation (4) and its solutions have some very interesting features, and naturally one wants to use these features to prove De Giorgi's conjecture. One obvious feature is the translation and rotation invariance of the equation. Hence any translations and rotations of the solutions are still solutions. The conjecture can also be interpreted as that the solutions to (4) indeed preserve the invariance of translation and rotation in a  $n - 1$  dimensional subspace. Here the main difficulty arises from the nonuniform behaviour of the solutions at infinity. The methods used so far can be roughly categorized as Liouville property methods, energy methods and maximum principle methods, although these methods are used in various forms and sometimes combined.

Modica and Mortola were first to obtain (partial) results for  $n = 2$ . In [20] they proved De Giorgi's conjecture for  $n = 2$  under the additional assumption that the level sets of  $u$  are the graphs of an equi-Lipschitzian family of functions. A classical version of the Liouville theorem is used in the proof.

Modica discovered the following pointwise gradient estimate for bounded entire solutions of (4).

$$(9) \quad |\nabla u(x)|^2 \leq 2F(u(x)), \quad \forall x \in \mathbf{R}^n.$$

He used the maximum principle and the translation invariance of the equation to prove this gradient bound. Later in [8], Caffarelli, Garofalo and Segala showed that De Giorgi's conjecture is true in any dimension provided that in (9) equality holds at some point  $x_0 \in \mathbf{R}^n$ .

Due to the translation invariance of (4), we can easily see that the derivative  $\sigma(x) = \frac{du}{dx_n}$  and any directional derivative  $\phi_\nu(x) = \nu \cdot \nabla u$  satisfy the linearized version of equation (4). Therefore the quotient  $h(x) = \frac{\phi_\nu(x)}{\sigma(x)}$  is a solution of the degenerate elliptic equation of the divergence form in the entire space

$$(10) \quad \operatorname{div}(\sigma^2 \nabla h) = 0, \quad x \in \mathbf{R}^n.$$

If  $\sigma$  is a positive constant, the classic Liouville theorem concludes that any bounded solution must be constant. To prove the De Giorgi Conjecture, it suffices to prove a Liouville theorem for this equation under appropriate conditions. Ghossoub and Gui [14] observed this and used a new Liouville-type theorem to prove De Giorgi's Conjecture for two-dimensional space.

A variation of De Giorgi's Conjecture is to replace (5) and (6) by the following uniform convergence assumption

$$(11) \quad u(x', x_n) \rightarrow \pm 1 \text{ as } x_n \rightarrow \pm\infty \text{ uniformly in } x' \in \mathbf{R}^{n-1},$$

and then ask whether

$$u(x) = g(x_n + T) \quad \text{for some } T \in \mathbf{R},$$

where  $g$  is the solution of the corresponding one-dimensional ODE.

This is also referred to as the Gibbons Conjecture, which was first established by Ghoussoub and Gui in [14] for  $n = 3$ , and later proved for all dimensions in [4], [6] and [13] independently. The proof of Gibbons Conjecture in dimension 3 in [14] is based on the Liouville property of degenerate elliptic operators of the divergence form in entire space. Later in [4] a general Liouville theorem for the divergence of degenerate operators was established and used to show that the De Giorgi Conjecture holds in all dimensions, provided all level sets of  $u$  are equi-Lipschitzian. The proofs in [6] and [13] use novel ideas involving applying the maximum principle in such a way that the translation and rotation invariance of (4) is essential (the so-called moving plane, sliding plane and rotating plane methods).

As we have seen before, equation (4) has a variational structure. It is, for any bounded domain  $\Omega$ , the Euler-Lagrange equation of the functional

$$(12) \quad E_{\Omega}(u) = \int_{\Omega} \left( \frac{1}{2} |\nabla u|^2 + F(u) \right) dx$$

defined on  $H^1(\Omega)$ . When  $\Omega$  is the ball  $B_R(0)$  centred at the origin with radius  $R$ , one may define

$$(13) \quad \rho(R) = \frac{E_{B_R}(u)}{R^{n-1}}$$

Modica proved that  $\rho(R)$  is increasing in  $R$ . Ambrosio and Cabre showed further that  $\rho(R)$  is bounded above by a finite positive constant. It is also known (see [19] and [2]) that solutions  $u$  of (4) and (5) are local minimizers of the functional  $E$  in the following sense: For any bounded smooth domain  $\Omega \subset \mathbf{R}^n$ ,

$$(14) \quad E_{\Omega}(u) = \min \left\{ E_{\Omega}(v); v = u \text{ on } \partial\Omega, |v| \leq 1, v \in C^1(\bar{\Omega}) \right\}.$$

Ambrosio and Cabre [3] obtained the above mentioned energy estimate and used it together with the Liouville property to settle the De Giorgi Conjecture in dimension 3.

Recently, Ghoussoub and Gui further developed a refined energy method which includes an estimate of the second-order term in the asymptotical expansion of  $\rho(R)$  when  $R$  goes to infinity.

One application of this approach is to prove De Giorgi's Conjecture in dimensions  $n = 4, 5$  under the additional assumption that the solutions also satisfy an anti-symmetry condition

$$(15) \quad u(y, z) = -u(y, -z) \text{ for } x = (y, z) \in \mathbf{R}^{n-k} \times \mathbf{R}^k,$$

where  $k$  is an integer with  $1 \leq k \leq n$ .

This approach generalizes a similar symmetry result for half-spaces and relaxes the growth condition of level sets at infinity.

On the other hand, it is natural to attempt to construct counterexamples with certain anti-symmetry, similar to those satisfied by the nonplanary minimal graphs which are counterexamples to the Bernstein problem in high dimensions. The above approach shows that such counterexamples do not exist for  $n = 4, 5$  for De Giorgi's Conjecture. However, there may still exist such counterexamples for  $n > 8$ .

I would like to add that in a recent preprint Savin showed De Giorgi's Conjecture for  $n \leq 8$ , under the reasonable condition (6). The main idea

is to prove a local version of improvement of flatness of the level sets for local minimizers, given that the level sets are sufficiently flat. Using the asymptotic flatness of the level sets of solutions to (4) for  $n \leq 8$ , he proved the slightly weaker version of the conjecture for dimensions less than or equal to 8. The details of the proof are under review.

## 4 Concluding Remarks & Open Problems

As explained before, De Giorgi's Conjecture is an interesting and important problem connecting applied fields to differential geometry. Several main tools in nonlinear PDEs are used in its investigation. In spite of major breakthroughs in recent years, the conjecture is still not completely solved. Even for the slightly weaker version of the conjecture with condition (6) and after Savin's result, it is still an open question whether or not there is a counterexample for dimensions higher than 8. Furthermore, one can replace condition (5) in the conjecture with the possibly weaker condition that  $u$  is a local minimizer, i.e., (14) holds for all bounded smooth domains. It is interesting to see whether or not conditions (5) and (14) are actually equivalent after a proper rotation.

Another related problem is the uniqueness of travelling wave solutions. If the double-well potential  $F$  does not have equal depth, i.e.  $0 = F(-1) < F(1)$  in (8), there always exists a unique one dimensional travelling wave solution (up to translation and after scaling) of (2) with fixed speed  $c_0 > 0$  which satisfies

$$(16) \quad g'' + c_0 g' - F'(g) = 0, \quad g' > 0, \quad s \in \mathbf{R}.$$

**Conjecture 2** Assume  $F(u)$  satisfies (8) with  $0 = F(-1) < F(1)$  and there is only one critical point of  $F$  in  $(-1, 1)$ , say  $u = 0$ . Consider

$$(17) \quad \Delta u + c \frac{\partial u}{\partial x_n} - F'(u) = 0, \quad \frac{\partial u}{\partial x_n} > 0, \quad |u| \leq 1, \quad x \in \mathbf{R}^n.$$

Then the above problem has solutions if and only if  $c \geq c_0$ . Furthermore, at least for  $n \leq 8$ , the level sets of these solutions must be hyperplanes.

We expect that De Giorgi's Conjecture and related problems will continue to be an exciting area of research.

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## Canada Won Two Gold and Three Bronze Medals at the 2003 International Mathematical Olympiad

Competing against students from 81 other countries, Canadian high school students have done extremely well, winning two gold medals and three bronze medals at the 44<sup>th</sup> International Mathematical Olympiad (IMO), Tokyo, Japan, July 11–18, 2003.

The six members of the 2003 Canadian IMO team were: Robert Barrington Leigh (Old Scona Academic High School, Edmonton), Olena Bormashenko (Don Mills Collegiate Institute, Toronto), Tianyi (David) Han (Woburn Collegiate Institute, Toronto), Oleg Ivrii (Don Mills Collegiate Institute, Toronto), János Kramár (U. Toronto Schools, Toronto), and Jacob Tsimerman (U. Toronto Schools, Toronto).

At the Awards Ceremony on July 18, 2003 in Tokyo, Gold Medals were awarded to Olena Bormashenko and Jacob Tsimerman; Robert Barrington Leigh, Tianyi (David) Han and Oleg Ivrii were awarded Bronze Medals. Olena Bormashenko is the first Canadian female student to win a gold medal and Jacob Tsimerman is the youngest Canadian student to win a gold medal.

The team was accompanied by the Team Leader, Andy Liu (U. Alberta), the Deputy Team Leader, Richard Hoshino (Dalhousie University), and the Observer, Robert Morewood (Burnaby South Secondary School).

*“Our young team performed extremely well. Canada finished in the top 13 and won five medals,”* said Andy Liu. *“I am immensely proud of their outstanding performance.”*

*“The IMO is the world championship high school mathematics competition. All six problems were very difficult and all of the Canadian students have done remarkably well. They have all demonstrated the essential problem solving skills, knowledge and creativity that is required to compete at this very high level,”* said Graham Wright, Executive Director of the Canadian Mathematical Society (CMS), the organization responsible for the selection and training of Canada’s IMO team.

Although students compete individually, country rankings are obtained by adding the teams’ scores. The maximum score for each student is 42 and for a team of six students the

maximum is 252. The Canadian team placed 12<sup>th</sup> out of 82 competing countries with a score of 119. “This ranking matches the best performance since Canada started competing at the IMO” said Graham Wright.

The top 13 teams and their scores are: Bulgaria (227); China (221); USA (188); Vietnam (172); Russia (167); Korea (157); Romania (143); Turkey (133); Japan (131); United Kingdom (128); Hungary (128); Canada (119) and Kazakhstan (119).

Since 1981, Canadian students have received a total of 14 gold, 27 silver, and 52 bronze medals. The six members of the Canadian IMO team were selected from among more than 200,000 students who participated in local, provincial and national mathematics contests. Prior to leaving for the 44<sup>th</sup> IMO, the team trained at the University of Calgary and the Banff International Research Station from June 24–July 10, 2003.

The 45<sup>th</sup> International Mathematical Olympiad will take place in Athens, Greece from July 4–18, 2004.

# Mahler's Measure

by Jeffrey D. Vaaler, University of Texas, Austin



*This article is based on the notes from the series of lectures Jeffrey Vaaler gave as a PIMS Distinguished Chair in June 2003 at Simon Fraser University.*

Let  $P(z)$  be a nonzero polynomial in  $\mathbb{C}[z]$  given by

$$P(z) = a_0 z^N + a_1 z^{N-1} + \cdots + a_{N-1} z + a_N$$

$$= a_0 \prod_{n=1}^N (z - \alpha_n), \quad \text{where } a_0 \neq 0.$$

Then the *Mahler measure* of  $P(z)$  is the positive number

$$(1) \quad \mu(P) = \exp \left\{ \int_0^1 \log |P(e(\theta))| d\theta \right\} = |a_0| \prod_{n=1}^N \max\{1, |\alpha_n|\},$$

where the second equality is Jensen's formula and we write  $e(\theta) = e^{2\pi i \theta}$ . It is immediate from the definition that the Mahler measure is multiplicative: if  $P(z)$  and  $Q(z)$  are nonzero polynomials in  $\mathbb{C}[z]$  then

$$(2) \quad \mu(PQ) = \mu(P)\mu(Q).$$

Of course Jensen's formula can be stated for more general classes of analytic functions. However, Mahler (see [18] and [19]) recognized that (1) and (2) could be used to prove basic inequalities about polynomials that are useful in problems of Diophantine approximation. Thus the term "Mahler's measure" is often used for (1) in research papers on arithmetical questions about polynomials. Today applications of Mahler's measure to problems in number theory, topology and dynamical systems extend far beyond the original setting of Mahler's earlier work. In this brief report we will discuss a few problems and results about Mahler's measure that are of particular interest to the author. A systematic treatment of further results on Mahler's measure can be found in the book of Everest and Ward [14].

There is an obvious generalization of Mahler's measure to polynomials in several variables. If  $P(\mathbf{z}) = P(z_1, z_2, \dots, z_N)$  is a nonzero polynomial in  $\mathbb{C}[z_1, z_2, \dots, z_N]$  then the Mahler measure of  $P$  is the positive number

$$\mu(P) = \exp \left\{ \int_{(\mathbb{R}/\mathbb{Z})^N} \log |P(e(\theta_1), e(\theta_2), \dots, e(\theta_N))| d\theta \right\}.$$

Again we find that the Mahler measure satisfies the multiplicative identity (2). But there is no simple analogue of the Jensen formula (1) for polynomials in two or more variables. However, for polynomials in two or more variables with integer coefficients some striking formulae for Mahler's measure are known. For example, C. J. Smyth [31] has shown that

$$\int_{(\mathbb{R}/\mathbb{Z})^3} \log |e(\theta_1) + e(\theta_2) + e(\theta_3)| d\theta = \frac{3\sqrt{3}}{4\pi} L(2, \chi),$$

and

$$\int_{(\mathbb{R}/\mathbb{Z})^4} \log |e(\theta_1) + e(\theta_2) + e(\theta_3) + e(\theta_4)| d\theta = \frac{7}{2\pi^2} \zeta(3),$$

where  $\chi$  is the nonprincipal Dirichlet character to the modulus 3,  $L(s, \chi)$  is the corresponding Dirichlet  $L$ -function, and  $\zeta(s)$  is the Riemann zeta-function. Further examples of this sort and related conjectures have been given in [2], [4], [10] and [25].

