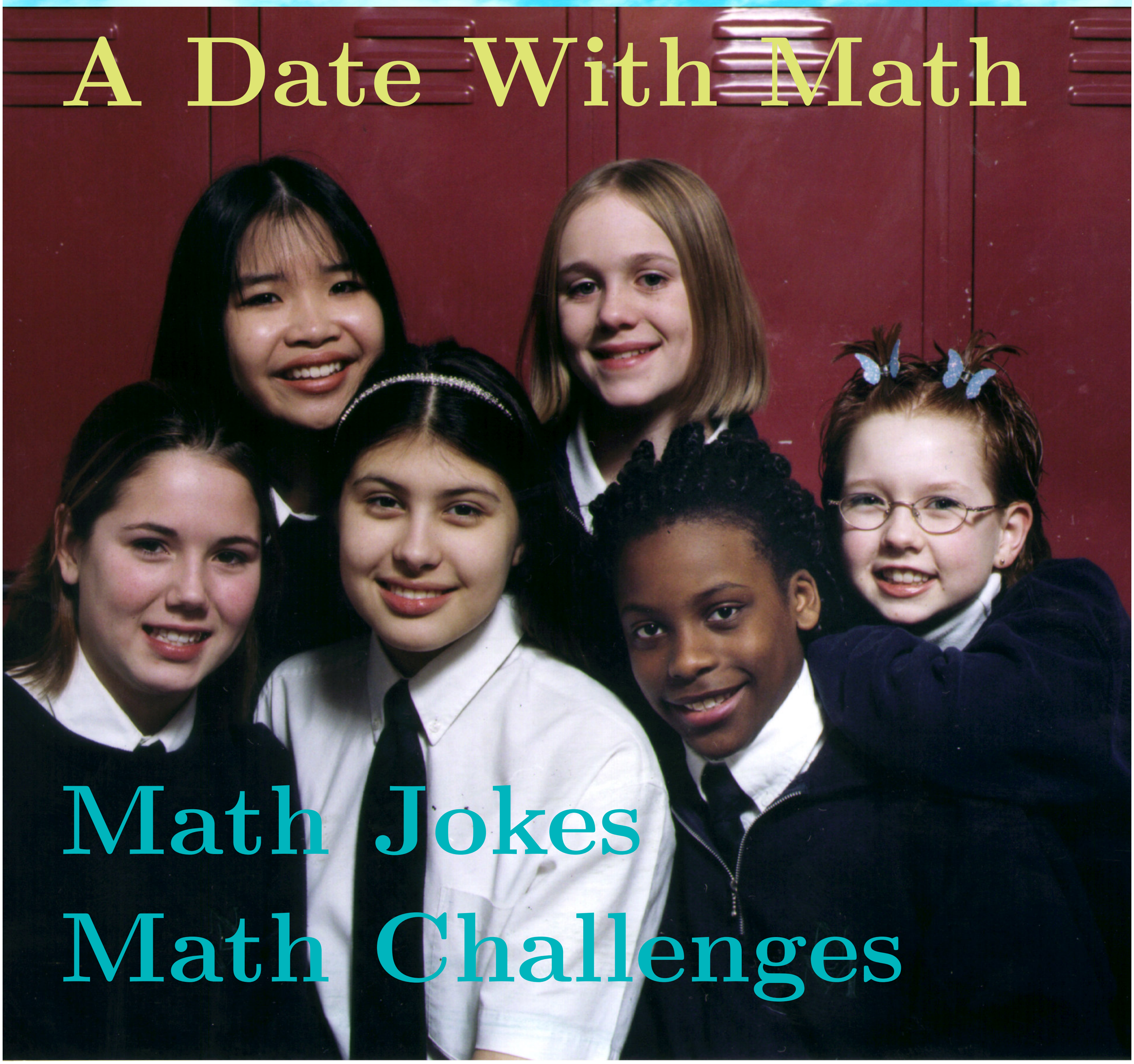
The Pacific Institute for the Mathematical Sciences

π in the Sky

June 2000

A Date With Math



Math Jokes
Math Challenges

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The Pacific Institute for the Mathematical Sciences

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This journal is devoted to cultivating mathematical reasoning and problem-solving skills, to prepare students for the challenges of the high-technology era.

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We also welcome **Letters to the Editor** from teachers, students, parents or anybody interested in math education (be sure to include your full name and phone number).

Cover page: The picture on the cover page (taken by Bill Brennan) features students from the Nellie McClung Girls' Junior High Program at Oliver School in Edmonton – courtesy of the principal Karen Linden.

A few words from the director of PIMS

I am very pleased to welcome *π in the Sky* as a new and groundbreaking initiative of the Pacific Institute for the Mathematical Sciences. This journal represents a major milestone in the mission of our institute: to promote research in, and applications of, the mathematical sciences, to be proactive in the training of young people, and to enrich education and public awareness in the mathematical sciences.

This journal is designed as a forum for dialogue and debate between academic mathematical scientists, educators, students, and the public at large. PIMS scientists are eager to share their love of mathematics, their discoveries, and the joy they bring. *π in the Sky* naturally complements the many other PIMS educational initiatives including Changing the Culture, an annual conference that forges ties between the math community and K-12 teachers, Math Fairs, Evenings in Mathematics, and Mathematics Unplugged workshops all focusing on presenting "fun" methods for doing math and computer science with children and their parents, and the Annual PIMS Elementary Grades Math Contest. I welcome all of you to explore the PIMS website at `http://www.pims.math.ca` and take part in our programs.

The birth of *π in the Sky* showcases the underlying principle on which PIMS is founded: that its main resource is the high-quality community of mathematical scientists in Western Canada. PIMS empowers its membership to create the greatest opportunities for realizing the collective vision of their favourite discipline. In this respect, PIMS has immediately recognized the importance of the vision of Wieslaw Krawcewicz, John Bowman and their colleagues as well as their commitment to give time and effort to achieve it. PIMS is proud to embrace, sponsor and support this initiative. Since the initial founding efforts of our colleagues at the University of Alberta, many mathematical scientists from all PIMS sites have joined in and committed towards the success of this enterprise.

Many thanks to the selfless efforts of our colleagues on the Editorial Board and to those who wrote the very interesting articles for this very first issue. My best wishes to all of you for an unqualified success of this wonderful initiative.

Nassif Ghoussoub, FRSC
Director, Pacific Institute for the Mathematical Sciences

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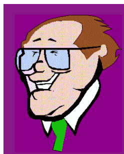

Math Opinion

The Perfect Education System for an Affluent Society

Andy Liu

Newcomers to North America, especially those with children of school age, are often appalled by the generally low academic standards of our schools. Because it is nominally free, it is not in the economic interest of the school boards to keep the children in schools for any longer than they are legally required to do. As a result, promotion from grade to grade is automatic, whether or not the children have accomplished anything at all. To make this seem accountable, the academic standard is set deliberately low so that failure is practically impossible.

However, it may be argued that this system is precisely what North America wants and needs. The academic standard in the former Soviet Union was certainly very high, and a large number of well-qualified people were produced. Unfortunately, the country did not have enough good jobs, so that most of them were under-employed. Their frustrations turned them into dissidents and eventually, the government was brought down.

While the fall of the Soviet Empire may be considered a very good thing in North America, such was certainly not the view of those who tumbled from power. This is a lesson not lost on the governments in North America. A country only needs a very small number of well-qualified people to fill the top jobs. It is important for the masses who will be frying hamburgers in McDonald's or doing other menial jobs to be content with their lot. In an affluent society, their livelihood is more than acceptable.