For large  $N$  Meyerson and Smyth [20] have established the estimate

$$(3) \quad \int_{(\mathbb{R}/\mathbb{Z})^N} \log |e(\theta_1) + e(\theta_2) + \cdots + e(\theta_N)| d\theta = \frac{1}{2} \log N - \frac{1}{2} \gamma + O\left(\frac{\log N}{N}\right)$$

as  $N \rightarrow \infty$ , where  $\gamma$  is Euler's constant. More generally, if  $L_{\mathbf{a}}(\mathbf{z}) = a_1 z_1 + a_2 z_2 + \cdots + a_N z_N$  is a nonzero complex linear form, then estimates for the logarithmic Mahler measure

$$\log \mu(L_{\mathbf{a}}) = \int_{(\mathbb{R}/\mathbb{Z})^N} \log |a_1 e(\theta_1) + a_2 e(\theta_2) + \cdots + a_N e(\theta_N)| d\theta,$$

have been given in [26]. For example, the following inequality is established in [26].

**Theorem** For all vectors  $\mathbf{a} \neq 0$  in  $\mathbb{C}^N$  we have

$$(4) \quad \log |\mathbf{a}| - \frac{1}{2} \gamma - 2 \leq \log \mu(L_{\mathbf{a}}) \leq \log |\mathbf{a}|,$$

where  $\gamma$  is Euler's constant and

$$|\mathbf{a}| = (|a_1|^2 + |a_2|^2 + \cdots + |a_N|^2)^{1/2}.$$

The upper bound on the right of (4) is immediate from Jensen's inequality, but the lower bound is more difficult. It suggests the following problem: Determine the increasing sequence of numbers

$$\delta_N = \sup \{ \log |\mathbf{a}| - \log \mu(L_{\mathbf{a}}) : \mathbf{a} \in \mathbb{C}^N, \mathbf{a} \neq \mathbf{0} \} \text{ and } \lim_{N \rightarrow \infty} \delta_N = \Delta.$$

We return now to consideration of polynomials in one variable, and especially polynomials with integer coefficients. In this setting there is great interest in the small values that  $\mu(P)$  takes for  $P(z)$  having integer coefficients. In view of (1) and (2) we may restrict attention to small values of  $\mu(P)$  for monic, irreducible polynomials with integer coefficients. There is a classical result of Kronecker [15] that identifies polynomials with integer coefficients and Mahler measure equal to 1.

**Theorem** If  $P(x)$  is a monic, irreducible polynomial in  $\mathbb{Z}[x]$ , then  $\mu(P) = 1$  if and only if  $P$  is a cyclotomic polynomial or  $P(x) = x$ .

In 1933 D. H. Lehmer [16], in connection with a method for identifying large prime numbers, asked the following question: For every  $\epsilon > 0$  does there exist a monic, irreducible polynomial  $P(x)$  in  $\mathbb{Z}[x]$  such that

$$(5) \quad 1 < \mu(P) < 1 + \epsilon ?$$



The numerical evidence on this question points in the direction of a negative answer. This has led to the formulation of a “Lehmer conjecture”, although in [16] Lehmer simply raised the question (5).

**“Lehmer Conjecture”** *There exists a positive constant  $c_1$  such that each nonzero polynomial  $P(x)$  in  $\mathbb{Z}[x]$  satisfies*

$$\mu(P) = 1, \quad \text{or} \quad 1 + c_1 \leq \mu(P).$$

In [16] Lehmer noted that

$$(6) \quad \mu(x^{10} + x^9 - x^7 - x^6 - x^5 - x^4 - x^3 + x + 1) = 1.176280821 \dots,$$

and stated that this was the smallest value of  $\mu(P)$  greater than 1 that he could find among monic, irreducible polynomials with integer coefficients. It is a remarkable fact that, in spite of intensive numerical searches, no monic irreducible polynomial with integer coefficients has been found having Mahler measure greater than 1 and smaller than the value 1.176280821... found by Lehmer in 1933. Thus the numerical evidence supports the following stronger conjecture.

**“Strong Lehmer Conjecture”** *Each nonzero polynomial  $P(x)$  in  $\mathbb{Z}[x]$  satisfies*

$$\mu(P) = 1, \quad \text{or} \quad 1.176280821 \dots \leq \mu(P).$$

Schinzel [28],[29] has observed that Lehmer’s problem is related to estimates for the number of irreducible factors of a polynomial, and some of his results have been sharpened and extended to number fields by Pinner and Vaaler [22], [23].

If  $P(x)$  is a monic, irreducible polynomial in  $\mathbb{Z}[x]$  of degree  $L \geq 1$  we say that  $P$  is *reciprocal* if it satisfies the identity

$$x^L P(x^{-1}) = \pm P(x).$$

C. J. Smyth has obtained a result that is analogous to the strong Lehmer conjecture for the restricted class of polynomials that are not reciprocal.

**Theorem** (C. J. Smyth [30]) *Assume that  $P(x)$  is a monic, irreducible polynomial in  $\mathbb{Z}[x]$  and  $P$  is not a reciprocal polynomial. Then the Mahler measure of  $P$  satisfies*

$$(7) \quad \mu(x^3 - x - 1) = 1.32471796 \dots \leq \mu(P).$$

For arbitrary monic, irreducible polynomials in  $\mathbb{Z}[x]$ , lower bounds for Mahler’s measure that depend on the degree of the polynomial were obtained by Blanksby and Montgomery [3] and by Stewart [32], (see also Schinzel [27]). However, the strongest result of this kind was obtained by E. Dobrowolski, who was able to exploit a well known congruence to a prime modulus in his argument.

**Theorem** (E. Dobrowolski [12]) *Let  $\epsilon > 0$ , then there exists a positive integer  $L_0(\epsilon)$  with the following property: if  $P(x)$  is a monic, irreducible polynomial in  $\mathbb{Z}[x]$ , if  $P$  is not cyclotomic and not  $x$ , and  $P$  has degree  $L \geq L_0(\epsilon)$ , then the Mahler measure of  $P$  satisfies*

$$(8) \quad 1 + (1 - \epsilon) \left( \frac{\log \log L}{\log L} \right)^3 \leq \mu(P).$$

Simpler proofs of (8), and with the factor  $(1 - \epsilon)$  replaced by  $(2 - \epsilon)$ , have been given by Cantor and Straus [8] and by Rausch [24]. And Louboutin [17] has given a very concise proof of (8) in which the factor  $(1 - \epsilon)$  is replaced by  $(9/4 - \epsilon)$ . It should be noted, however, that these slight improvements all make use of the basic method introduced by Dobrowolski. In a different direction, Amoroso and Zannier [1] have shown that Lehmer’s conjecture holds for polynomials  $P(x)$  which split in an Abelian extension of  $\mathbb{Q}$ .

Attempts to prove or to disprove the Lehmer conjecture have led to several interesting Diophantine problems about  $\mathbb{Z}[x]$ . Write  $\Phi_1(x), \Phi_2(x), \Phi_3(x), \dots, \Phi_n(x), \dots$  for the sequence of (monic) cyclotomic polynomials in  $\mathbb{Z}[x]$ . Let  $F(x)$  in  $\mathbb{Z}[x]$  be a product of cyclotomic polynomials:

$$F(x) = \prod_{1 \leq n} \Phi_n(x)^{e(n)}, \quad \text{where} \quad \deg(F) = \sum_{1 \leq n} \varphi(n)e(n) = 2N.$$

Then  $\mu(F) = 1$  and so in favourable cases one might expect the Mahler measure of  $F(x) \pm x^N$  to be only slightly larger than 1. In fact the Lehmer polynomial occurring in (6) has this form:

$$x^{10} + x^9 - x^7 - x^6 - x^5 - x^4 - x^3 + x + 1 = \Phi_1^2(x)\Phi_2^2(x)\Phi_3^2(x)\Phi_6(x) - x^5.$$

Here is a list of the first five monic, irreducible polynomials with the smallest known Mahler measure greater than 1 expressed in this way:

$$\begin{aligned} 1.176280 \dots &= \mu(\Phi_1^2(x)\Phi_2^2(x)\Phi_3^2(x)\Phi_6(x) - x^5) \\ 1.188368 \dots &= \mu(\Phi_1^2(x)\Phi_2^2(x)\Phi_3^2(x)\Phi_4(x)\Phi_6(x)\Phi_9(x) + x^9) \\ 1.200026 \dots &= \mu(\Phi_1^2(x)\Phi_2^2(x)\Phi_4(x)\Phi_6(x)\Phi_7(x) + x^7) \\ 1.201396 \dots &= \mu(\{\Phi_1^2(x)\Phi_5^2(x)\Phi_7(x)\Phi_{10}(x) + x^{10}\}/\Phi_6(x)) \\ 1.202616 \dots &= \mu(\Phi_1^2(x)\Phi_2^2(x)\Phi_3(x)\Phi_4(x)\Phi_6(x)\Phi_{12}(x) + x^7) \end{aligned}$$

More extensive tables of this sort have been given in [21]. While such numerical evidence is suggestive, at present there are no results known which establish that monic irreducible polynomials in  $\mathbb{Z}[x]$  with small Mahler measure greater than 1 must be of this form. And it is not known if the Lehmer conjecture holds for this restricted class of polynomials.

Another device for working with polynomials having small Mahler measure is to express them as factors of polynomials with coefficients restricted to the set  $\{-1, 0, +1\}$ . This is always possible if the Mahler measure is smaller than 2.

**Lemma** (E. Bombieri, J. V. [5]) *Let  $0 < \delta$  and assume that  $P(x)$  is a nonzero polynomial in  $\mathbb{Z}[x]$  with  $1 \leq \mu(P) \leq 2 - \delta$  and  $\deg(P) = L$ . Then there exists a polynomial*

$$(9) \quad Q(x) = x^N + \epsilon_1 x^{N-1} + \epsilon_2 x^{N-2} + \dots + \epsilon_N,$$

where  $\epsilon_n \in \{-1, 0, +1\}$ ,

such that  $P(x)|Q(x)$  and  $N \ll_\delta L^2 \log L$ .

The method used in [5] to establish the existence of the polynomial  $Q(x)$  can be applied in more general situations, however, it does not provide information about  $Q(x)$  beyond what is reported in the Lemma. For applications to problems of Diophantine approximation it would be useful to have some further knowledge of the roots of  $Q(x)$ . For example, it would be useful to know if there exists a polynomial  $Q(x)$  of the form (9), satisfying the conclusion  $P(x)|Q(x)$ , and also satisfying  $\mu(Q) = \mu(P)$ .

Here is another problem, somewhat related to the Lemma, which is open: For every positive integer  $M$  does there exist a polynomial

$$(10) \quad Q(x) = x^N + \epsilon_1 x^{N-1} + \epsilon_2 x^{N-2} + \dots + \epsilon_N,$$

where  $\epsilon_n \in \{-1, 0, +1\}$ ,

such that  $Q$  has a root at  $\alpha$  of multiplicity at least  $M$  and  $|\alpha| > 1$ ? Because

$$(1.176280821\dots)^4 < 2\dots \quad \text{but} \quad (1.176280821\dots)^5 > 2,$$

the Lemma implies that such a  $Q$  exists with  $M = 4$ , but fails to produce a polynomial  $Q$  with  $M = 5$ . If Lehmer's conjecture is false, then the Lemma implies that such a polynomial  $Q(x)$  exists for arbitrarily large values of  $M$ .

Next we describe a method for estimating the number of polynomials  $P(x)$  having integer coefficients, degree less than or equal to  $N$  and Mahler measure less than or equal to a positive parameter  $T$ . In doing so it will be convenient to modify our notation and adopt a more geometric language. We consider Mahler's measure to be the function  $\mu : \mathbb{C}^{N+1} \rightarrow [0, \infty)$  given by

$$\mu(\mathbf{x}) = \exp \left\{ \int_0^1 \log |x_0 e(Nt) + x_1 e((N-1)t) + \dots + x_{N-1} e(t) + x_N| dt \right\},$$

where, as before, we write  $e(t) = e^{2\pi i t}$ . Thus  $\mu(\mathbf{x})$  is the Mahler measure of the polynomial  $x_0 z^N + x_1 z^{N-1} + \dots + x_N$ , which we now identify with its vector of coefficients. The function  $\mu$  satisfies the following basic properties:

1.  $\mu(\alpha \mathbf{x}) = |\alpha| \mu(\mathbf{x})$  for all  $\alpha \in \mathbb{C}$  and all  $\mathbf{x} \in \mathbb{C}^{N+1}$ ,
2. the function  $\mathbf{x} \rightarrow \mu(\mathbf{x})$  is continuous,
3.  $\mu(\mathbf{x}) = 0$  if and only if  $\mathbf{x} = \mathbf{0}$ ,
4.  $\mathcal{S}_{N+1} = \{\mathbf{x} \in \mathbb{R}^{N+1} : \mu(\mathbf{x}) \leq 1\}$  is a bounded, symmetric star body.

In particular,  $\mu$  is a symmetric distance function in the sense of the geometry of numbers. We also define the *monic* Mahler measure  $\nu : \mathbb{C}^N \rightarrow [1, \infty)$  by

$$\nu(\mathbf{y}) = \mu\left(\begin{pmatrix} 1 \\ \mathbf{y} \end{pmatrix}\right).$$

Thus  $\nu(\mathbf{y})$  is the Mahler measure of the monic polynomial  $z^N + y_1 z^{N-1} + \dots + y_N$ . Let  $\lambda_N$  denote Lebesgue measure on the Borel subsets of  $\mathbb{R}^N$ . Using standard estimates for  $\mu$  and  $\nu$  we find that

$$\int_{\mathbb{R}^N} \nu(\mathbf{y})^{-\sigma} d\lambda_N(\mathbf{y}) < \infty \quad \text{if and only if} \quad N < \sigma,$$

and

$$\int_{\mathbb{C}^N} \nu(\mathbf{y})^{-2\sigma} d\lambda_{2N}(\mathbf{y}) < \infty \quad \text{if and only if} \quad N < \sigma.$$

Let  $s = \sigma + it$  be a complex variable. Then define  $s \rightarrow F_N(s)$  and  $s \rightarrow G_N(s)$  in the half plane  $\{s \in \mathbb{C} : N < \Re(s)\}$  by

$$F_N(s) = \int_{\mathbb{R}^N} \nu(\mathbf{y})^{-s} d\lambda_N(\mathbf{y}) \quad \text{and} \quad G_N(s) = \int_{\mathbb{C}^N} \nu(\mathbf{y})^{-2s} d\lambda_{2N}(\mathbf{y}).$$

It follows that  $F_N$  and  $G_N$  are analytic in this half plane. Now define  $f_N : (0, \infty) \rightarrow [0, \infty)$  and  $g_N : (0, \infty) \rightarrow [0, \infty)$  by

$$f_N(\xi) = \lambda_N \{\mathbf{y} \in \mathbb{R}^N : \nu(\mathbf{y}) \leq \xi\},$$

and

$$g_N(\xi) = \lambda_{2N} \{\mathbf{y} \in \mathbb{C}^N : \nu(\mathbf{y}) \leq \xi\}.$$

In the half plane  $\{s \in \mathbb{C} : N < \Re(s)\}$  we have

$$F_N(s) = \int_0^\infty \xi^{-s} df_N(\xi) = s \int_1^\infty \xi^{-s-1} f_N(\xi) d\xi,$$

and

$$G_N(s) = \int_0^\infty \xi^{-2s} dg_N(\xi) = 2s \int_1^\infty \xi^{-2s-1} g_N(\xi) d\xi.$$

At this point it is clear that information about the analytic functions  $F_N(s)$  and  $G_N(s)$  could be used to determine, or to approximate, the distribution functions  $f_N(\xi)$  and  $g_N(\xi)$ . The following result shows that  $F_N(s)$  and  $G_N(s)$  are relatively simple, rational functions.

**Theorem** (S-J. Chern, J. V., [9]) *The functions  $F_N(s)$  and  $G_N(s)$  extend by analytic continuation to rational functions on  $\mathbb{C}$ . In particular, we have*

$$(11) \quad F_N(s) = \mathcal{C}_N s^{M+1} \prod_{m=0}^M (s - N + 2m)^{-1},$$

where

$$M = \left\lfloor \frac{N-1}{2} \right\rfloor \quad \text{and} \quad \mathcal{C}_N = 2^N \prod_{m=1}^M \left( \frac{2m}{2m+1} \right)^{N-2m},$$

and

$$(12) \quad G_N(s) = \{N!\}^{-1} (\pi s)^N \prod_{n=0}^{N-1} (s - N + n)^{-1}.$$

From this theorem and the Mellin inversion formula we are able to determine the distribution functions  $f_N(\xi)$  and  $g_N(\xi)$ . If  $1 \leq \xi < \infty$  then

$$(13) \quad f_N(\xi) = \mathcal{C}_N 2^{-M} \{M!\}^{-1} \sum_{m=0}^M (-1)^m (N-2m)^M \binom{M}{m} \xi^{N-2m},$$

where  $M$  and  $\mathcal{C}_N$  are determined as in the Theorem, and

$$(14) \quad g_N(\xi) = \pi^N \{N!\}^{-2} \sum_{n=0}^N (-1)^n (N-n)^N \binom{N}{n} \xi^{2(N-n)}.$$

We can also determine the volume of the star bodies  $\mathcal{S}_{N+1}$  in  $\mathbb{R}^{N+1}$  and  $\mathbb{C}^{N+1}$ .



**Corollary** For each positive integer  $N$  we have

$$\begin{aligned} \lambda_{N+1} \{ \mathbf{x} \in \mathbb{R}^{N+1} : \mu(\mathbf{x}) \leq 1 \} \\ = 2^{N+1} (N+1)^M \prod_{m=1}^M \left\{ \frac{(2m)^{N-2m}}{(2m+1)^{N+1-2m}} \right\}, \end{aligned}$$

where  $M = \lfloor (N-1)/2 \rfloor$ , and

$$\lambda_{2N+2} \{ \mathbf{x} \in \mathbb{C}^{N+1} : \mu(\mathbf{x}) \leq 1 \} = \frac{\pi^{N+1} (N+1)^{N+1}}{((N+1)!)^2}.$$

Now let  $\mathcal{M}(N, T)$  denote the number of polynomials having integer coefficients, degree at most  $N$ , and Mahler measure less than or equal to  $T$ . Let  $\mathcal{M}_1(N, T)$  denote the number of *monic* polynomials having integer coefficients, degree equal to  $N$ , and Mahler measure less than or equal to  $T$ . When  $T$  is large compared with  $N$ ,  $\mathcal{M}(N, T)$  is equal to the number of integer lattice points in the dilated star body  $T\mathcal{S}_{N+1} \subseteq \mathbb{R}^{N+1}$ , and we expect this to be approximately equal to the volume of the star body  $T\mathcal{S}_{N+1}$ . We define

$$V_{N+1} = 2^{N+1} (N+1)^M \prod_{m=1}^M \left\{ \frac{(2m)^{N-2m}}{(2m+1)^{N+1-2m}} \right\}$$

where  $M = \lfloor (N-1)/2 \rfloor$ . It can be shown that the boundary

$$\partial\{\mathcal{S}_{N+1}\} = \{ \mathbf{x} \in \mathbb{R}^{N+1} : \mu(\mathbf{x}) = 1 \}$$

consists of  $2N+2$  smooth surfaces each of which is a connected, compact subset of the zero set of a polynomial in  $N+1$  variables having *integer* coefficients. In particular, the boundary of  $\mathcal{S}_{N+1}$  is rectifiable. These observations lead to the following estimate

**Theorem** (S-J. Chern, J. V., [9]) For each positive integer  $N$  we have

$$(15) \quad \mathcal{M}(N, T) = V_{N+1} T^{N+1} + O_N(T^N) \quad \text{as } T \rightarrow \infty.$$

Related results have been obtained by DiPippo and Howe [11] and by Dubickas and Konyagin [13].

The constant implicit in the  $O$ -notation on the right of (15) depends on the  $N$ -dimensional Hausdorff measure of the boundary of  $\mathcal{S}_{N+1}$ . In fact it is possible to give a somewhat weaker estimate for  $\mathcal{M}(N, T)$  which is *uniform* in both  $N$  and  $T$ . Such an estimate depends on the following inequality.

**Theorem** (S-J. Chern, J. V., [9]) If  $\mathbf{x}$  and  $\mathbf{y}$  are in  $\mathbb{C}^{N+1}$  then

$$(16) \quad | \mu(\mathbf{x})^{1/N} - \mu(\mathbf{y})^{1/N} | \leq 2 | \mathbf{x} - \mathbf{y} |_1^{1/N},$$

where  $|\mathbf{x}|_1 = |x_0| + |x_1| + \dots + |x_N|$  is the  $l^1$ -norm on vectors in  $\mathbb{C}^{N+1}$ .

Using the inequality (16) we obtain (see [9]):

**Theorem** Assume that the integer  $N \geq 2$  and the real parameter  $T$  satisfy

$$8^N N^{2N} \leq T,$$

then

$$(17) \quad | \mathcal{M}(N, T) - V_{N+1} T^{N+1} | \leq 8N^2 V_{N+1} T^{N+1-1/N}.$$

**Theorem** Assume that the integer  $N \geq 2$  and the real parameter  $T$  satisfy

$$5^N N^{2N} \leq T,$$

then

$$(18) \quad | \mathcal{M}_1(N, T) - f_N(T) | \leq 5N^2 \mathcal{C}_N (N/2)^M (M!)^{-1} T^{N-1/N}.$$

These results provide information about the distribution of values of Mahler's measure when  $T$  is much larger than  $N$ . It would be of interest to have corresponding information when  $N$  is larger than  $T$ . Thus we may formulate the following problem: estimate  $\mathcal{M}(N, T)$  and  $\mathcal{M}_1(N, T)$  with  $1 \leq T$  fixed and  $N \rightarrow \infty$ . Such an estimate in the special case  $T = 1$  and  $N \rightarrow \infty$  has been carried out by Boyd and Montgomery [7].

We have described how the Mellin transform

$$F_N(s) = \int_{\mathbb{R}^N} \nu(\mathbf{y})^{-s} d\lambda_N(\mathbf{y})$$

defines an analytic function in the right half plane  $\{s \in \mathbb{C} : N < \Re(s) = \sigma\}$  and extends by analytic continuation to a rational function. In a similar manner the generalized Dirichlet series

$$(19) \quad s \rightarrow \mathcal{F}_N(s) = \sum_{\mathbf{l} \in \mathbb{Z}^N} \nu(\mathbf{l})^{-s}$$

defines an analytic function in the right half plane  $\{s \in \mathbb{C} : N < \Re(s) = \sigma\}$ . Of course  $\mathcal{F}_N(s)$  is sensitive to the values that Mahler's measure takes at integer lattice points, that is, at polynomials with integer coefficients. Therefore knowledge of the analytic functions  $\mathcal{F}_N(s)$  may provide useful information about small values of Mahler's measure at polynomials with integer coefficients. We conjecture that the functions  $\mathcal{F}_N(s)$  defined by (19) extend by analytic continuation to meromorphic functions on  $\mathbb{C}$ . If this conjecture is correct then it raises the following problem: What analytic properties of the functions  $\mathcal{F}_N(s)$  are equivalent to Lehmer's conjecture?

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## The SMAI Celebrates its 20th Anniversary

Contributed by Michel Thera, President SMAI



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The SMAI now has a membership of about 1,200, both from the academic and industrial communities. Members include experts in scientific computing, numerical analysis, partial differential equations, applied probability theory, statistics, control, automatic control, optimization and discrete mathematics, among others.

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# A Simple Multiresolution Technique for Diffraction Image Recovery

by D. Russell Luke, PIMS Postdoctoral Fellow, Simon Fraser University



In this article a Fourier-based multiresolution technique is described. It is used to speed up algorithms for recovering diffraction images from noisy and aberrated data. The method has been used to achieve a 17-fold speed up of an adaptive optics algorithm developed by the author for an early prototype of the James Webb Space Telescope, which is due to replace the Hubble Space Telescope in 2011. The technique, simple and frequently rediscovered, is based on windowed Fourier transforms. While a natural strategy for these purposes, the method is not specific to this setting and can be employed in any application that uses a combination of far field scattering data and spatially dependent physical constraints.

## 1 Introduction

While debate is heating up over the fate of the ageing Hubble Space Telescope [1], progress continues on Hubble's replacement, the James Webb Space Telescope (JWST) [2]. A central design feature of the JWST is *adaptive optics* capabilities that will allow the telescope to fine-tune itself during observations. To accomplish this, the telescope will combine computational wavefront sensing techniques together with deformable mirrors to detect and correct for system aberrations. In the past, wavefront sensing was performed by an intricate labyrinth of hardware. The JWST marks the beginning of a shift toward computational techniques that are a dividend of efforts to find and correct for the manufacturing defect that plagued Hubble's early days.

The problem of wavefront sensing is just one instance of a common problem in imaging known as phase retrieval. In broad terms, one is faced with recovering, from intensity measurements alone, the geometric path of propagation of a wave that has interacted with some scattering material or medium. In mathematical terms, the phase problem amounts to finding the real and imaginary parts of a complex-valued scalar function from knowledge of its amplitude before and after being acted upon by a unitary linear operator. The problem has been around at least since the 1890's [3], and breakthroughs in the early 1950's for scattering from periodic structures [4–6] earned Hauptman and Karle the Nobel Prize in Chemistry in 1985. The phase problem is far from solved, however, in any analytical sense for general settings. Nevertheless the problem has been solved *numerically* for over three decades [7, 8] even though there is still no proof that the most popular techniques should work at all, let alone so well [9, 10]. It is to this happy circumstance that Hubble owes some of its success, since without phase retrieval algorithms it might have been impossible to pinpoint fabled flaw in Hubble's original primary mirror [9].

We show the flavor of this problem by way of a discussion of a Fourier-based multiresolution technique for decreasing the cpu-time for compu-

tational wavefront sensing from noisy and aberrated data. We use this method to achieve a 17-fold speed up of an adaptive optics algorithm developed by the author for an early prototype of the JWST. Though often forgotten, the technique is a well understood tool based on windowed Fourier transforms. A natural strategy for our purposes, the method is not specific to our setting and can be employed in any application that uses a combination of far field scattering data and spatially dependent physical constraints. For a more detailed review of diffraction imaging we refer the reader to [9] and references therein. The extension of these ideas to obstacle scattering as described in [11] is a topic of current research.

## 2 Phase retrieval

The *forward imaging* model is formulated on the space of square integrable functions mapping  $\mathbb{R}^2 \rightarrow \mathbb{C}$ . The model input  $u : \mathbb{R}^2 \rightarrow \mathbb{C}$  is an electromagnetic field generated by the object we are trying to observe. We call the domain of  $u$  the *physical domain*. The device through which the wave travels is characterised by a modified Fourier transform  $\mathcal{F}_m$  of the form

$$(1) \quad (\mathcal{F}_m u)(\xi) \equiv \int_{\mathbb{R}^2} u(x) e^{i(2\pi x \cdot \xi + \tilde{\theta}_m(x))} dx$$

The function  $\tilde{\theta}_m : \mathbb{R}^2 \rightarrow \mathbb{R}$  for  $m = 1, 2, \dots, M$  is a phase *aberration* that models a known device tuning such as defocus. The model output, or data, corresponding to the  $m$ th tuning of the device are amplitude measurements,  $\psi_m : \mathbb{R}^2 \rightarrow \mathbb{R}_+$ , where  $\mathbb{R}_+$  denotes the non-negative orthant. Due to the relations between the image  $\psi_m$  and the Fourier transform, we refer to the domain of the image as the *frequency domain*. The data  $\psi_m$  are often referred to as the frequency domain *magnitude constraints* of the imaging model.

Just before the wave passes through our instrument, it is assumed that  $|u| = A$  where  $A : \mathbb{R}^2 \rightarrow \mathbb{R}_+$  and the modulus  $|\cdot|$  is the *pointwise Euclidean* magnitude. This is known as the *entrance pupil constraint*. For convenience, we denote

$$(2) \quad \mathcal{F}_0 \equiv \mathcal{I} \quad \text{and} \quad \psi_0 \equiv A$$

where  $\mathcal{I}$  is the identity operator. The imaging model is then given by

$$(3) \quad |\mathcal{F}_m u| = \psi_m, \quad m = 0, 1, \dots, M.$$

The *inverse problem* with which we are concerned is to solve (3) for  $u$  given  $\mathcal{F}_m$  and  $\psi_m$ . Whatever method one has for solving this problem, the next section details a strategy for obtaining low resolution estimates first, and using these as a bootstrap to higher resolution solutions.