It is therefore the job of the schools to make sure that the majority of the students will not be qualified for anything better. The motto is that everyone has the God-given right to be a fool. Children in many other cultures do not enjoy this right. If they ever aspire in that direction, they will be beaten back to the straight and narrow. Here, students and teachers, as well as children and parents, can lead a relatively peaceful and democratic coexistence.

How does North America maintain its eminence in scientific and technological advances? Recent history showed that it simply stole the best that were produced by other countries. However, its own education system, discredited though it might have been, had also produced talents that take second place to no others in the world. How is this possible?

The answer lies in the total absence of pressure in the schools. Subjects that arouse no passion in the students can safely be ignored. For those who have no passion in anything, this leaves a lot of time to play or sleep. For those who are sufficiently self-motivated, the time can be spent on subjects of their interest, with the result that they could devote much more effort on them than students from other countries. Granted, the number of such students is very small, but then, we only require enough of them to meet the needs of society.

Our final point is that our society is increasingly dominated by commercial concerns. Science and technology have become subservient to business needs. The economy is largely driven by advertisements. If our schools teach our students properly, they will begin to see that the advertisements are a load of utter nonsense. This will certainly bring about the end of civilization as we know it. Is that what schools are supposed to do?

About the Author: Andy Liu's ambition was to be an elementary school teacher, so he completed a Bachelor of Science degree at McGill University and then came to the University of Alberta to get a degree in Education. He ended up doing a Ph.D. in mathematics as well and because he was so highly qualified he found it difficult to get a permanent position as an elementary teacher. He eventually joined the Mathematical Sciences Department at the University of Alberta in 1980. He proceeded to establish a reputation as a top teacher of mathematics and as a world-wide spokesman for contests in mathematics (which are a very big deal in many countries). He also started a program called SMART (Saturday Mathematical Activities, Recreation and Tutorials) for Junior High students, entirely on his own, funded principally out of his own pocket. Over the years he accumulated an impressive list of awards: Faculty of Science Teaching Award, University of Alberta Rutherford Teaching Award, Faculty of Engineering Teaching Award. He and Murray Klamkin coached the U.S. Mathematical Olympiad Team to top standings in several International Mathematical Olympiads. Then in 1996 the national and international awards started to arrive:

- 1996: The David Hilbert International Award for the promotion of of Mathematics World Wide (awarded by the World Federation of National Mathematics Competitions).
- 1998: Distinguished Educator of the Year (awarded by the Ontario Institute for Studies in Education, University of Toronto).
- 1999: Canadian University Professor of the Year (awarded by The Council for the Advancement and Support of Education, and the Canadian Council for the Advancement of Education).
- 1999: Named a 3M Teaching Fellow, one of up to 10 national awards given annually by the 3M Corporation.
- 1999: The Michael Smith Award for the Promotion of Science, awarded annually by Industry Canada in honour of

Canadian Nobel Prize winner Michael Smith (Chemistry, 1993).

No University Professor in Canada can claim such a list of honours. Andy is a demanding, but popular teacher, exactly as he prefers. His respect for students is immense, but he demands that they learn to think, and he does it in a firm and supportive manner. We all benefit by paying attention to the perceptions of a person who has devoted so much thought and energy to the teaching and learning of mathematics. (Jack Macki)

Math Jokes

As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality. Albert Einstein (1879-1955)

I've heard that the government wants to put a tax on the mathematically ignorant. Funny, I thought that's what the lottery was! Gallagher

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Young man, in mathematics you don't understand things, you just get used to them. John von Neumann (1903-1957)

"You do not really understand something unless you can explain it to your grandmother." Albert Einstein

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Life is complex. It has real and imaginary components. Tom Potter

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Copernicus' parents: Copernicus, young man, when are you going to come to terms with the fact that the world does not revolve around you?! Erin Leonard

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An astronomer is on an expedition to darkest Africa to observe a total eclipse of the sun, which will only be observable there, when he's captured by cannibals. The eclipse is due the next day around noon. To gain his freedom he plans to pose as a god and threaten to extinguish the sun if he's not released, but the timing has to be just right. So, in the few words of the cannibals' primitive tongue that he knows, he asks his guard what time they plan to kill him.

The guard's answer is "Tradition has it that captives are to be killed when the sun reaches the highest point in the sky on the day after their capture so that they may be cooked and ready to be served for the evening meal."

"Great," the astronomer replies.

The guard continues, though, "But because everyone's so excited about it, in your case we're going to wait until after the eclipse."

Edward Ruden

Statistics are like a bikini - what they reveal is suggestive, but what they conceal is vital. Aaron Levenstein

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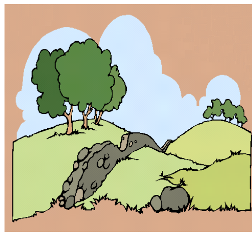


Math Stories

Solving Problems can be Fun

Ambikeshwar Sharma

Sometimes simple mathematical problems are put to you at a time and at a place when you least expect them. I believe that it pays to face them and to try to understand them and if possible to solve them. As a student of mathematics it is not fair to refuse to attend to the problem or to pass it off with a contemptuous wave of the hand. This conviction came to me by an event that happened to me several years ago when I was going by an evening overnight train from Lucknow to Allahabad, India.

Later this conviction became all the more strongly entrenched in my mind when I came to Edmonton about 36 years back. Here I met for the first time Prof. Leo Moser who was a simple, courteous and soft spoken person full of anecdotes, humor and problems. Although the department was small and had no separate building to itself, Leo Moser was the centre of activity discussing problems with any one who cared to listen. He had a photographic memory and was an excellent chess player. I learned that he could play chess against 20 or 30 teams of students from different schools at the same time and would win against all of them. In the faculty lounge in the department, he would discuss problems or tell anecdotes to his students. Anyone who came in and wanted to listen was welcome.

One day I heard him talking to a student about Blichfeldt's Lemma. I had heard about it but I did not know what it was and why it was important. Professor Moser then explained to me how this lemma was proved by an American mathematician, Blichfeldt. It states that if there is a plane region R of any shape with an area more than n units, then it is always possible to translate (i.e., slide without turning) it in such a way that it covers $n+1$ lattice points (with integer coordinates). In particular, he explained by a sketch on the blackboard that for $n=1$, there is a pair of distinct points in the area R that can be translated into two distinct points A and B with coordinates (x_1, y_1) and (x_2, y_2) such that $x_2 - x_1$ and $y_2 - y_1$ are both integers. He explained to me another time how this lemma was used by Minkowski to solve a problem of Hilbert about an orchard called 'the orchard problem.'

Like the Scottish Problem Book of problems in Warsaw (Poland), he had started a book of problems in which visitors and anyone who has a problem could put his problem in black and white. His contacts with people like Martin Gardner, Prof. P. Turán, Prof. P. Erdös, Prof. D.J. Newman and many others brought us the visits of some of these well known people. Once I showed him a poem called "Song of a Ph.D.," a parody written on the lines of Gilbert and Sullivan, which I had heard at Cornell. He read the poem and could recite it the next day. I still recall the first stanza, which runs like this:

When I was a kid and went to school,
Arithmetic was taught by rote and rule,
I did long division and I did cube roots,
At the Rule of Three, I was specially astute,
I was so astute at the Rule of Three,
That now I am the holder of a Ph.D.

Professor Moser was very hospitable and the evening parties at his home were always a treat. His passing away at an early age due to a heart attack was a serious blow to the Department. One of his last students Prof. David Klarner is known for his work on Polyominoes.

To return to the circumstance of the event that happened to me in India, when I was working at the University of Lucknow: I wanted to go to Allahabad by the overnight evening train and to consult the University library there during the day and return the next evening back to Lucknow. I could not afford the luxury of a first class or second class ticket and so bought a third class return ticket. I arrived at the railway station half an hour earlier than the scheduled departure time in order to acquire an upper berth (if possible) and at best a comfortable seat away from the tumult and rush of passengers. I decided to take a seat that looked promising but noticed that a gentleman was ambling outside with an eye on his suitcase. He had already occupied the upper berth and had spread his blanket there for the night and so I had to occupy the lower berth.

A few minutes later the gentleman came in and I learned from him that he was a businessman who was going to Allahabad on some business. He had a big store in a fashionable area in Aminabad. I told him that I was a lecturer at the University and taught mathematics. He seemed happy to learn this and asked me if I could solve two questions for him that his son had asked him and that he could not do. I told him that I would give his problems a try and invited him to state them. My companion started telling me the first problem:

Problem 1. A man was badly in need of an honest, hard working servant to look after his cows and do all the household work, as his wife was sick and could not manage the job. He was willing to pay him food and lodging and a dollar a day, paid monthly, but the servant must do all the jobs. He confided his problem to a close friend of his who promised to look around and find a good chap. In a few days his friend, a goldsmith by profession, brought him a young sturdy fellow who was willing to do all the work for the salary offered, except that there would be a condition that he wanted the master to accept. The condition required by this servant was that if he decided to leave on a certain day, he must get the exact salary up to that day. If the master were unable to pay the exact amount up to that day, the master would have to pay a severe penalty of losing some body part (nose and ears). Since the man needed a servant badly, he agreed to the terms without much thought. The servant did prove to be excellent and he did all the jobs well without murmur or dissent. But the master began to worry about the terms imposed by the servant and this worry made him sick. He again told his difficulty to his friend, the goldsmith, who asked him to be of good cheer. He asked him to give him \$31 and in return he gave him five gold rings, which he was asked to put on his fingers. My companion asked me to tell him the price of each of those golden rings with which he could pay his strange servant if he decided to leave on any day of the next month.

I went over the problem with him again to get some time to think. By this time other passengers were streaming in and our compartment was getting filled up. After a few minutes I was lucky to get the solution for my friend and when I told him the price of each of the five rings, he was happy. His second problem was as follows:

Problem 2. Three men with a monkey bought some mangos, but decided to eat the mangos next morning after the night's sleep. At night one of the men got up and saw that if he gave one mango to the monkey, he could divide the rest of the mangos into three equal groups. So he ate his share and gave one mango to the monkey. Later a second person got up and he also noticed that if he gave one mango to the monkey he could divide the remaining mangos into three equal groups. So he gave one mango to the monkey and ate his share of the mangos and went to sleep. Finally the third man got up and gave one mango to the monkey and ate his share of the mangos and went to sleep. When all of them got up in the morning, they again found that if they gave one mango to the monkey, they could divide the rest equally among themselves. The problem is to determine the smallest possible number of mangos that the men had bought.

The train had now started. My companion insisted that I occupy the upper berth while he would share his lower berth with two others. By the time the train arrived at the next station I was able to announce to my friend that the smallest number of mangos in the second problem was 79. Although we had no pen or paper, I could explain to him how I obtained the solution and he could verify the result. The next morning as the train steamed in at Allahabad, my friend woke me up and we parted as good friends. I was happy to have earned a friend by my effort to solve his problems.

Problem 3. If you can find the day of the week from the date of birth of a person, you can make a good impression in any company and it becomes great fun to demonstrate this to the guests of the evening. This kind of problem is called a Calendar Problem. We recall that the calendar that we use now is the Gregorian Calendar started by the Pope Paul Gregory in 1582 who fixed the days in the year as 365 but that every fourth year would be a leap year except when it is divisible by 400. Thus 1700, 1800, 1900 are not leap years, while 2000 is a leap year. If we keep in mind that the days of the week recur every 7^{th} day, all calculations in calendar problems are based on congruence modulo 7[1] To give a convenient and easy formula for calculating the day of the week, when a particular date r falls, we make the following conventions:

The days of the week are numbers as follows:

0	–	Sunday
1	–	Monday
2	–	Tuesday
3	–	Wednesday
4	–	Thursday
5	–	Friday
6	–	Saturday

We shall number the months in the following way:

1	–	March
2	–	April
3	–	May
4	–	June
5	–	July
6	–	August
7	–	September
8	–	October
9	–	November
10	–	December
11	–	January
12	–	February

This curious numbering is chosen because in each leap year February gets an extra day. So it is convenient to begin a year with March and to close it with February. Then February 28, 1999 will be considered as the last day of 1998. If someone is born on the r^{th} day of the m^{th} month of the year $N = 100C + D$, $0 \le D \le 99$, then we can obtain d the day of the week by the following:

$$d = r + \left[\frac{13m-1}{5}\right] - 2C + D + \left[\frac{C}{4}\right] + \left[\frac{D}{4}\right] \pmod 7$$

where $[x]$ = integral part of x.

Let us calculate the day of the week for July 13, 1938. Here $r = 13$, $m = 5$, $C = 19$, $D = 38$. Then

$$\begin{aligned} d &= 13 + \left[\tfrac{13\times5-1}{5}\right] - 2\cdot 19 + 38 \\ &\quad + \left[\tfrac{19}{4}\right] + \left[\tfrac{38}{4}\right] \pmod 7 \\ &= 13 + \left[\tfrac{64}{5}\right] - 38 + 38 \\ &\quad + \left[\tfrac{19}{4}\right] + \left[\tfrac{38}{4}\right] \pmod 7. \end{aligned}$$

Since $13 = 7 + 6$, we say that $13 \equiv 6 \pmod 7$, $\left[\frac{64}{5}\right] = 12 = 7 + 5 \equiv 5 \pmod 7$, $\left[\frac{19}{4}\right] = 4$, $\left[\frac{38}{4}\right] = 9 \equiv 2 \pmod 7$. Then $d = 6+5+4+2 = 17 \equiv 3 \pmod 7$. Therefore July 13, 1938 falls on a Wednesday.

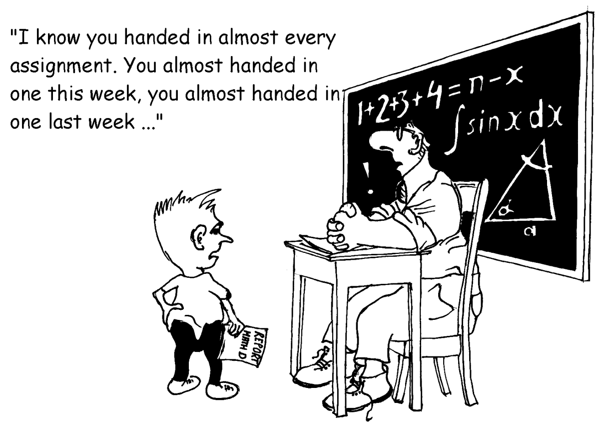
[1] Two integers are congruent modulo 7 if they have the same remainders when divided by 7.

Answer to Problem 1: \$1, \$2, \$4, \$8, \$16.

You can find more information about the author at the Internet address:

http://www.math.ualberta.ca/People/Facultypages/Sharma.A.html

Math Jokes



"Two things are infinite: the universe and human stupidity; and I'm not sure about the universe." Albert Einstein

A famous statistician would never travel by airplane, because he had studied air travel and estimated the probability of there being a bomb on any given flight was 1 in a million, and he was not prepared to accept these odds.

One day a colleague met him at a conference far from home. *"How did you get here, by train?"*

"No, I flew."

"What about your the possibility of a bomb?"