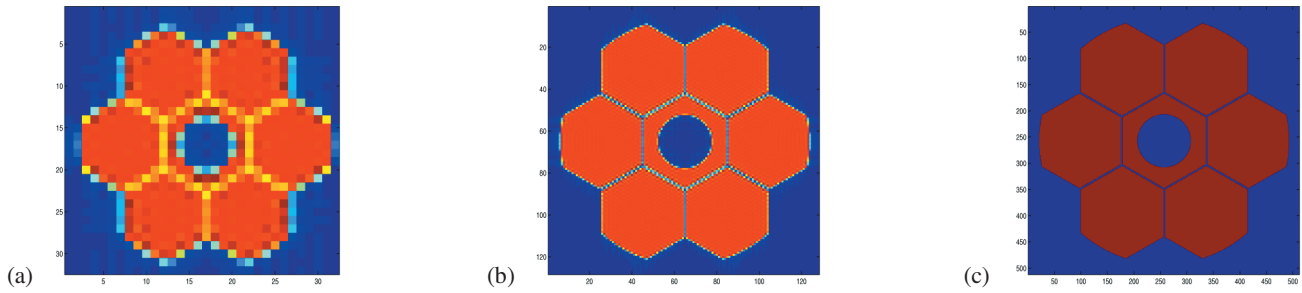


Figure 1: Multi-Resolution entrance pupil constraints, at (a) 32 x 32 resolution corresponding to the 32 x 32 image data shown in Fig.(2), (b) 128 x 128 resolution corresponding to the 128 x 128 image data, and (c) 512 x 512 resolution.

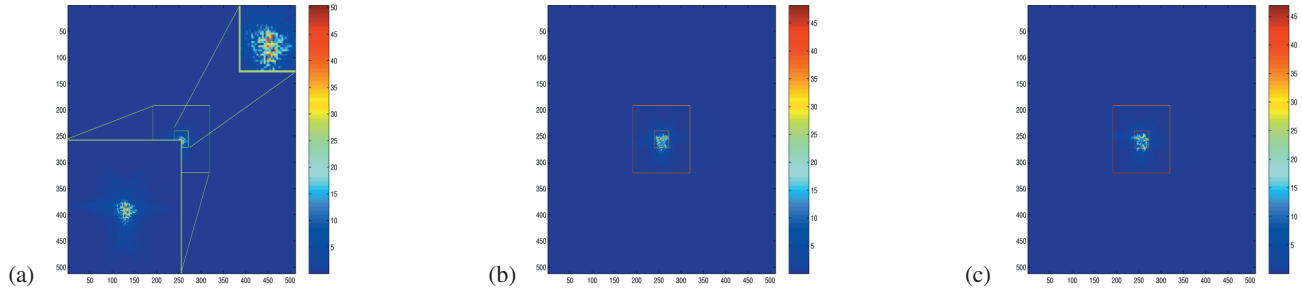


Figure 2: Multi-resolution image data. Three levels of multiresolution windowing operations are depicted for each diversity image. Frame (a) shows close-ups of each of the three resolution levels. The centre 32 x 32 pixels of each diversity image, together with the corresponding low-resolution entrance pupil constraint Fig. (1.a) are used to generate the approximate solution shown in Fig. (3.a). This solution is used to initialize the same problem with the centre 128 x 128 pixels and the corresponding mid-resolution entrance pupil constraint (Fig. (1.b)) as data. The solution to this problem, shown in Fig. (3.b), is used to initialize the full resolution problem. The progress of the error metric versus iteration of the multiresolution implementation is shown in Fig. (5).

### 3 A multiresolution strategy

We motivate this method with a discussion of filtering, and show the corresponding interpretation as a multiresolution analysis.

Noise often dominates high frequency components of an image  $\psi_m(\xi)$ . A common technique for reducing or *filtering* noise is to truncate the images and set all high frequency components to zero, that is, to set  $\psi_m(\xi) = 0$  for  $\|\xi\|_\infty > a$ , for some cutoff  $a > 0$ . For the imaging model given by Eq. (3), the image  $\psi_m$  is the magnitude of the modified Fourier transform of the field  $u$ . Thus the filtering operation just described amounts to multiplying the Fourier transform of  $u$  by a *window* or characteristic function of the box of length  $2a$  in the frequency domain. Suppose the domain is sampled on square pixels of length  $\Delta x$ . Let  $\mathcal{X}_n$  denote the indicator function for the  $n \times n$  box of pixels centred at zero, where, for convenience,  $n$  is a multiple of 2. For a sampled image  $\psi_m$  centred at zero we have the following system of equations for the windowed image

$$(4) \quad \mathcal{X}_n \odot |\mathcal{F}_m u| = \mathcal{X}_n \odot \psi_m, \quad m = 1, \dots, M.$$

where  $\odot$  represents the discrete Hadamard matrix product and  $\mathcal{F}_m$  ( $m = 1, \dots, M$ ) are the discrete counterparts of the continuous operators defined in Eq. (1). Note that the window is *not* applied to the physical domain equation ( $m = 0$ ) given by Eq. (2) and Eq. (3). This has to do with the relation between the truncation of high frequency Fourier modes and blurring in the physical domain. This discussed in more detail below.

The multiresolution approach relies on our ability to write the left hand side of Eq. (4) as a localized average of nearby pixels of  $ue^{i\theta_m}$ . Since the Hadamard product commutes with the pointwise modulus function we can write the windowed function on the right-hand side of Eq. (4) as  $\mathcal{X}_n \odot |\mathcal{F}_m u| = |\mathcal{X}_n \odot \mathcal{F}_m u|$ . For  $m \geq 1$ , by the Discrete Convolution Theorem, we have

$$|\mathcal{X}_n \odot \mathcal{F}_m u| = \left| \left( \mathcal{X}_n^\vee * \left( ue^{i\theta_m} \right) \right)^\wedge \right| \quad m = 1, \dots, M.$$

Here  $\wedge$  and  $\vee$  indicate the discrete Fourier transform and its inverse respectively.

In 2-dimensions, the Fourier transform of the window function is a product of sinc functions of each of the components separately.

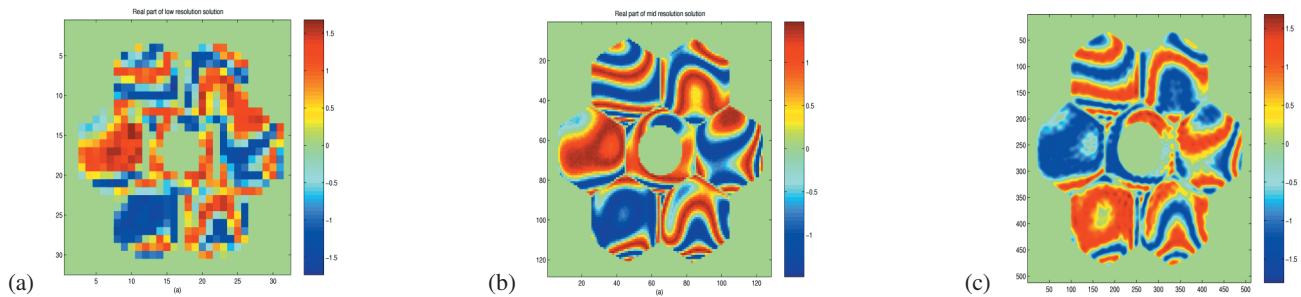


Figure 3: Aberrated wavefront for the segmented pupil recovered from 3 diversity point source images on successively finer grids. The real part of the low resolution wavefront (a) is generated from a truncation of the image data to the centre 32 x 32 pixels. This solution is used as a first guess for the next resolution, 128 x 128. The 128 x 128 pixel resolution solution (b) is used as a first guess for the full resolution problem whose solution is shown in (c).

For  $x \in \mathbb{R}^2$

$$\text{sinc}((n\Delta x)x) \equiv \frac{\text{sin}((\pi n\Delta x)x_1)}{(\pi n\Delta x)x_1} \times \frac{\text{sin}((\pi n\Delta x)x_2)}{(\pi n\Delta x)x_2} = \frac{1}{n\Delta x} \mathcal{X}_n^\vee.$$

Convolution against a sinc function,  $\mathcal{X}_n^\vee$ , can be approximated by a localized discrete linear operator,  $\mathcal{A}_n$ , that averages blocks of adjacent pixels. For the moment we leave the definition of  $\mathcal{A}_n$  ambiguous – many different averaging operators are possible. For  $m \geq 1$  the convolution on the right hand side of Eq. (4) can therefore be approximated by  $\mathcal{X}_n^\vee * (ue^{i\hat{\theta}_m}) \approx \mathcal{A}_n (ue^{i\hat{\theta}_m})$ . This yields the following approximation of Eq. (4)

$$(5) \quad \left| \left( \mathcal{A}_n (ue^{i\hat{\theta}_m}) \right)^\wedge \right| \approx \mathcal{X}_n \odot \psi_m, \quad m = 1, \dots, M.$$

The filtering operation applied to the images  $\psi_m$  ( $m = 1, \dots, M$ ) cannot be directly applied to the physical domain constraint represented by the “image”  $\psi_0$ . The analog in the physical domain is an averaging operation. To see this consider the (discrete) Fourier dual of the physical constraint  $|u|^\wedge = \psi_0^\wedge$ . Now, apply the window  $\mathcal{X}_n$  to get  $\mathcal{X}_n \odot |u|^\wedge = \mathcal{X}_n \odot \psi_0^\wedge$ . Again, by the Discrete Convolution Theorem the Fourier dual of the windowing operation, *i.e.* the filtering operation in the physical domain, is given by  $\mathcal{X}_n^\vee * |u| = \mathcal{X}_n^\vee * \psi_0$ . We approximate the right hand side of this equation by  $\mathcal{X}_n^\vee * |u| \approx \mathcal{A}_n |u|$ . This yields the approximate physical domain relation corresponding to the application of a window in the Fourier domain,

$$(6) \quad \mathcal{A}_n |u| \approx \mathcal{X}_n^\vee * \psi_0.$$

Equations (5) and (6) constitute a low resolution imaging system. The averaging operator  $\mathcal{A}_n$  blurs information in adjacent pixels of the wavefront estimate  $u$ , smoothing out edges as well as noise. It is not necessary, therefore, to maintain a high pixelization for the wavefront estimate  $u$  since fine detail is lost by averaging. In Eq. (5) only the centre  $n$  pixels of the image are kept in the calculation. Our implementations rely on the Fast Fourier Transform Algorithm (FFT) to calculate the discrete Fourier transforms. These require square arrays with dimensions that are powers of 2. Our computations take advantage of the lower resolution image by using a pixelization of  $u$  that is consistent with the size of the window  $\mathcal{X}_n$ . This dramatically reduces the dimensionality of the optimization problem and thus computation time. It cannot be expected that the solution to the low resolution problem will be as good as the high resolution, however, we use the low resolution solutions as a bootstrap to higher resolution estimates. Ideally, all of the hard work is done at low resolution and relatively few iterations are necessary to achieve a solution at the highest resolution. This is indeed what we achieve (see Fig. (5)).

## 4 Numerical Results

The aperture design for one of several prototype telescopes studied at NASA’s Goddard Space Flight Center for the JWST is shown in Fig. (1).

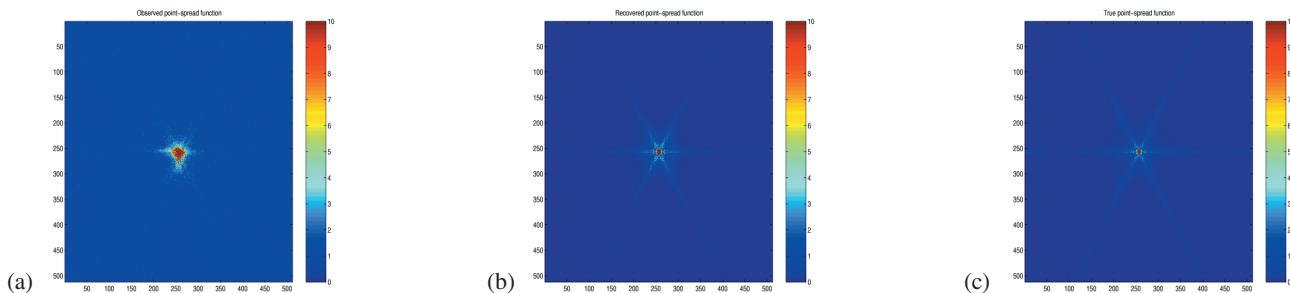


Figure 4: Noisy point-spread function (a) for a segmented pupil on a 512 x 512 grid. The recovered point-spread function (b) was first filtered with a Fourier window filter before processing by the wavefront reconstruction algorithm. Frame (c) shows the true, unaberrated point-spread function.



This consists of seven, meter-class panels that constitute the physical domain constraint for our problem. To recover the phase, three *diversity* images are used, two out of focus and one in focus image shown in Fig. (2).

In Fig. (2.a) a series of windowing operations is depicted for the three  $512 \times 512$  diversity images. First, the centre  $32 \times 32$  pixels of each diversity image are kept, and the remaining pixels are set to zero, that is for  $m = 1, 2, 3$ , we set

$$\tilde{\psi}_m = \mathcal{X}_{32} \odot \psi_m.$$

The corresponding physical domain operation is to smooth the entrance pupil constraint by convolution with the sinc function. This is achieved by setting

$$\tilde{\psi}_0 = (\mathcal{X}_{32} \odot \psi_0^\wedge)^\vee.$$

The resulting entrance pupil constraint is depicted in Fig. (1.a). For  $m = 1, 2, 3$ , the dimension reduction of the images  $\tilde{\psi}_m$  is straight forward. One simply ignores the zero pixels outside of the window. In the physical domain the reduction of dimension is achieved by assigning single values to blocks of  $16 \times 16$  adjacent pixels. In our implementations the value that is assigned is the average of the  $16^2$  pixels. The corresponding wavefront reconstruction problem is  $1/256$  the original problem size. The solution corresponding to this resolution is depicted in Fig. (3).

The next step is to use the solution depicted in Fig. (3) as an initial guess for the next resolution, which in this example is  $128 \times 128$  pixels. To do this, one simply divides the pixels of the low resolution solution into 16 sub-pixels. the image and physical domain data are treated the same as with the  $32 \times 32$  case. The solution to the  $128 \times 128$  problem is then used as the initial guess for the full resolution problem. The conjugate of the phase recovered in Fig. (3.c) is generated via deformable mirrors in the telescope in order to achieve the nearly diffraction limited *point spread function* shown in Fig. (4.b).