Well, I began thinking that if the odds of one bomb are 1:1,000,000, then the odds of TWO bombs are $\frac{1}{1{,}000{,}000} \times \frac{1}{1{,}000{,}000}$. This is a very, very small probability, which I can accept. So, now I bring my own bomb along!" (Philip Clarke)

MATH GAMES

Against all odds

Byron Schmuland

In 1975, when I was a high school student, the Canadian government held its first lottery, to help cover the costs of the 1976 Olympic Games in Montreal. In those days lotteries were unheard of, and this one offered one Canadian the chance of winning a million dollars at the price of ten dollars per ticket. The actual draw was televised live, as the climax of an hour-long variety show. I was very excited. I didn't win.

Nowadays, of course, lotteries are no big deal. They have become part of our daily life, with chances to win prizes big and small on practically every day of the week. Ever since blowing ten bucks in 1975, I've been fascinated with calculating the odds in games and lotteries. I'd like to show you how to calculate the chances in some different games, and finish off with a discussion of Lotto 6-49.

In solving this kind of problem, the numbers involved are very large, but the idea is quite simple: the chance of an event is the ratio of favorable outcomes to the total number of outcomes. A branch of mathematics called combinatorics helps us count these big numbers.

> The Probability Formula.
>
> $$\text{The chance of an event} = \frac{\text{The number of favorable outcomes}}{\text{The total number of outcomes}}$$

For instance, the total number of outcomes in Lotto 6-49 is 13 983 816. So, your odds of hitting the jackpot are about one in fourteen million! But how do they come up with the number 13 983 816? Did someone have to sit down with a paper and pencil, and write out all the different things that could happen?

The answer is no, to find the number of outcomes you don't have to list them out; you just use some basic mathematics. Let me show you, but before we try to tackle the Lotto 6-49 problem we'll warm up with some simpler problems.

Rolling the dice.

What's the chance of rolling a ⚅, if you roll a fair die? Since all possible outcomes are ⚀, ⚁, ⚂, ⚃, ⚄, ⚅, and there is only one favorable outcome ⚅, the chance is 1/6.

What's the chance of rolling at least one ⚅ if you roll two fair dice? The set of possible outcomes is illustrated here:

⚀⚀	⚀⚁	⚀⚂	⚀⚃	⚀⚄	⚀⚅
⚁⚀	⚁⚁	⚁⚂	⚁⚃	⚁⚄	⚁⚅
⚂⚀	⚂⚁	⚂⚂	⚂⚃	⚂⚄	⚂⚅
⚃⚀	⚃⚁	⚃⚂	⚃⚃	⚃⚄	⚃⚅
⚄⚀	⚄⚁	⚄⚂	⚄⚃	⚄⚄	⚄⚅
⚅⚀	⚅⚁	⚅⚂	⚅⚃	⚅⚄	⚅⚅

Counting these up we find that there are 36 possible outcomes. The outcomes that have at least one ⚅ are ⚀⚅, ⚁⚅, ⚂⚅, ⚃⚅, ⚄⚅, ⚅⚀, ⚅⚁, ⚅⚂, ⚅⚃, ⚅⚄, ⚅⚅. Since there are eleven of these, the chance of rolling at least one ⚅ with two fair dice is 11/36.

If we want to go any farther, we need ways of counting the number of outcomes without actually listing them all out. Luckily, the multiplication rule comes to the rescue.

> The multiplication rule. The number of pairs is equal to the number of choices for the first object times the number of choices for the second object. This also works for triples, quadruples, etc.

The multiplication rule says that, since there are 6 choices for each roll, the number of pairs is $6 \times 6 = 36$. Hey! This is exactly how many we counted above. How about the number of pairs with at least one ⚅? Here we can't apply the multiplication rule directly. Let's look at the opposite problem, and count the number of pairs with no sixes. If we don't allow ⚅, we have five choices for the first die and five for the second, so there are $5 \times 5 = 25$ pairs with no ⚅s. So the number of pairs with at least one six must be $36 - 25 = 11$. The chance of getting at least one six with a pair of dice is 11/36.

Do you see what happened? We just solved the two dice problem without listing the outcomes. The nice thing is that the same idea works no matter how many dice we use.

Suppose we roll ten dice. The total number of outcomes is

$$6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 60\,466\,176.$$

The number of outcomes with no ⚅s is $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 9\,765\,625$, so there are $60\,466\,176 - 9\,765\,625 = 50\,700\,551$ outcomes with at least one ⚅. The chance of rolling at least one ⚅ with ten dice is $50\,700\,551/60\,466\,176 \approx 0.8385$: about an 84% chance.

Birthdays.

Do you know anyone in your class that has the same birthday as you? You might be surprised to find that matching birthdays are not that unusual. Let's figure out the chances of a shared birthday.

I am going to use the number 30 as the class size; you could find the chance in the same way for any other class size.

We ignore leap years and assume that there are 365 possible birthdays. An outcome is just a list of 30 birthdays, so the multiplication rule says that the total number of outcomes is $365^{30} = 7.392 \times 10^{76}$ points.

The number of outcomes without a shared birthday is $365 \times 364 \times 363 \times \cdots \times 336 = 2.171 \times 10^{76}$. Why? There are 365 choices for the first person's birthday. Since the second person's birthday must be different, there are only 364 choices. The third person's birthday must be different than both of the first two, leaving 363 choices, etc.

By subtracting we find that the number of outcomes with a shared birthday is $7.392 \times 10^{76} - 2.171 \times 10^{76} = 5.221 \times 10^{76}$, and so the chance of a shared birthday is $(5.221 \times 10^{76})/(7.392 \times 10^{76}) = 0.7036$, about 70%.

If you wonder where the formulas come from...

The number of ways to order n distinct objects is called "n factorial" and is written $n! = n \times (n-1) \times \cdots \times 1$. If you don't use the whole set, the number of ordered sets of r objects chosen from n distinct objects is $n \times (n-1) \times \cdots \times (n-r+1) = n!/(n-r)!$.

Each unordered set of size r corresponds to $r!$ distinct ordered sets, so we must divide by $r!$ to get the number of such unordered sets. This value is called "n choose r" and written $\binom{n}{r} = n!/r!(n-r)!$.

For instance, suppose that $r = 3$. Then all six ordered sets ABC, ACB, BAC, BCA, CAB, CBA correspond to the same unordered set $\{A, B, C\}$. The number of unordered sets is one-sixth the number of ordered sets.

The 52-card shuffle.

Shuffling a deck of cards puts them into a particular order. What is the probability that someone else, sometime in the past, has shuffled a deck of cards into the exact same order? The number of different ways to order a deck of cards is $52! = 8.0658 \times 10^{67}$. To understand how big this number is, let us suppose that everyone on earth, from the beginning of time, has been doing nothing but shuffling cards and checking what order the deck is in. Let's overdo it, and suppose that mankind has been on earth for a million years, and that the population of the earth was constantly equal to 10 billion. If we can do one shuffle per second, then the total number of shuffles in history is $10\ 000\ 000\ 000 \times 1\ 000\ 000 \times 365 \times 24 \times 60 \times 60 = 3.1536 \times 10^{23}$. This is less than one billionth of one billionth of one billionth of the total number of possible shuffles. By shuffling an ordinary deck of cards, you've created something that has never existed before in the history of the universe!

Try this at home!

Now that I've convinced you that the number of possible shuffles for a deck of cards is incredibly large, you may find the following game quite interesting. Take two decks of cards and shuffle both of them thoroughly. Give one deck to a friend and place both your decks face down. Now, at the same time, you and your friend turn over your top card. Are they the same card? No? Then try again with the second card, the third card, etc. If you go through the whole deck, do you think you and your friend will ever turn over the same card? Try it and see!

Lotto 6-49.

The most popular lottery in Canada is Lotto 6-49. Six distinct numbers are randomly chosen from 1 to 49 and your prize depends on how many of these match the numbers on your ticket. If you match three numbers you win ten dollars, and if you match all six numbers you win the jackpot. Of course, there are other prizes for matching 4 or 5 numbers, as well. What are your chances?

The number of possible ticket combinations is $\binom{49}{6} = 13\ 983\ 816$. Your chance of winning the jackpot therefore is one out of 13 983 816, which is 7.15×10^{-8}.

As for matching three numbers, consider the numbers from 1 to 49 as divided into two groups: the six numbers on your ticket, and the forty-three numbers that aren't on your ticket. To win ten dollars you need exactly three from the first group, and three from the second group. The number of Lotto 6-49 drawings of that type is

$$\binom{6}{3} \times \binom{43}{3} = \frac{(6)(5)(4)}{(3)(2)(1)} \times \frac{(43)(42)(41)}{(3)(2)(1)} = 20 \times 12\ 341 = 246\ 820.$$

Thus, the chance of matching exactly three numbers is $246\ 820/13\ 983\ 816 = 0.017650$.

We can find all the Lotto 6-49 probabilities in the same way. The bottom of the ratio is always equal to the total number of Lotto 6-49 draws: $\binom{49}{6}$. The top of the ratio always has two terms, 43 choose something times 6 choose something. The term with 43 represents the number of ways to choose from the 43 values not on your ticket, and the other term represents the number of ways to choose from the 6 values on your ticket. If you think of the numbers as "good" or "bad" according to whether they are on your ticket or not, then $\binom{43}{6}\binom{6}{0}$ is the number of draws that result in 6 bad numbers and 0 good numbers. Similarly, $\binom{43}{5}\binom{6}{1}$ is 5 bad numbers and 1 good number, and so on. The following table gives the complete lowdown on Lotto 6-49.

Matches	Probability
0	$\frac{\binom{43}{6}\binom{6}{0}}{\binom{49}{6}} = 0.43596\,49755$
1	$\frac{\binom{43}{5}\binom{6}{1}}{\binom{49}{6}} = 0.41301\,94505$
2	$\frac{\binom{43}{4}\binom{6}{2}}{\binom{49}{6}} = 0.13237\,80290$
3	$\frac{\binom{43}{3}\binom{6}{3}}{\binom{49}{6}} = 0.01765\,04039$
4	$\frac{\binom{43}{2}\binom{6}{4}}{\binom{49}{6}} = 0.00096\,86197$
5	$\frac{\binom{43}{1}\binom{6}{5}}{\binom{49}{6}} = 0.00001\,84499$
6	$\frac{\binom{43}{0}\binom{6}{6}}{\binom{49}{6}} = 0.00000\,00715$

Adding up the first three probabilities in the table shows that there is a better than 98% chance of losing your dollar. The odds of winning ten dollars (matching three numbers) is $0.01765 \approx 1/56$, so on average you spend 56 dollars to win 10 dollars. A last bit of Lotto 6-49 trivia: If you play twice a week, every week for a thousand years, the chances are better than 99% that you never, ever win the jackpot!

An unsolved Lotto 6-49 problem.

One way to win Lotto 6-49 is to buy 13 983 816 tickets: one of each type. Of course this costs \$13 983 816 and probably isn't worth it.

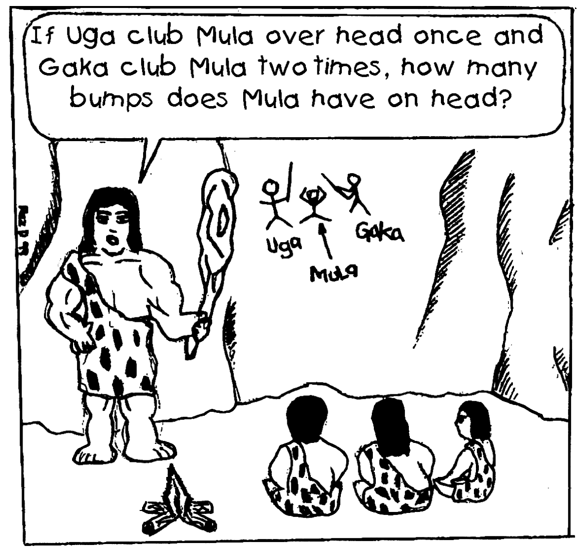
Suppose you'd settle for ten dollars. What's the smallest number of Lotto 6-49 tickets you need to buy to guarantee matching at least 3 numbers? Even with today's supercomputers and advanced mathematics, the answer is: nobody knows.

You can find out about the author at the following web site:

http://www.stat.ualberta.ca/people/schmu/dept_page.html

You can also send your comments directly to the author at `schmu@stat.ualberta.ca`

Math Jokes



Prehistoric Word Problems

Two statisticians were travelling in an airplane from LA to New York. About an hour into the flight, the pilot announced that they had lost an engine, but don't worry, there are three left. However, instead of 5 hours it would take 7 hours to get to New York. A little later, he announced that a second engine failed, and they still had two left, but it would take 10 hours to get to New York. Somewhat later, the pilot again came on the intercom and announced that a third engine had died. Never fear, he announced, because the plane could fly on a single engine. However, it would now take 18 hours to get to new York. At this point, one statistician turned to the other and said, *"Gee, I hope we don't lose that last engine, or we'll be up here forever!"* (S.A. Maas)

It is proven that the celebration of birthdays is healthy. Statistics show that those people who celebrate the most birthdays become the oldest. (S. den Hartog, Ph D. Thesis Universtity of Groningen.)

Ernst Eduard Kummer (1810-1893), a German algebraist, was rather poor at arithmetic. Whenever he had occasion to do simple arithmetic in class, he would get his students to help him. Once he had to find 7×9. *"Seven times nine,"* he began, *"Seven times nine is . . . er . . . ah . . . ah . . . seven times nine is . . . ."* *"Sixty-one,"* one student suggested. Kummer wrote

61 on the board. *"Sir,"* said another student, *" it should be sixty-nine."* *"Come on gentlemen, it can't be both,"* Kummer exclaimed, *"It must be one or the other."*

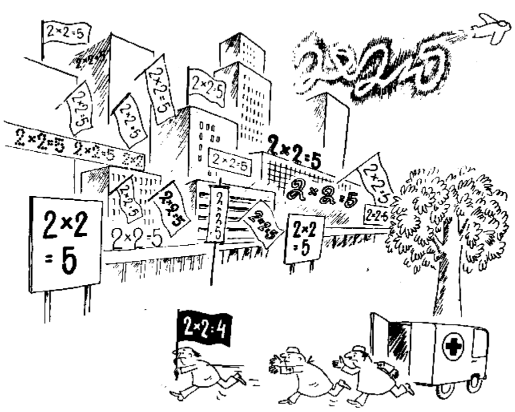
My geometry teacher was sometimes acute, and sometimes obtuse, but always, he was right.

"I do not feel obliged to believe that the same God who has endowed us with sense, reason, and intellect has intended us to forego their use." Galileo Galilei

"It is a miracle that curiosity survives formal education." Albert Einstein

A retired mathematician took up gardening, and is now growing carrots with square roots. (Zdislav V. Kovarik)

The retired mathematicians house was called aftermath. (Brian Skinner)



Einstein was feeling gloomy. A friend asked him, "*What's the matter?"* Einstein replied: *"My wife just doesn't understand me!"* (David Seppala)

"How do I love thee? Let me count the ways!" by Laurent W. Marcoux

Remember when you were young, and you fell in love for the first time? There you were, double original burger combo in hand, wiping the mustard stains from your sweetheart's chin, when the object of your affections (hereafter referred to as the OOYA) uttered the magic words:

"I kinda love you, I guess."

You immediately replied, "I love you more."

Not to be outdone, the OOYA countered with, "No, I love you twice as much."