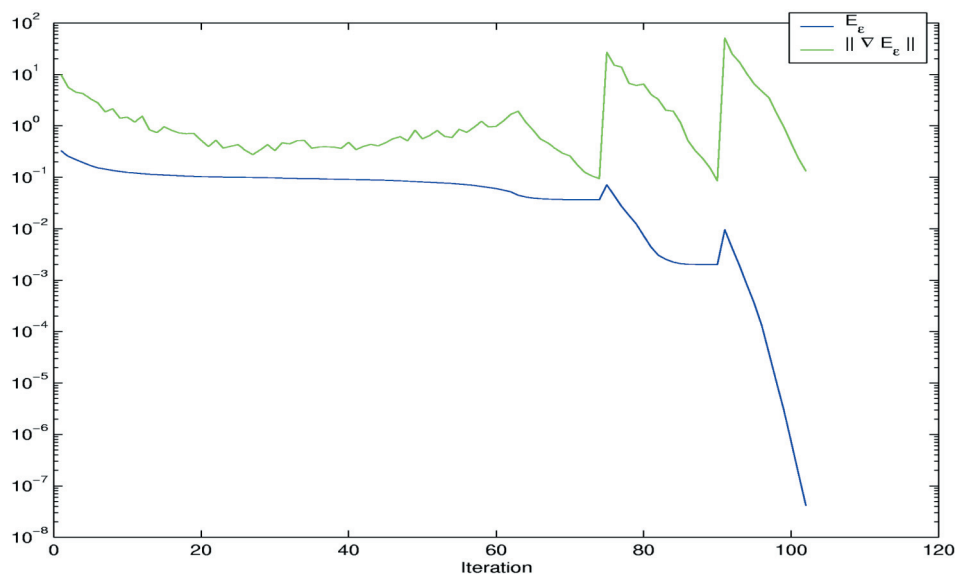
In Fig. (5) the squared set distance error versus iteration for a multiresolution implementation of a limited memory BFGS algorithm with trust regions to solve an extended least squares problem developed by the author [9, Sec. 5.3 and Alg.6.2] is shown. Notice that the flat region of the error metric indicating algorithm stagnation (typical for this problem) is encountered at low resolution. The higher resolution runs are started in

a neighbourhood of the solution and very few iterations are required for convergence. All of the hard work is accomplished cheaply at low resolutions. Starting from an initial phase guess of zero, the multiresolution implementations reduced cpu-time by a factor of 17 over the full resolution run.

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Download: [www.pims.math.ca/publications/preprints/](http://www.pims.math.ca/publications/preprints/).

Figure 5: Error metric and corresponding norm of the gradient versus iteration for a multiresolution implementation of the limited memory BFGS algorithm with trust regions applied to an extended least squares problem discussed in [9, Sec. 5.3 and Alg. 6.2]. The jumps in the objective value and gradient indicate transitions from low to higher dimension iterations. The flat region of the iterations is handled at low resolutions. Only when the estimate is in the neighbourhood of a solution does the algorithm switch to higher resolution calculations.



# The National Programme on Complex Data Structures News

*Contributed by Jamie Stafford, University of Toronto*

The National Programme on Complex Data Structures (NPCDS) activity began in earnest shortly after funding was released by NSERC in April of this year. This includes workshops at Le Centre de Recherches Mathématiques and the Fields Institute, a successful MITACS application, appointments to research positions, a successful call for proposals and planned activity for the annual meeting of the Statistical Society of Canada and the Pacific Institute for the Mathematical Sciences. In addition, NPCDS will seek opportunities at the Banff International Research Station and also seek to strengthen ties with the Statistical and Applied Mathematical Sciences Institute in the United States.

NPCDS uses its resources to seed activity that has the potential to lead to national interdisciplinary projects. A two-stage mechanism is used that involves inaugural workshops and, if successful, subsequent two-year projects. Within this context, NPCDS currently supports one project in complex survey data analysis for population health and social science, and will host an inaugural workshop in statistical genomics/bioinformatics. Descriptions are given below. A recent request for proposals for inaugural workshops generated sufficient interest that at least one workshop is currently planned to take place at PIMS and/or BIRS.



*Participants of the First Canadian Workshop on Statistical Genomics: over 30 students received full travel support to attend!*

## Statistical Methods for Complex Survey Data

Survey data are now being collected and analyzed by many government, health and social science organizations with subsequent analysis being used to identify the determinants of health and to influence public policy. Surveys used have increasingly complex structures in both longitudinal and cross-sectional forms, and new statistical methods are needed to make the best use of this data. Canada is a world leader in sample survey methodology and many of Canada's top researchers in this area are on this team. The researchers on this team have partnered with Statistics Canada, and their affiliated Research Data Centres across the nation, the Toronto Rehabilitation Institute, UNESCO and WestStat. They carry out research in the general areas of modelling of survey data, missing data in the survey response file, and variance estimation under complex survey designs. One of the team's activities is the placement of ongoing PhD and postdoctoral students in collaborative research positions at Statistics Canada and at the Toronto Rehabilitation Institute. The students are supervised jointly by researchers at these institutions and by professors at universities throughout Canada. The team has successfully sought further support from MITACS and held an inaugural workshop hosted and supported by the Centre de Recherches Mathématiques.



*David Bellhouse leads an NPCDS project on Complex Survey Data*

## Statistical Genomics

The fields of Genetics and Molecular Biology are undergoing an unprecedented revolution triggered by mappings of the genomes of various organisms, including humans. While sequencing the genome is aimed at answering the questions "What?" and "Where?", functional genomics addresses the important questions of "Why?" and "How?". Modern functional genomics utilizes biochips and other high-throughput modalities in increasingly large experiments generating huge quantities of data. These data is quite varied depending on modality and technology used or the nature of experiments carried out. It also presents many broad statistical challenges, which together represent a unique set. The field of Statistical Genomics is being born to work with molecular biologists, geneticists and bioinformaticians to answer these challenges. To this end NPCDS and the Fields Institute hosted the First Canadian Workshop on Statistical Genomics Sept 3–5, 2003, which was attended by leaders in these various fields. The availability of travel awards for students met with a tremendous response with 40 students from across the nation receiving support. A proposal for a national project in this area is being drafted by a subset of the participants and progress is being made in seeking partnerships in industry and at various research institutes.



*Rafal Kustra is heading activity in Statistical Genomics*

## Design and Analysis of Computer Experiments for Complex Systems

The rapid growth in computing power has made the computational simulation of complex systems feasible and helped avoid physical experimentation. Consequently, the design and analysis of computer experiments has become an integral part in the exploration of scientific, and industrial, processes as well as creating new and important challenges. The *National Program on Complex Data Structures* is pleased to host an inaugural workshop that aims to address three main problems in this area: screening experiments, function fitting in high dimensions and integration of physical and computer experiments. The workshop will receive partial support from the *Los Alamos National Lab*, will involve the *Statistical and Mathematical Sciences Institute* in the US and novel aspects include a reading list distributed in advance with which participants are expected to be intimately familiar. The workshop will be held at *BIRS* in 2004 with Derek Bingham (SFU) as Principal Organizer. A formal announcement is forthcoming.

Information about the programme may be found at [www.pims.math.ca/NPCDS/](http://www.pims.math.ca/NPCDS/).

# MITACS' News

Contributed by Karen Booth, MITACS



## MITACS Quebec Interchange November 13–14, 2003

Mathematics of Information Technology and Complex Systems (MITACS) currently hosts Interchanges in different cities across Canada. This regional event features university research in mathematics and provides networking opportunities for student researchers, companies and government organizations. Interchanges have talks from expert university and industry speakers on a particular theme and hold a poster competition for student research. Our next event, the Quebec Interchange, will be held in Montreal on November 13–14, 2003. The theme of the conference is **Data Mining**, and will be attended by students, faculty, and industry from Quebec.

Here are a few facts about this event:

- Attendance is free for students and industry pay \$100.
- Travel and accommodation reimbursements will be provided to all graduate students and post-docs.
- Students and post-docs can present posters or demos of their work in a judged poster competition.
- There will be speakers and sessions throughout the day, discussing data mining issues in industry and academia.

- Undergraduate students are invited to participate, with a limited number of travel subsidies available to each university.
- Two “Short Courses” will be offered to students on the second day.
- Students have the opportunity to present their research in a lecture setting to their peers and researchers.
- Undergrad students can attend a presentation by a variety of Schools of Graduate Studies.

The MITACS Quebec Interchange is an excellent opportunity for students and faculty to highlight their work, while making key contacts with research organizations, professors and other students in this region. There are important benefits that can come from participating in this event. They include the potential for new networking opportunities with industry and the development of new research collaborations with an applied or industrial focus. In addition, students can be made aware of opportunities available in the Canadian data mining sector; and companies will be given a chance to meet the best and brightest students in the region, and in some cases to recruit those students and keep them in Canada.

## MITACS Engages in a Broad Range of Biomedical Research

MITACS biomedical projects span research across a broad spectrum, from studies on the spread of infectious diseases in response to the SARS outbreak and to climate change, to the impact of introduced species on our fisheries, agriculture and aquaculture industries. In another MITACS led biomedical project, mathematical advances are creating new methods of detecting heart disease.

In the project *Statistical Genetic Modelling and Analysis of Complex Traits*, leader **Shelley Bull** of Toronto and her team sta-

tistically models complex traits in individuals, families, and populations, including human and forest populations, to efficiently analyse high-dimensional data from genetic studies. **Leon Glass** of McGill and his team, on the project *Control of Atrial Fibrillation*, develop new methods to control atrial fibrillation and have produced specialized software to detect atrial activation times. The team is using mathematical models to analyse cardiac data, identify changes in heart rhythms and predict arrhythmias.

*Biomedical Models of Cellular and Physiological Systems and Disease* led by **Leah Keshet** of UBC, focuses on a variety of application areas, including Diabetes, stem cell biology, and Alzheimer's disease. An initial investigation of diabetes is ongoing in our Western division (UBC and SFU). Our Montreal-based group has focused on cell division kinetics, Hematopoietic Stem Cells, and oscillations in Periodic Chronic Myelogenous Leukemia.

**Samuel Shen** of Alberta leads the project *Analysis of Climate Change Signals*. Global warming and adverse climate changes have become a major threat to environments, forestry, and agriculture, and have a major impact on the global spread of infectious diseases. The team looks at regional outbreaks of malaria and dengue fever using dynamic modelling, statistical analysis and regional ground precipitation data from NASA satellites and stations, and tracks the geographical progression of these diseases.

The SARS research team, led by **Jianhong Wu** of York, is developing a mathematical model to describe the dynamics of SARS transmission, including an examination of the role of the hospitals in the spread of the disease. *Transmission Dynamics and Spatial Spread of Infectious Diseases: Modelling, Prediction and Control* will also be addressing the absence of a rapid diagnostic test or vaccine and the impact of quarantine.

MITACS continues to be a leading force for scientific progress in the use of mathematics to improve systems and technologies that lead to advances in biomedical research.



Arvind Gupta, Nassif Ghoussoub and Ed Perkins at the MITACS Forth Annual Conference in Ottawa, which took place in May 2003. A tribute to Nassif was part of the conference.



# 6th PIMS-IMA Graduate Mathematics Modelling Camp

## BIRS, Banff, Alberta, May 17–22, 2003

Contributed by Rachel Kuske, University of British Columbia

The 6th Annual PIMS-IMA Graduate Mathematics Modelling Camp (GIMMC) took place at BIRS this year. Thirty-four graduate students from North America participated in the programme. It was co-sponsored by the Institute for Mathematics and its Applications (IMA).

GIMMC is designed to give graduate students in the Mathematical Sciences an opportunity to learn techniques of mathematical modelling under the supervision and guidance of experts in the field.

GIMMC is the first leg of the PIMS-IMA Industrial Mathematics Forum which also includes the PIMS-IMA Industrial Problem Solving Workshop (IPSW).

In a first session, the mentors presented the problems, and for the remainder of the week, they guided a group of graduate students through to a resolution, this culminated in a group presentation and a written document at the end of the week.

The Mentors & Problems were:

**Emily Stone** (Utah State U.): *Modelling PCR Devices for Fun and Profit*

The group developed models based on differential equations for Polymerase Chain Reaction, used to amplify sequences of DNA. The goal was to identify the initial concentration of DNA, using only observations of the process over time.

**Richard Braun** (U. Delaware): *Thin Fluid Film Drainage Mathematical Models of a Boundary of a Thin Fluid*



Some of the camp mentors take a hike: Richard Braun, Rachel Kuske, David Misemer, Emily Stone, Robert Piché, Fadil Santosa.

A mathematical model for the evolution of a thin film boundary was developed and studied analytically and numerically to get a description of the thin film boundary decay rate. This quantity is important in quality control for the production of surfactants.

**Sonja Glavaski** (Honeywell): *Stability of Hybrid Systems using Sum of Squares (SoS) Programming Approach: VCCR System Example*

Hybrid systems are human controlled dynamic processes, such as air-conditioning systems and car transmissions, which have different states. Stability and control of these systems was studied with Lyapunov functions and Sum of Squares methods.

**David Misemer** (3M): *Modelling Polymer Purification by Counter-current Extraction*

The purification process in the production of adhesives is necessary for avoiding side-effects, as in medical patches, or malfunctions, as in electrical devices. The group built two models of purification via counter-current exchange, analysed both models and compared their results with experimental data.

**Fadil Santosa** (IMA & U. Minnesota): *Solar Car Racing Strategy*

The team developed models for power consumption in a car powered by solar energy. Using optimal control methods, they developed optimal racing strategies for a variety of weather, road, and racing conditions.

**Robert Piché** (Tampere U. Technology, Finland): *Converting Machine Tool Measurements into a CAD Model*

Manufacturers of machine tools often rebuild and modify existing machine tools, incorporating new technology to meet customer requirements at sig-



Graduate students Thalya Burden, Ying Han, Samet Kadioglu, Xinghua Deng, Lin Zhou, and Tzvetalin Vassilev worked on the problem that Robert Piché brought, namely, how to generate 3D geometric models using data from a measurement “arm”.

nificantly lower cost. A geometrical algorithm based on level set methods was developed to give a mathematical description of the measurements, which can then be input into a CAD package.

For more information please see [www.pims.math.ca/industrial/2003/gimmc/](http://www.pims.math.ca/industrial/2003/gimmc/).