Predictably, you responded "No, no, I love you ten times as much as that." (**Editor's note:** *Could happen!*)

Triumphantly, the OOYA slammed a fist on the table and exclaimed: "No way, I love you infinity times more than that!"

Oh oh. You started to sweat. Beads of perspiration forming on your brow were somewhat less than cool. How do you top that? What's bigger than infinity? A nervous smile betrayed your anxiety, as your teeth chattered and an uncontrollable twitch set your whole body in break dance motion. Maybe you shouldn't have said "Ok, let me get back to you on this," but who could blame you? What could you have done?

Enter mathematics laughing, stage right.

Of course, you always knew math was great as a breakfast supplement, but who would have thought that it could come to the rescue in your most desperate hour? (Ok, other than maybe Euler, Leibniz and Weierstraß.) Why is this? Well, because with the grecian formula of mathematics gently massaging the grey regions of your cerebral cortex, you *could say*: "But *which infinity* do you mean? *Countably infinite*?" Of course, you'd need to know that there was more than one infinity. And to know that, you'd need to know how to compare two different infinities. And that, you'll remember, is why you came to me in the first place.

Alright then. Let's not be overambitious in our first steps into the great big world of infinite sets. Being humble, gentle souls, let's try to decide how to compare two regular numbers, shall we? For instance, how do we know that 700 is bigger than 400? (**Editor's note:** 700 *is* bigger than 400, by the way.) A mathematician's tools are:

- their off-beat good looks
- logic

Since your own good looks are what got you into this mess, let's try to appeal to logic to get you out.

Let's suppose that you are at a dance. There are Guys, there are Girls, there's Alanis Morrissette blaring on the stereo. Not her CD, mind you, the *real* Alanis Morrisette. Just your luck. You're a Will Smith fan, and you're jiggy with that. But we digress. There are Guys, lots of them. There are Girls, lots of them. They're dancing around, and moving, and they're impossible to count, mostly because you keep forgetting if you've counted those two goofy looking Guys standing in the corner. But maybe you want to know if there are more Guys than Girls. What to do? What to do?

It's time for one of those breathtaking moments of mathematical inspiration. Here it comes. It's a good one. Wait for it! Ok, how about this? You start to **pair them up**. That's it. C'est tout. Das ist alles. Eso es todo. Who'da thunk it? You tell each Girl to choose 1 (and ONLY 1) Guy to dance with. If those two goofy Guys, or heck, if any other Guys are left debating the finer merits of Pepsi vs. Coke, there must have been more Guys than Girls. On the other hand, if there are still Girls standing about talking about how the Atlanta Falcons blew it bigtime in the Superbowl, there must have been more Girls. Somehow, this is too simple, n'est-ce pas? But it works. In fact, it works so well, we're going to milk the living Begeezuz out of it. What we're going to do next is to steal... make that BORROW this idea to help us compare two *infinite sets*.

Consider the *natural numbers* $\mathbb{N} = \{1, 2, 3, \ldots\}$.

Being very considerate, consider also the *even* natural numbers, $\mathbb{E} = \{2, 4, 6, 8, \ldots\}$. Most of us would agree that there are infinitely many elements in each set. But which is bigger? To a mathematician, the trick is to take both sets to a dance. Indeed, we are mean, lean dancing machines. You may have noticed what snappy dressers we tend to be. Then again, maybe not.

So, suppose we have infinitely many Guys, each wearing a T-shirt with a natural number on it. Snazzy, eh? No two Guys are allowed to have the same number. Suppose we have infinitely many Girls, each wearing a T-shirt with an *even* natural number on it. Sounds like a great party already, you're thinking? No two Girls are allowed to have the same number.

The Question is:

Is it *possible* to have every Guy dancing with *exactly* one Girl, and every Girl dancing with *exactly* one Guy, with no Guys or Girls sitting alone by the coke machine?

Here's one way: Suppose, just suppose that we pair them up like this:

Guy No.	1	2	3	4	5	6	7	8	9	...
Girl No.	2	4	6	8	10	12	14	16	18	...

Of course, Guy 1 and Guy 2 could switch partners. That would still work. No one asked you if there is only *one* way of pairing them up. Nope. The question was, is it *possible* to pair them up?

BUT, you say, wait a second! What if we pair them up like THIS????

Guy No.	2	4	6	8	10	12	14	16	18	...
Girl No.	2	4	6	8	10	12	14	16	18	...

Then, oh sure, all of the Girls are dancing, but there's lots of Guys left over!!

Well, do I look particularly nervous to you? No. Is there a reason? Yes. I never asked if *every* pairing would work. I only asked if it is *possible* to *find* a pairing! This isn't one of them, but we found one, in fact two, different pairings above. That's more than enough.

For a mathematician, the fact that we can pair off all of the even numbers with all of the natural numbers means that the two sets, even though they are infinite, must be the same size. But this doesn't sound too literate, so instead we impress the media types by saying that the sets $\mathbb{N}$ and $\mathbb{E}$ have the same *cardinality*. We call the cardinality of $\mathbb{N}$ (or of $\mathbb{E}$, for that matter) ALEPH NOUGHT, and we write $|\mathbb{N}| = \aleph_0$. Unless I've been lied to all of my life by my teachers and colleagues, $\aleph$ is the first letter of the Hebrew alphabet. Whenever we can pair up the elements of a set $\mathbb{X}$ with $\mathbb{N}$ like we just did with $\mathbb{E}$, we say that $\mathbb{X}$ is *denumerable* or *countable*.[2] That is because we can use $\mathbb{N}$ to help us "count" the elements of $\mathbb{X}$. Still with me?

Let's look at the *integers*, $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$. How big is $\mathbb{Z}$? Here we go again. I'll pair them up this way:

Guy No.	1	2	3	4	5	6	7	8	9	...
Girl No.	0	1	-1	2	-2	3	-3	4	-4	...

Hopefully you'll agree that each Girl has one Guy, each Guy has one Girl, and everyone is happily dancing to the Best of Billy Idol (ask your grandpa who he was). Cool. We've just seen that $\mathbb{Z}$ is countable. It has cardinality $\aleph_0$.

Oooh, ooooh, I've got one! The *rationals*, $\mathbb{Q}$!!!! $\mathbb{Q}$ is just the set of all fractions, so

$$\mathbb{Q} = \left\{ \frac{p}{q} :\ p, q \in \mathbb{Z},\ q \neq 0 \right\}.$$

[2]Technically, *countable* means either *denumerable* or *finite*.

There are *infinitely many* rationals between each pair of integers, right? For example, between 1 and 2 you have er…hold on…oh yeah, you have: $\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}$, etc.

Again: between two SINGLE integers, there are INFINITELY MANY rationals! Certainly I'm not going to tell you that $\mathbb{Q}$ and $\mathbb{N}$ are the same size! Well, to make a long story short, *yes I am*!

So, I've got to go dancing again. This time, as you might imagine, choosing the pairs is more delicate. (Like, maybe it's a slow dance and certain Girls prefer certain Guys or something. Hmm, maybe not.) Here's how I'll write out the combinations this time. I'll write them as ordered pairs: the first coordinate of my ordered pair will be the "Guy Number" (the Guys are playing the role of $\mathbb{N}$ in this scenario), and the second coordinate is the "Girl Number" (the Girls are playing the role of $\mathbb{Q}$, and doing a fine job of it, I might add). So, for example, the ordered pair $(14, \frac{3}{2})$ means that Guy 14 is dancing with Girl $\frac{3}{2}$. Keeping this in mind, here is my pairing:

$$
\begin{array}{llllll}
(1,0) & & & & & \\
(2,\frac{1}{1}) & (3,\frac{2}{1}) & (7,\frac{3}{1}) & (8,\frac{4}{1}) & (16,\frac{5}{1}) & \dots \\
(4,\frac{-1}{1}) & (6,\frac{-2}{1}) & (9,\frac{-3}{1}) & (15,\frac{-4}{1}) & (18,\frac{-5}{1}) & \dots \\
(5,\frac{1}{2}) & (10,\frac{2}{2}) & (14,\frac{3}{2}) & (19,\frac{4}{2}) & (27,\frac{5}{2}) & \dots \\
(11,\frac{-1}{2}) & (13,\frac{-2}{2}) & (20,\frac{-3}{2}) & (26,\frac{-4}{2}) & (33,\frac{-5}{2}) & \dots \\
(12,\frac{1}{3}) & (21,\frac{2}{3}) & (24,\frac{3}{3}) & (34,\frac{4}{3}) & (42,\frac{5}{3}) & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}
$$

By drawing a line from Guy 1 to Guy 2 to Guy 3 to Guy 4 and so on, you should be able to see the pattern for choosing pairs. Of course, it's harder than before, but it is still "doable" (—that's a word, isn't it?), and after all, the rationals are a pretty complicated set of numbers. So, $\mathbb{Q}$ *is* countable. Every natural number is dancing with some rational and vice-versa, and no one is left over.

Hmm. One thing you may have noticed, if you are really perspicacious (and there are treatments for this nowadays). You may have noticed that the rational number $\frac{2}{2}$ is the same as the rational number $\frac{1}{1}$, or $\frac{3}{3}$, or $\frac{4}{4}$ for that matter. In other words, Girl $\frac{1}{1}$ is dancing with Guys 2, 10, 14 and a whole lot more! That's a technicality we can get around by simply not writing down "$\frac{2}{2}$" if it already comes up before as "$\frac{1}{1}$." Similarly, we wouldn't write down "$\frac{3}{3}$," or "$\frac{-12}{-12}$." I *would have* done that, honest, but the list gets ugly to write. Of course, you might be tempted to conclude that jeez, there must be more Guys that Girls, since every Girl is dancing, and in fact, the Girls even have more than one partner, although each Guys are only dancing with a single Girl!

There are people who worry about these things. Some of them come up with ideas. Good ideas. So good, in fact, we call these ideas "Theorems." Here is a Theorem due to two Guys called **Schröder** and **Bernstein.** They should not be confused with the piano-playing kid in the Peanuts comic strip and the former orchestra conductor, although it is tempting to do so. Here is one way of interpreting their Theorem—probably a way they never thought of.

Schröder-Bernstein *Suppose you have a bunch of Guys and a bunch of Girls together in this HUGE room. We won't even pretend to say what we mean by a "bunch." Ok, suppose you are able to find* some *way of getting every Girl to dance with one, or even more than one Guy. Maybe some Guys aren't even dancing at this point. Maybe they're eating potato chips and drinking carrot juice, nectar of the gods of rabbits.*

Suppose that in the NEXT song, you find a NEW way of getting every Guy to dance with one, or even more than one Girl. This time, maybe some Girls are hitting the taco bar.

Then we, Mssrs. Schröder and Bernstein, GUARANTEE (and trust us, we're doctors) you that there is some THIRD way of pairing them up in the third dance so that each Guy has one Girl, each Girl has one Guy, and EVERYONE is dancing.[3]

This is truly marvelous. Why? Think about it. When we had the natural numbers dancing with the rationals, each Girl had (at least one) partner. By Schröder–Bernstein, we're half way there. If, in the second song, we pair up Guy 1 with Girl 1, Guy 2 with Girl 2, etc., then every Guy is dancing. That's the other half. Although these gentlemen refuse to tell us how to do it, they nevertheless GUARANTEED that there is some way of pairing them up so that EVERYONE is dancing with exactly one partner. That means that $\mathbb{Q}$ is countable. Cool.

Let's review a bit. The infinite sets $\mathbb{N}$, $\mathbb{E}$, $\mathbb{Z}$ and $\mathbb{Q}$ are all countable. They are all "the same size." This isn't helping. Your love life is at stake, the OOYA has just told you they love you infinitely many times more than you love them, and it seems as though infinity, while it can

[3]For the technically inclined, I agree that replacing injections by surjections may involve using the Axiom of Choice. Like you've never used the Axiom of Choice in your lives! Like, maybe every vector space has a basis, and you didn't use the Axiom of Choice to prove it. Sorry for the digression. In fact, I've just been informed by a Zermelo-Fraenkel-wise friend of mine (thanks, Ross) that in 1982, it was still an open question whether or not this version of the Schröder–Bernstein Theorem was equivalent to the Axiom of Choice. Anyhow, since it is common practice to use the Axiom of Choice (or one of its equivalent formulations, I don't even know why I brought it up).

wear lots of different disguises, can only be found in the "one size fits all" bin at your local department store.

Here's the advice you've been waiting for. Don't call Sue Johansen. Just tell the OOYA: "But I ℝeally love you." This is going to melt the OOYA's heart. The reason?

Remember that the *real numbers* are just the numbers we can write as infinite decimals. Yeah, there are lots of these. Every natural number can be written as a decimal, for instance $4 = 4.000\ldots$. You may already know that every rational number (i.e. every fraction) can be written as a decimal with a *repeating* term at the end: for example, $1/7 = .142857142857142857\ldots$. But there are others, like $\pi = 3.141592653\ldots$, that are still real numbers, but can't be written as a fraction no matter how hard you try. They are real numbers, but they are not rational. We call 'em, are you ready? …*irrational numbers*. (Well, what on earth would you have called them?) Let us write $\mathbb{R}$ for real numbers. Nifty notation, eh? How big is $\mathbb{R}$? Infinite? Sure, it contains $\mathbb{N}$, and $\mathbb{N}$ is infinite. Same size as $\mathbb{N}$? Let's think. Hmmm, not so obvious…not obvious that it's not, either …hmmm …er …uh …hmmmm …

Fear not. We can do this. The idea is cool. ℝeal cool. I shouldn't even be showing you something so cool at your age, but I can't help myself. It's weird though, at least the first time you see something like this. ℝeal weird. That's what makes it so cool. Watch.

We're going to see that there are so many real numbers even between 0 and 1 (and we'll let the Girls wear a T-shirt with a real number between 0 and 1 on the back) that no matter how we try to pair up the Guys (each wearing a T-shirt with a natural number on it) with the Girls, there'll always be some Girl who's not dancing. How? Here goes:

Suppose I'm wrong. That is, suppose one can find a Guy for each Girl. Let's check out the pairing (it's good to be nosy when you're in math). First, let's try an example to get a feeling for what's going to go wrong.

Guy No.	Girl No.
1	.231678…
2	.114632…
3	.048175…
4	.011364…
⋮	⋮

I claim that some Girl is not dancing. Which Girl? Here's how I'll find her.

Look at Guy 1. The first digit of his partner's number is 2. I'll pick 8.

Look at Guy 2. The second digit of his partner's number is 1. I'll pick 8.

Look at Guy 3. The third digit of his partner's number is 8. I'll pick 4.

Look at Guy 4. The fourth digit of his partner's number is 3. I'll pick 8. And so on.

Look at Guy k. If the k^{th} digit of his partner's number is 8, I'll pick 4. Otherwise, I'll always pick 8.

In this case, my choice is $.8848\ldots$.

You might say: "how do you know she's not further down the list?" Go ahead and say it. I'll answer:

She's not Guy 1's partner. Her first digit is wrong.
She's not Guy 2's partner, her second digit is wrong.
She's not Guy 3's partner, her third digit is wrong.

You might say: "Ok, I'll just add her to my list." Go ahead and say it. I'll use the same trick to find a NEW Girl who's not dancing. The point is not that there is a *single* Girl who never dances, but rather that no matter how you try to pair them up, in any given dance, at least one Girl is warming the bench.