### Upcoming Industrial Activities

#### 7<sup>th</sup> Annual PIMS GIMMC

U. Victoria, May 10–14, 2004

Outstanding graduate students at both the Masters and PhD level in the fields of mathematics, applied mathematics, statistics, and computer science, or related disciplines, from all Canadian and US universities are invited to apply. The deadline for applications: March 15, 2004, and early application is encouraged. For more information or to apply please see [www.pims.math.ca/industrial/2004/gimmc/](http://www.pims.math.ca/industrial/2004/gimmc/).

#### 8<sup>th</sup> Annual PIMS IPSW

UBC, May 17–21, 2004

For more information please see [www.pims.math.ca/industrial/2004/ipsw/](http://www.pims.math.ca/industrial/2004/ipsw/).

# 7th PIMS-IMA Industrial Problem Solving Workshop

## University of Calgary, May 25–29, 2003

Contributed by Rachel Kuske, University of British Columbia

The 7th PIMS-IMA Industrial Problem Solving Workshop was held at the University of Calgary, May 25–29, 2003. Approximately 55 people registered for the event, including the 34 graduate students who attended the GIMMC the previous week.

The participants divided into seven groups to work on the industrial problems which are described below.

**Lalitha Venkataram** (Schlumberger): *Solving Fredholm integral of the first kind in two dimensions*

One problem of interest to Schlumberger, which provides a wide range of products and services for the energy industry, is the determination of average rock pore size through spectral analysis by Nuclear Magnetic Resonance (NMR). Mathematically, the problem presented is an inversion of a 2-D Fredholm Integral of the first kind in order to solve for the underlying relaxation density. The problem is ill-posed in the classical sense both in continuous and discrete formulation. The group focused on three directions of attack. Since the problem is ill-posed, regularization methods are natural. They studied both SVD truncation and Tikhonov regularization with good success, significantly reducing data size by using factored form rather than the Kronecker

product form. The group also investigated the possibility of higher order regularization in the Tikhonov problem. Solution of this problem was achieved only for the Kronecker form as the factorization did not appear to be consistent with the higher order regularization. A number of other investigations were initiated, including the Galerkin method and an analysis of the Butler, Reeds and Dawson (1981), with some limited success.

**Veena B. Mendiratta** (Lucent): *Modelling Quality and Warranty Cost*

The main aim of this project was to begin a modelling effort directed at optimizing the warranty and quality costs associated with the production of a system with both hardware and software components. This optimization is constrained by the need to maintain reliability of the product, while staying within an operational budget. The aim was to identify important aspects of the modelling, focusing on identifying the major quality-related attributes of interest, modelling the key reliability indicators (failure rate and the severity level) and modelling the building and warranty costs for a product with a certain quality level. The optimization model then minimizes the sum of the quality and warranty costs over the entire class of admissible quality-related attribute vectors.

The project focused on the modelling function, and it was run on test data. Progress relied heavily on communication between the team and Dr. Mendiratta, in order to model reality as closely as possible without being able to compare with real data.

**Bruce McGee** (McMillan-McGee Corp): *The Thermodynamic Bubble Problem for the In-Situ Thermal Remediation of Contaminated Soils*

In the remediation process used by McMillan-McGee, over a period of several weeks electrical energy is introduced to the contaminated soil using a multi-

tude of finite length cylindrical electrodes. Current is forced to flow through the soil by the voltage differentials at the electrodes. Water is also pumped into the soil via the injection well and out of the ground at the extraction well. The soil is heated up by the electrical current and the contaminated liquids and vapours are produced at the extraction well. The reason for using the electrical current is that “flushing” the soil using water alone is not effective for removing the contaminants. By heating up the soil and vaporizing the contaminated liquid, it is anticipated that rate of extraction will increase as long as the recondensation is not significant.

In the two-phase zone, since vapour bubbles tend to rise due to the buoyancy force, and the temperature decreases along the vertical path of the bubbles out of the heated region, it is possible that the bubbles will recondense before reaching the extraction well. The main objective of this modelling exercise is to determine the necessary vacuum pressure (pressure drop from the electrodes to the extraction well) so that the chemical bubbles are removed at the extraction well before they rise too high and condense to the liquid state.

**John R. Hoffman** (Lockheed-Martin): *Problems associated with the Probability Hypothesis Density Function approach for multi-target tracking*

The team focused on the state estimation problem for tracking single and multiple targets. The Probability Hypothesis Density (PHD) makes the tracking problem computationally feasible by propagating only the first-order multi-target statistical moments by using a particle filter implementation for the PHD. The problem then becomes one of estimating the targets’ state based on the output of the PHD when using a particle filter implementation. The approach used in this paper, based on the Expectation-Maximization (EM) algorithm, views the PHD distribution as a mixture distribution, and the particles as an i.i.d. sampling from the mixture distribution. Using this, a maximum likelihood estimator for the parameters of the distribution can be generated.



Lalitha Venkataramanan of Schlumberger Doll Research explains NMR remote sensing to graduate students Ying Han, Lin Zhou, Quingguo Li, Qian Wang and Xinghua Deng.



**Edward Keyes** (Orisar, formerly Semiconductor insights): *Methods to localize inadvertent power and ground connections on integrated circuits*

Orisar Inc. (formerly SemiConductor Insight) provides reverse engineering services to integrated circuit IC manufacturers. The process produces a circuit diagram from a chip and allows the manufacturer to learn about a competitor's product or to determine if intellectual properties infringements have been committed by their competitor. Reverse engineering of integrated

circuits is made difficult by the shrinking form factor and increasing transistor density. Electron microscope photography captures a detailed image of each IC layer. A noise removal algorithm is then applied to the pictures, which are then passed to pattern recognition software in order to transfer the layer design into a polygonal representation of the circuit.

In the current work-flow, workers at Orisar Inc. visually inspect the polygonal representation of the circuit in order to find errors. However this process, performed manually, is very time consuming. The team proposed a method which can do better by automating it and thus saving valuable workers time and accelerating the process of reverse engineering.

**Carlos Tolmasky** (Cargill): *Correlation structures corresponding to forward rates*

While it has become common to model a single stock using the Black-Scholes formulation, the modelling of bond prices requires simulations of the change of interest rates as a function of their maturity, and thus the entire yield curve. The spectral decomposition of the correlation matrix for the spot rates from this curve shows that the top three components can explain nearly all the data. Similar structure is observed for bonds and commodities. As suggested by Lekkos, instead the forward rates should be analyzed. The team performed a spectral analysis of the forward rates, and investigated a model for the associated structure. Principal component analysis together with yield curve modelling formed the basis of this study.

**Wei Lu** (Manifold Data Mining Inc.): *Product-Driven Data Mining*



*"Handwaving explanations" in a conversation between Carlos Tolmasky (Cargill) and Rachel Kuske (UBC).*

Understanding how complicated and interrelated factors drive the consumer is the primary goal of Manifold Data Mining. They have developed innovative demographic and household spending pattern databases for six-digit postal codes in Canada. Their collection of information is expressed through thousands of individually tracked factors. This large collection of information about consumer behaviour is typically referred to as a data mine. The question posed to the group was to 1) find an algorithm that predicts the likelihood of consumers to respond favourably to a given product, based on very few factors. In addition, once this prediction is made for a given consumer the group was also asked to 2) develop a second algorithm that infers other statistical information regarding the consumer. The first algorithm identifies a few factors in the data mine which differentiate customers in terms of a particular product preference. Then the second algorithm builds on this information by looking for patterns and correlations in the data mine which identify related areas of consumer spending.

For more information please see [www.pims.math.ca/industrial/2003/ipsw/](http://www.pims.math.ca/industrial/2003/ipsw/).

The annual prize for the best "PIMS-slip" went to **Charles Bergeron** (Ecole Polytechnique de Montreal) with the following statement:

*"I can't believe it took us half an hour to show that the time derivative of position is velocity."*

## IAM-PIMS Distinguished Colloquium Series 2003/2004

The Institute of Applied Mathematics (IAM) and PIMS at UBC are sponsoring their 7th Annual Distinguished Colloquium Series.

The speakers are:

**Marco Avellaneda** (Courant Institute): *Reconstructing Volatility*

**Andrea Bertozzi** (Mathematics, UCLA): *Higher-Order PDEs in Fluid Dynamics and Image Processing*

**Stephen Boyd** (Electrical Engineering, Stanford): *Recent Advances in Convex Optimization*

**Chris Bretherton** (Atmospheric Science & Applied Mathematics, U. Washington): *Understanding the Circulation of the Tropical Atmosphere Using Simple Mathematical Models*

**Jorge Nocedal** (Electrical & Computer Engineering, Northwestern U.): *The New Faces of Nonlinear Optimization*

**Harry Swinney** (Physics, U. Texas Austin): *Spatial Patterns and Shock Waves in Sand*

These lectures are being videotaped and will be available from the page [www.pims.math.ca/industrial/2003/iampims\\_lect/](http://www.pims.math.ca/industrial/2003/iampims_lect/).

## Lunchbox Lectures Continue in Calgary

The PIMS/Shell Lunchbox Lecture Series is continuing in fall 2003. The series takes place at the Shell Centre in downtown Calgary. The lectures focus on mathematical techniques and applications relevant to the oil and gas industry, and demonstrate the utility and beauty of applied mathematics.



The fall 2003 lectures are:

**Gerald Cole** (Kinesiology, U. Calgary)

**Donald M. Henderson** (Biological Sciences, U. Calgary): *Dinosaurs and differentials: using mathematics to study ancient life*

**Jedrzej Sniatycki** (Math and Statistics, U. Calgary): *Purposefulness and Determinacy in Mechanics*

**Hugh Cowie Williams** (Math and Statistics, U. Calgary): *Cryptography, Pseudosquares and Number Sieves*

Please see [www.pims.math.ca/industrial/2003/lunchbox/](http://www.pims.math.ca/industrial/2003/lunchbox/).



# Numeracy & Beyond: Developing a Mathematical Habit of Mind in K–12

by Sharon Friesen, Galileo Educational Network

## Background

The ability to reason with numbers; ask questions of numbers; assess situations and identify problems; recognize the human, interpersonal, technical, scientific and mathematical dimensions of a problem; be creative and innovative in exploring possible solutions; check to see if a solution works; and act on opportunities for improvement are essential dispositions and skills for today's society (Conference Board of Canada [1]). These fundamental skills that the Conference Board identifies fall under a broad definition sometimes called quantitative literacy or numeracy. *"Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, behaviours, and processes"* [8, p.14].

In July 2003, PIMS sponsored the first part of a two part workshop intended to address the question: *what minimum numeracy is required of the average citizen in this computer age?* In addressing this question, the committee established as its first priority the need to identify key principles to guide the teaching of school mathematics. These principles should be simple, widely acceptable, practical and yet fundamental. School mathematics enters the discussion of numeracy because it is the primary source of numeracy for most people.

The introduction of numeracy, like the introduction of any new initiative, into the school classroom presents some challenges. In considering how to best go about both formulating and introducing the changes to make numeracy a reality in our schools, it is important to remember that to date most new initiatives have failed to enter the classroom walls (Friesen, [6]; Hiebert, [7]). Eisner [4] states that there are nine reasons for this failure:

- Internalized images of teachers' roles
- Attachment to familiar pedagogical routine
- Rigid and enduring standards for appropriate behaviour
- Teacher isolation

- Inadequacies of in-service education
- Conservative expectations for the function of schools
- Distance between educational reformers and teachers implementing change
- Artificial barrier between disciplines and between teachers
- Feckless piecemeal efforts at reform

Cuban [2], speaking of mathematics teaching and learning in the U.S. states that:

*"High school graduates in 2000 took more mathematics and science courses, did more mathematics and science homework, and read from "better" mathematics and science textbooks than did their forebears. Today's teachers who are certified to teach mathematics and science are familiar with the new mathematics and science curriculum standards that began appearing in the late 1980s. And test scores have improved on national and international standardized tests in mathematics and science, although not to the degree desired.*

*"Second, this reform agenda of binding public schools to the nation's economy has led inexorably to producing traditional schools and classrooms that in decorum, subject matter, and teaching style would make the grandparents of today's students feel at home. Within this overall climate of heightened concern for preparing students for college and information-based workplaces and increased emphasis on the newest technologies, mathematics and science teachers still lecture, require students to take notes, assign homework from texts, and give multiple-choice tests. If anything, in the past few years mathematics and science classrooms, while awash in graphing calculators and computers, have largely experienced a resurgence of traditional ways of teaching and learning."*

What is clear to us is that just increasing the amount of mathematics that is taught will not help us achieve a numerate citizenry. If anything, that seems to have catapulted us backwards. In order for our efforts with numeracy to be realized within K–12, we will need to find different ways to work with teachers to address matters of content knowledge, class-

room pedagogy, resources and assessment. We will need to create new ways to engage teachers in professional learning opportunities through which they, themselves, can learn mathematics differently, increase their effectiveness in the teaching of mathematics, engage with and develop more robust mathematical problems, and connect with colleagues and mentors who are also attempting to increase numeracy in their classrooms. The task before us is daunting. What we are attempting to create with numeracy and beyond is a mathematical habit-of-mind (Costa & Kallick [3])—a habit-of-mind that develops within citizens the dispositions that the Conference Board of Canada identifies.

## Getting Started: That's A Good Problem

A small group of mathematicians, math educators and teachers, supported by PIMS, Mt. Royal College and the Galileo Educational Network, have started to address the problem of numeracy in K–12 in Alberta. While we know that there is policy work needed, we have taken a different approach. We have started our work in the classroom. Dr. Jean Springer and Dr. Indy Lagu, mathematicians from Mt. Royal College and Kelly McKie Grenier and I, math educators from the Galileo Educational Network have started an initiative which we call *That's A Good Problem*. This initiative provides teachers, students and parents with an opportunity to engage with mathematics, increases the mathematical understanding and competence of teachers, provides opportunities for deep engagement with mathematics and provides teachers with the opportunity to work with and learn from mathematicians and math educators within the context of their own classrooms.

Schools are invited to send a team of 4 or 5 teachers to a half-day professional development day. The focus of this meeting is on: teaching mathematics through math explorations and investigations by working through a number of math explorations, providing suggestions

for introducing math explorations to other teachers in the school and providing suggestions for introducing and organizing a school Math Fair. As teachers work together on math problems, we provide them with the type of mentorship, support and encouragement that we want them to adopt in their own classrooms.

Teachers return to their schools with a copy of *the Math Fair booklet* created by Dr. Ted Lewis of the University of Alberta, which contains a number of good math explorations that they introduce to their school staffs and to their students. Jean, Indy, Kelly and I go to each of the schools for a full day to work with teachers and their students as they work on problems from *the Math Fair booklet* and prepare themselves to host a Math Fair. This provides teachers with an opportunity to learn in the context of their own classrooms. As a culmination to the work that the students and teachers do, each school hosts a Math Fair at which students set up a display of their math problems, but not the solutions to the problems. Students entice their parents and invited guests to work through their math problems. Jean, Indy, Kelly and I attend the Math Fair evenings to speak with parents and invited guests about numeracy and mathematics education.

We provide a follow-up half-day professional development session for 4–5 teachers from each school. The focus of this session is to assist teachers in how to identify and create good mathematics explorations for their students. We use the following guideline to start working on additional problems for the classroom:

1. Make every exploration begin with a detailed “story”.
2. Allow group work, but encourage individual effort.
3. See that students work with mathematical ideas in an active manner.
4. Choose tasks which can be successfully explored at many levels.
5. Design activities which permit innovative solutions by students.
6. Include a rapid evolution from the simple to the profound.
7. Expose the frontiers of knowledge when exploring ideas.
8. Select fun activities which deal with important, useful mathematics.
9. Ensure participation requires the communication of original thought.

10. Provide opportunities for interpretation, multiple correct solutions.

Friesen & Stone [5]

Math Fairs along with the accompanying professional development provide teachers with some definitive guidance on the numeracy needs of tomorrow’s students and citizens. Teachers are often surprised at students’ ability to engage with the math investigations. Students are often surprised that they have the ability to assist an adult solve their math problem.

*“I enjoyed the math fair because it was fun solving the difficult problems. My mom thought they [people] were confused on jumping chips and my mom got frustrated and skipped jumping chips. I felt good because we helped them [parents] instead of them helping us. Math can be fun, exciting and interesting. I would like to have a math fair because we can do better in math and want to do math. We did this because we wanted to see how our parents solve the problems, because they solve them in a more advanced way.”* -Joel

*“The math fair was a success because we all worked together. I enjoyed making a problem and working in a group. It was hard for my parents to figure out the problem. Helping my parents was good because then it would be easier to make them finish the problem. We should have a Math Fair every year so other people and our parents can learn more math and to give us different ways to do math. It also shows us math is fun and to improve math. Math can be exciting and we can be better problem solvers.”* - Emmett

*“I feel math is fun again. I went with my uncle and he thought it was really nice. I felt really smart helping my uncle. At first he didn’t get it then I told him to read it again. I would want a math fair every year because we can see how smart our parents are.”* - Sarah

*“I think the Math Fair was fun because I have all the games to myself. I enjoyed when I made the hint cards and made the heads and tails for our game. My mom was confused of my game and when she finished playing she went to Randy’s house. When I helped my mom she got better luck of playing. I like the Math Fair because our brain gets smarter and our parents too. Doing different ways to do math is fun. I want to do a Math Fair each year because we will be better at math. Math can be exciting and I can be better at math.”* - Chi

In addition to creating and finding more

engaging mathematical problems and resources to work with, *That’s A Good Problem* helped us all, mathematicians, math educators and teachers, to see that what we needed was some way to continue to address matters of curriculum, classroom pedagogy and the continued development of robust resources in a more deliberate and concentrated way. The Math Fair opened the crack to permit teachers and students to see what mathematics learning could be; however, teachers did not have enough mathematical or pedagogical ease to hold the space open or to open it wider on their own. So this year we have added the second component to *That’s A Good Problem—Lesson Study*.

## Lesson Study

*Lesson Study* is a professional development process that Japanese teachers engage in to systematically examine their practice, with the goal of becoming more effective. This examination centres on teachers working collaboratively on a small number of “study lessons”. Working on these study lessons involves planning, teaching, observing, and critiquing the lessons. To provide focus and direction to this work, the teachers select an overarching goal and related research question that they want to explore. This research question then serves to guide their work on all the study lessons. (Lesson Study Research Group [9]).

It is our hope that through *Lesson Study* we will be able to continue to support teachers to continue to address matters of content knowledge, classroom pedagogy, robust classroom and resources. In addition to expanding the initial work that we began with *That’s A Good Problem*, we know that we also need to find ways to help teachers address matters of assessment and integrate numeracy into other subject disciplines.

In life, mathematics is everywhere, and the responsibility for fostering quantitative literacy should be spread broadly across the curriculum. Developing a mathematical habit-of-mind should be regarded as much more than an affair of the mathematics classroom alone. However, for now that has been our starting place because we felt that it was important to get our own house in order first. Our next steps will be to explore the ways in which numeracy can be best achieved through interdisciplinary study.

## Conclusion

There continues to be much work needed to address the question: what minimum numeracy is required of the average citizen in this computer age? It has been our stance, that while committees meet and policy papers emerge, one of the ways to identify key principles, which are simple, widely acceptable, practical, yet fundamental, which should guide the teaching of school mathematics can be found when mathematicians, math educators and teachers start to work together in the place where all of this agenda will get worked out—the classroom.

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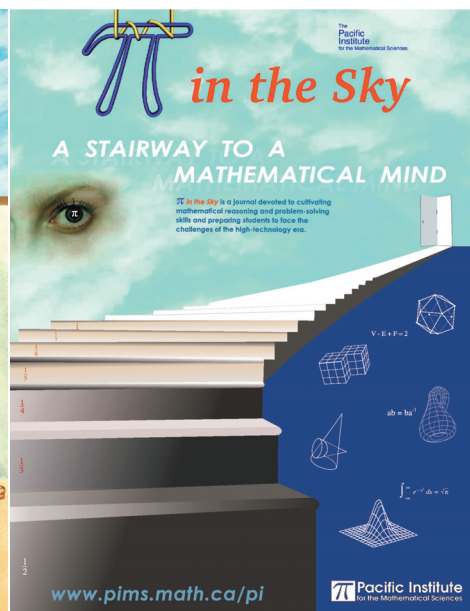
# Pi in the Sky Magazine: The New Issue and Poster

In Fall 2003 PIMS is producing a poster advertising *Pi in the Sky Magazine*. This poster was designed by Sarah Bentz who is in her final year of a Bachelor of Design degree in Visual Communication Design at the University of Alberta. The poster was financed by the Vice President Research at the University of Alberta Gary Kachanoski.

The seventh issue of the PIMS educational magazine *Pi in the Sky* came out recently. The cover was created by Czech artist Gabriela Novakova. The scene depicted was inspired by the article on mathematical biology written by Jeremy Tatum, *Maths and Moths*, which is published in the issue. Prof. Zmodtwo is again featured on the cover page, this time doing research on moths and butterflies.

The Math Opinions section features *Reckoning and Reasoning or The Joy of Rote* by Klaus Hoehsmann. This article talks about the D'Amore Test.

*A. N. Kolmogorov and His Creative Life* by Alexander Melnikov is included in the Math Biography section. Andrei Nikolaevich Kolmogorov was the foremost mathematician of the 20th century, and the article was written to mark the 100th anniversary of his birth.



The cover of the latest *Pi in the Sky Magazine* (left) and the new *Pi in the Sky* poster (right)

Other articles include *Its All For the Best: How looking for the best explanations revealed the properties of light* by Judith V. Grabiner, *Why I Don't Like Pure Mathematics* by Volker Runde, and *Shout-*

*ing Factorials!* by Byron Schmuland.

This issue, as well as all the previous issues, may be downloaded from [www.pims.math.ca/pi/](http://www.pims.math.ca/pi/).



# BIRS Math Fair Workshop

April 10–12, 2003

Contributed by Ted Lewis and Andy Liu, University of Alberta

The BIRS Math Fair workshop was unusual for BIRS in that its focus was Education rather than research. The participants were teachers from elementary schools, junior high schools, colleges and universities, and also people from other institutions and organizations that have a deep interest in Mathematics Education.

The purpose of the workshop was to help teachers learn how to run a successful math fair, to exchange information about math fairs, and to put the members of this diverse group in contact with each other. The deeper purpose was to change the mathematical culture in the classroom, and after five years of experience we believe that this is beginning to happen.

It must be stressed that the sort of math fair that we are talking about is radically different from a typical science fair. Without going into too much detail, the four main tenets are that the math fair be non-competitive (no prizes), that it be all-inclusive (not just for the elite students), that it be interactive (not a poster session) and that it be based on problem-solving.

The problem that we have now is to disseminate the news about the success of math fairs. Workshops are one way of helping teach-

ers learn about math fairs, helping them sustain their efforts, and letting them share experiences with co-workers. As well, workshops build trust between teachers and other educators.

Teachers were invited to the workshop on the condition that they subsequently hold a math fair in their own schools. All participants received a booklet that contains the underlying principles for the math fair.

The workshop dealt with what constitutes a good problem for a math fair, included several examples, and described several different types of math fairs that are based on the guidelines. Many of the participants had already organized math fairs at their schools, and although there was great variation in the details, all followed the guidelines set out in our booklet.

How does a teacher find problems that are suitable for the math fair? Do you begin with a curriculum topic and design an appropriate puzzle, or do you start with a challenging puzzle and try to fit it to the curriculum? The workshop advocated the latter approach, and spent some time having the participants find ways to adapt a good puzzle to the curriculum. A few days ago we visited a math fair organized by

one of the workshop participants, and saw that this adaptation was taking place.

One of the most valuable and spontaneous aspects of the workshop occurred when the teachers who had already conducted math fairs began sharing information about their experiences. The ones who had not yet had a math fair asked many questions and picked up the enthusiasm from those that did. There were some common fears experienced by teachers who had done the math fair for the first time: They want their students to succeed and have a tendency to intervene when students are presented with an unfamiliar task. The math fair works best when, as one teacher put it, you let the students take ownership of their problems. This is a difficult thing for teachers to do, especially when they know that the result is going to be on public display.

Because of the uncertainty of a new venture, many teachers will limit either exposure or participation on their “first-time” math fair. Discussions about this indicated that subsequent math fairs would be greatly expanded, and that the math fair would become a regular part of the students’ math activities.

# Changing the Culture 2003

SFU Harbour Centre, May 2, 2003

Contributed by Malgorzata Dubiel, Simon Fraser University

The sixth annual *Changing the Culture* conference sponsored by PIMS, and **Malgorzata Dubiel** was the main organizer. The conference brought together over 90 school teachers, college and university faculty and graduate students, to discuss issues related to teaching mathematics at all levels. In response to concerns raised earlier in the 2002/2003 school year, both by people teaching “Principles of Mathematics” 11 and 12 in BC high schools and people teaching first year calculus courses in BC colleges and universities, about algebraic skills of students, the theme of this year’s conference was

## *Do We Need To Teach Algebra?*

The conference started with a plenary talk by **Bernice Kastner** (Towson University, Maryland) entitled *Algebra: The Language of Mathematics*. Kastner talked about the impact of technology on high school mathematics curricula. She identified positive gains, like new tools for developing intuition through visualization, and negative aspects, like a loss of ability amongst many students to “speak the language of Mathematics—algebra”. She introduced the term “learned disability” to describe students’ inability to think mathematically and

go beyond routine algorithmic approaches to problem solving.

The problems connected with students’ transitions from secondary to post-secondary mathematics courses were revisited during the panel discussion, with **Lorraine Baron** (Mount Boucherie Secondary) and **Kanwal Neel** (Steveston Secondary) representing high school mathematics teachers, and **Wayne Matthews** (Camosun College) and **Brian Wetton** (UBC) representing the post-secondary sector. The panelists did agree that the algebraic skills of

*continued on next page*

high school graduates have declined in recent years. The possible reasons for this situation were discussed: overloaded high school curriculum, (mis)use of technology, statistics replacing big parts of algebra in the Principles of Mathematics 12 course, and others.

In between the talks and discussions, the participants could choose to attend two out of a possible three workshops:

1. *Helping Students Learn Calculus using Problem Solving Workshop*, by **Joanne Nakonechny** and **Roger Donaldson** (UBC)

about how the UBC Mathematics Department is attempting to help students to succeed in first year calculus courses.

2. *Can Algebra be made Lively?*, by **Malgorzata Dubiel** (SFU) and **Klaus Hoechsmann** (PIMS) - on using pictures and visualization as an aid in algebraic proofs.

3. *Roots and Routes to Algebra*, by **Peter Liljedahl** (SFU) on how to aid pre-algebra students towards algebraic thinking.

The conference was concluded on a lighter note by a very entertaining and thought pro-

voking talk by **Rina Zazkis** and **Peter Liljedahl** (SFU) titled *Hollywood Perceptions of Mathematics: Cultural truth or Mathematical Fiction?* By analyzing selected fragments of popular movies, the speakers and the audience examined the movie industry's (and the general public's) view of mathematics and people who are good at it.

*Changing the Culture 2004* is entitled *Mathematics Curriculum: Could Less Be More?* and will take place on Friday April 23, 2004.

## Connecting Women in Mathematics Across Canada (CWIMAC) University of Alberta, June 12–13, 2003

Contributed by *Malgorzata Dubiel, Simon Fraser University*

The first workshop *Connecting Women in Mathematics Across Canada* (CWIMAC) was organized jointly by the Committee for Women in Mathematics of the Canadian Mathematical Society and PIMS.

Participation in the conference was by invitation: the applicants had to submit a statement of interest, a title and abstract of a talk about their work and/or research interests, and a letter of support from their supervisor. Twenty nine women graduate students from fifteen Canadian universities were selected to attend. They spent two intensive and exciting days, attending talks and presentations, and sharing experiences with eleven women faculty members—speakers and mentors at the conference.

Two plenary talks were given:

**Christiane Rousseau** (U. Montreal): *An Academic Career: A Fantastic Opportunity to Mix Harmoniously Teaching and Research*

**Priscilla Greenwood** (Arizona State U. & UBC): *Mathematical Versatility*

Christiane Rousseau's talk focused on her work on applications of divergent series, while Priscilla Greenwood described her most recent work on applications of stochastic processes in Biology.

The conference included two panel discussions, which were followed by small group discussions, led by the members of the panels: Panel I: *Balancing a Career and a Personal*



*The Participants of the Connecting Women in Mathematics Across Canada Workshop at the wrap-up session*

and Family Life, with **Rachel Kuske** (UBC), **Judith McDonald** (Washington State U.), **Ortrud Oellerman** (U. Winnipeg) and Gerda de Vries (U. Alberta).

Panel II: *Career Strategies: How to Survive a Graduate School and Get a Job You Want*, with **Susan Cooper** (Queens U.), **Leah Edelstein Keshet** (UBC), **Kathy Heinrich** (U. Regina) and **Dorette Pronk** (Dalhousie U.).

Participants' presentations of their research formed an important part of the conference. They were divided by research interests into three groups: Combinatorics and Algebra, Mathematical Biology and Applied Mathematics.

The organizing committee was: Malgorzata Dubiel (Chair, SFU), Judith McDonald

(WWU), Rachel Kuske (UBC), Mateja Sajna (U. Ottawa), Dorette Pronk (Dalhousie U.), Ortrud Oellermann (U. Winnipeg), Gerda de Vries (U. Alberta), Leah Edelstein Keshet (UBC) and Susan Cooper (Queens U.).

Additional support was provided by the Department of Math and Statistics, University of Alberta. Nelson. A Thomson Company co-hosted a dinner reception for conference participants.

For more information and the detailed schedule of the conference, see [www.cms.math.ca/bulletins/2003/cwimac03.e](http://www.cms.math.ca/bulletins/2003/cwimac03.e).

All participants declared the conference a great success, and worth continuing. The next such workshop is being planned to take place in 2005 at BIRS.

# Statistics and Probability in Action

## University of Victoria, April 1–2, 2003

Contributed by Eric Agyekum and Angus Argyle, Math Grad Students, University of Victoria

*Statistics and Probability in Action* was a poster session held at the University of Victoria. At the start of the winter academic term, the instructors for Statistics 260 (an introductory course in probability and statistics) asked their students to interview one or more individuals who use statistics and/or probability on a daily basis. The students constructed profiles of the individuals and displayed their work in posters. They explained how the people used statistics/probability in their jobs, what training and education they received, what the joys and challenges of their work were, and much more.

Over the course of two days in the last week of classes, the posters were displayed in the Student Union Building (SUB) where the students answered questions about their posters.

Popular subjects for the posters were actuaries and insurance companies, weather fore-

casters, scientists, and participants in the gambling industry. Some posters also investigated the usage of statistics by baseball scouts, the customer purchasing patterns and statistics in 7-Eleven stores, and how Sony determines the production levels for new lines of electronics. One student profiled a promotions manager who uses statistics and probability to estimate the costs, alcohol sales and viability of future events at night clubs; the imaginative student displayed the information on a bright orange T-shirt; and, the promotions manager agreed to wear the shirt at the poster session.

Overall, the students thought the poster session was a positive and enjoyable experience. In addition to exposing the students to the uses of statistics and probability in the real world, it gave them the opportunity to gain insight into potential co-op jobs and future careers.

# Forever Annual Math Exhibition (FAME)

## S.J. Willis Educational Centre, Victoria, May 15, 2003

Contributed by Kelly Choo, PIMS Website Manager

Students in School District #61 (Greater Victoria) took part in FAME, the *Forever Annual Math Exhibition* at S.J. Willis Educational Centre on May 15, 2003. A total of 85 students participated in the event, with 7 senior entries, 16 junior and 13 elementary. Students from the following schools entered the exhibition: Lansdowne (21); Esquimalt (1); Lambrick (1); Hillcrest (3); Monterey (1); Frank Hobbs (6); and Royal Oak (3). In total, six Distinction awards (90+%), 12 First Class (80+%), 15 Runner-ups (70+%) and three Participants



The elementary exhibit at FAME

were given out. The winning schools (scores for top 3) were Frank Hobbs (elementary) and Lansdowne (junior and senior).

The event was organized by mathematics teachers Betty Doherty of Lansdowne and Wendy Swonnell of Lambrick Park. FAME is sponsored by PIMS, BCAMT, the Greater Victoria Teacher Association and School District #61.



The junior high winners of FAME

# PIMS Math Fair

## Project Developer

Contributed by Wendy Nielsen, PIMS BC Math Fair Project Developer 2002/03

The year 2002/03 saw the delivery of seven PIMS Math Fair workshops to schools in Richmond, North Vancouver, West Vancouver and Abbotsford. Several of these were a direct result of interaction with the Math Fair Project Developer at the annual *British Columbia Association of Mathematics Teachers (BCAMT)* conference, where a workshop for teachers was offered. PIMS also had an information booth in the exhibit hall. Student workshops, intended to stimulate interest in developing Math Fair projects, provided background information, as well as help in getting started. Students from Grades 7, 8, 10 and 11 developed projects as a result of the workshops held in their classrooms. A total of 20 projects were submitted in the Computing and Mathematical Sciences category of the Greater Vancouver Regional Science Fair (GVRSF), held at UBC in May, 2003. Most all of the projects submitted were produced in classrooms where Math Fair workshops were held, and 12 of these won awards at the GVRSF.

Enthusiasm continues to grow for the “Math Fair” as more and more teachers are made aware of the potential of entering projects in the GVRSF. The position of Math Fair Project Developer remains a key factor in this regard, as do promotional activities such as workshops at the BCAMT conference. Efforts to continue promoting “Math Fair” are sure to continue to produce results as more and more students have fun developing their own projects in computing sciences and mathematics.

The 2003/04 PIMS BC Math Fair Project Developer is **Ilija Katic**. The Greater Vancouver Regional Science Fair is taking place at UBC on April 1–3, 2004.

**Janet Martin**, the 2001/2002 PIMS BC Math Fair Project Developer, received a UBC Faculty of Science Graduate Teaching Award in 2003.



# SIMUW Debuted in Summer, 2003

Contributed by *Sándor Kovacs, University of Washington*

The Summer Institute for Mathematics at the University of Washington (SIMUW) is an exciting program for students who have not yet completed their final year of high school. The first class of enthusiastic, talented students arrived at the campus of the University of Washington on June 22, 2003 for a six-week program of classroom activities, special lectures, field trips, social activities, and intense interaction with faculty, TA's, and fellow students.



*Building 3-dimensional structures*

Some topics such as elliptic curves and computer graphics were studied in two-week segments and other topics such as Markov chains and the mathematics of movement were discussed in special half-day sessions. Students in the program deepened their understanding and appreciation of many active areas of mathematics. Perhaps the most important aspect of the program was social — friendships and contacts were made that will last long after the final session.



*Throwing boomerangs: Pi in the Sky*

This programme has been totally funded by a gift from a generous anonymous couple. Students from Washington, British Columbia, Oregon, Alaska, and Idaho are encouraged to apply for SIMUW 2004.

Details about the 2004 program will soon appear on the website [www.math.washington.edu/simuw/](http://www.math.washington.edu/simuw/).



*Deep in thought*

## 2003 ESSO-CMS-PIMS Math Camp

Simon Fraser University, June 23–27, 2003

Contributed by *Malgorzata Dubiel, Simon Fraser University*

The 2003 ESSO-CMS-PIMS Math Camp for High School students took place at the SFU Burnaby campus. This was the third time SFU hosted the camp, which was organized by Malgorzata Dubiel and Justin Gray.

Thirty grade 9 and 10 students from 18 Lower Mainland schools were selected from over seventy applications sent by their teachers. For five days, these exceptional students participated in exciting and challenging activities and problem sessions. The activities were organized by the SFU faculty and graduate students, and three invited speakers: **Lily Yen** (Capilano College), **Branko Curgus** (Western Washington University) and **Rob Scharein** (CECM, WestGrid & NewMIC).

The presentations included: **Keith Promislaw** (Math, SFU, & Ballard Powersystems): *Fuel Cells: Where Does all the Water Go?*

**Eirikus Palsson** (Biosciences, SFU): *Modelling Cell Movements and Communications Using a 3D Model*

**Rob Scharein** (CECM SFU, WestGrid & NewMIC): *Knots and Pretzels*

**Rina Zazkis & Peter Liljedahl** (Faculty of Education, SFU): *Mathematics in the Movies*

Several problem sessions and challenge problems given every day culminated in a contest written Friday morning. And, since the quality and the enthusiasm of the participants was extremely high, everybody left with a prize — and some with more than one!



*Participants of the June 2003 ESSO-CMS-PIMS Math Camp*

For more information and pictures from the camp, see the camp website at [www.cecm.sfu.ca/~lisonek/MathCamp.htm](http://www.cecm.sfu.ca/~lisonek/MathCamp.htm)

# 5<sup>th</sup> Annual PIMS Elementary Grades Mathematics Contest

University of British Columbia, May 24, 2003

Contributed by Fanny Lui, PIMS

The PIMS Elementary Grades Math Contest, which was designed for students ranging from Grades 5–7, provides an opportunity for students to express their interests and talents in mathematics and to experience mathematics as an exciting sport.

A total of 283 students participated in the 5th annual contest, with 108, 93 and 82 in grades 5, 6 and 7 respectively. The format was the same as in previous years. There were 3 rounds, and the written part came first with the Sprint and Target rounds. The top 10 students from these rounds went on to the Countdown round where students duelled against each others. It started with the 9th and 10th ranking students, and the winner of that contest then went on to “duel” with the 8th place holder. As a result the person who ranked 10th had the potential of winning the contest by beating the 9 people ahead of him/her one by one. The dueling consisted of answering math questions against the clock and sounding a buzzer.

The top 10 in each grade received a T-shirt and medal. The top 3 also received a trophy. Certificates of participation were available for all students on the day. The top 10 winners in each grade were:

**Grade 5:** 1. Clara Hwang (Bayview Community) 2. Jeffrey Yeh (Vancouver Montessori) 3. Deshin Finlay (Braemar) 4. Duncan Dauvergne (Queen Mary) 5. Roger Zhang (Sir Wilfred Laurier Annex) 6. Kevin Fang (Dr Annie B Jamieson) 7. Anson Wong (Lord Byng) 8. Sandra Long (Sir William Osler) 9. Gloria Chu (White Rock) 10. Anne Zhu (Harold Bishop)

**Grade 6:** 1. Jeffrey Choi (John T Errington) 2. Denny Choi (St John's) 3. Juno Jung (Marlborough) 4. Jonathan Leung (James Whiteside) 5. Timothy Wai (Our Lady of Perpetual Help) 6. Yiyi Wang (Dr Annie B Jamieson) 7. Bill Xia (Tomekichi Homma) 8. Eric Shen (Maple Grove) 9. Keith Lui (St Francis Xavier) 10. Sophie Jisoo Kwak (Canyon Heights)

**Grade 7:** 1. Aram Ebtakar (Maple Creek Middle) 2. Joel Li (David Lloyd George) 3. Qi Liu (Queen Mary) 4. Alarica Tang (Kitchener) 5. Yuan Liang (Pitt River Middle) 6. Tae Ken Kim (Holly) 7. Daniel Park (Kwayhquiltum Middle) 8. Aaron Lo (Lord Byng) 9. Bryan Huang (Sir William Osler) 10. Elliot Hoyt (Fort Langley)



The top 3 ELMACON winners in each grade.

## Upcoming Education Activities

- November 18, 2003: **Alberta High School Mathematics Competition**, Part I of the 2003–04 Season
- January 9–12, 2004: **PIMS Graduate Information Weekend**, PIMS-UBC, PIMS-SFU, PIMS-UVic
- February 14–15, 2004: **IAM-CSC-PIMS Senior Undergraduate Math Modelling Camp**, PIMS-UBC, PIMS-SFU
- April 1–3, 2004: **Greater Vancouver Regional Science Fair**, UBC
- April 23, 2004: **Changing the Culture VII, Mathematics Curriculum: Could Less Be More?**, SFU Harbour Centre
- May 29, 2004: **6<sup>th</sup> PIMS Elementary Grades Mathematics Contest**, UBC
- June 20–July 31, 2004: **Summer Institute for Mathematics**, U. Washington
- July 12–15, 2004: **First Joint Canada-France Meeting on the Mathematical Sciences**, Toulouse, France
- December 4–9, 2004: **Numeracy and Beyond**, BIRS

## A Contest for Epsilons U. Victoria, June 2, 2003

Contributed by David J. Leeming

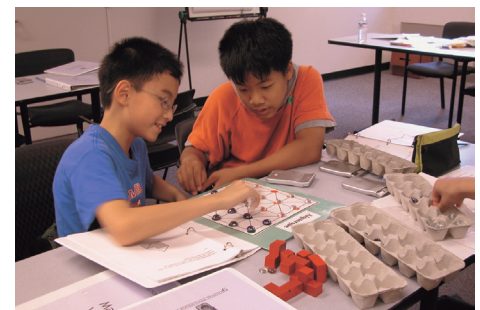
The students in the Math and Stat Course Union at the University of Victoria organized a half-day event entitled *Contest for Epsilons* for students in grades 5–7 in the Greater Victoria area. The event consisted of two contests and seminars on some mathematical topics. One hundred and thirty students from ten schools took part in the event. The students in the Course Union prepared the competition questions, did the registration and marketing of the event, and gave the presentations.

Students David Hosick and Michael Kim organized and ran the event with the assistance of students and faculty from the Department of Mathematics and Statistics. Some financial support was provided by the PIMS and the BC Association of Mathematics Teachers and refreshments were provided by Thrifty Foods.

The first contest was such a success that it is now to become an annual event.

## MathClick Workshop PIMS-UBC, Aug 29, 2003

MathClick workshops are full-day mathematics immersion experiential events for students in grades 5–7. The main intention is to awaken children's latent talent and interest by showing them that mathematics can be also playful and intriguing. It was the 3rd year that PIMS has organized and hosted one of these workshops. Math Circles is a 17-week follow-up to MathClick. It takes place for 1.5 hours every second Saturday during the fall of 2003, and is based on the Singapore Grade 6 programme with modifications.



Student playing a game at MathClick.





*The Banff Centre from the air. Photo courtesy of Robert Piche.*

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### PIMS is supported by:

- ◆ The Natural Sciences and Engineering Research Council of Canada
- ◆ The Alberta Ministry of Innovation and Science
- ◆ The British Columbia Ministry of Competition, Science and Enterprise
- ◆ Simon Fraser University
- ◆ University of Alberta
- ◆ University of British Columbia
- ◆ University of Calgary
- ◆ University of Victoria
- ◆ University of Washington
- ◆ University of Northern British Columbia
- ◆ University of Lethbridge

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