Ok. That works great for this example, but we really want to know that it will work for *any* pairing, not just a rearrangement of this particular one. We'll start with an arbitrary list:

Guy No.	Girl No.
1	$.a_{11}\, a_{12}\, a_{13}\, a_{14}\, \ldots$
2	$.a_{21}\, a_{22}\, a_{23}\, a_{24}\, \ldots$
3	$.a_{31}\, a_{32}\, a_{33}\, a_{34}\, \ldots$
4	$.a_{41}\, a_{42}\, a_{43}\, a_{44}\, \ldots$
⋮	⋮

Here, a_{ij} refers to the j^{th} digit in the number on the back of the T-shirt for the Girl dancing with the i^{th} Guy. (I hope you got that. Think about it for a while if you have to. It's worth the effort.)

Like the great Victor von Frankenstein before me, I'll build my lonely Girl as follows:

If $a_{kk} = 8$, I'll choose a number $b_k = 4$.
If $a_{kk} \neq 8$, I'll choose a number $b_k = 8$.

The Girl who's not dancing is the Girl wearing the number $.b_1 b_2 b_3 b_4 \ldots$ on her T-shirt. The argument is the same as before. She's not Guy 1's partner. Her first digit is wrong.

She's not Guy 2's partner, her second digit is wrong.
She's not Guy 3's partner, her third digit is wrong, etc.

She's NOT DANCING!!! (Who would dance with a Girl with so many wrong digits?)

So, no matter how hard we try, we can never get all of the Girls to dance, because there are just *too many Girls*. There are more Girls than Guys. Lots more. Infinitely many more! The Guys are surrounded!

The moral of the story is (as if a story like this deserves a moral), the cardinality of $\mathbb{R}$ is *greater* than the cardinality of $\mathbb{N}$. We say that $\mathbb{R}$ is *uncountable*. What is the cardinality of $\mathbb{R}$? Who knows? It is big. We sometimes call it c. What we do know is that there are as many real numbers as there are *subsets* of the natural numbers. This is a huge set. But we don't know if there is any infinite cardinal between the size of $\mathbb{N}$ and c. That would rule. But this is still just what we needed. We have now shown that there is more than one "infinity." (In fact, there are infinitely many, but we'll leave this to another time with another OOYA.)

For the time being, when the OOYA says: "I love you infinitely many times more than that," just smile and say:

"Oh, you just love me a countable number of times more than that. My love is $\mathbb{R}$eal, c?"

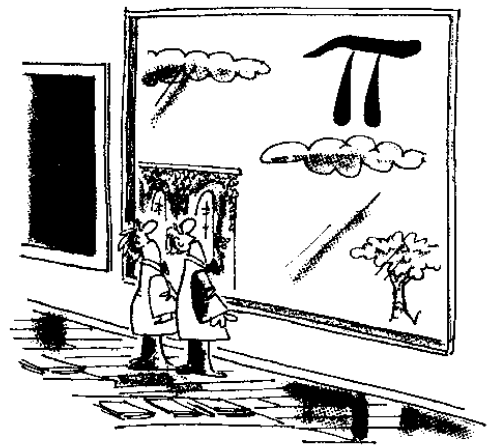
About the author. No one ever invites him to dances and he has no idea why. It's not like he talks about this stuff in public or anything. Ok, like, maybe a little. But only if someone asks him, like you did. I've got to face it. He just lives vicariously through cardinal numbers. You can send him an E-mail at: `L.Marcoux@ualberta.ca` . Check out his web page at: http://www.math.ualberta.ca/~lmarcoux/lmarcoux.html

Math Jokes

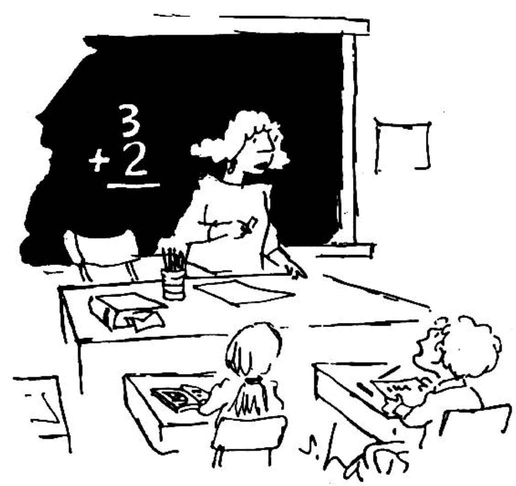
Weiner was in fact very absent minded. The following story is told about him: When they moved from Cambridge to Newton his wife, knowing that he would be absolutely useless on the move, packed him off to MIT while she directed the move. Since she was certain that he would forget that they had moved and where they had moved to, she wrote down the new address on a piece of paper, and gave it to him. Naturally, in the course of the day, an insight occurred to him. He reached in his pocket, found a piece of paper on which he furiously scribbled some notes, thought it over, decided there was a fallacy in his idea, and threw the piece of paper away. At the end of the day he went home (to the old address in Cambridge, of course). When he got there he realized that they had moved, that he had no idea where they had moved to, and that the piece of paper with the address was long gone. Fortunately inspiration struck. There was a young girl on the street and he conceived the idea of asking her where he had moved to, saying, "Excuse me, perhaps you know me. I'm Norbert Weiner and we've just moved. Would you know where we've moved to?" To which the young girl replied, "Yes daddy, mommy thought you would forget."

The capper to the story is that I asked his daughter (the girl in the story) about the truth of the story, many years later. She said that it wasn't quite true – that he never forgot who his children were! The rest of it, however, was pretty close to what actually happened....

I will never forget the day in statistics when, the Professor, who had all of the traditional looks of one (white hair, tweed jacket with leather elbow patches) was writing on the board $X_i \quad Y_j$; when one of the students asked, *"Don't you mean $X_j \quad Y_i$?"* The Prof looked at the board a bit, then erased the marks with his sleeve, and said; *"Yes, you are correct. Quite often I will say one thing, write another, and be thinking a third. What I am thinking is correct, and you will be tested on."* Every jaw in the classroom hit the floor! (Bert Tagge)



A researcher tried jalapenos on a stomach of an ulcer patient, and the ulcer went away. The researcher published an article "Jalapenos Cure Stomach Ulcers." The next patient subjected to the same treatment died. The researcher published a follow-up article "More Detailed Study Reveals That Jalapenos Cure 50% Of Stomach Ulcers." Zdislav V. Kovarik



"Do we need this even if we're not planning to go to college?"

Similarity and Dimensionality

Mark Solomonovich

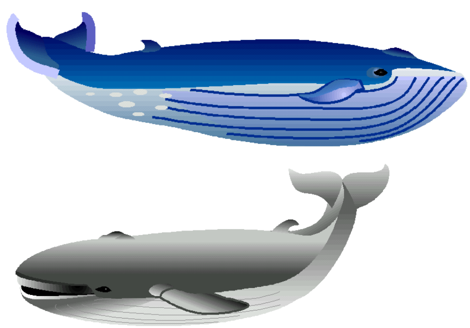
I. Real life problems

It would not surprise anyone that mathematical methods are applied in such areas as physics and electrical engineering. Quite unexpectedly, mathematical methods can sometimes provide a solution to a problem in an area that does not seem to allow a rigorous formulation.

The following problems, in spite of non-rigorous formulations, do allow rigorous solutions and qualitative substantiation based on certain mathematical ideas called Similarity and Dimensionality.

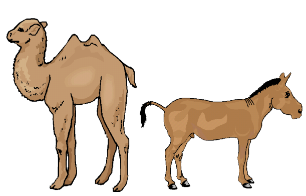
Try to solve these problems. If you cannot propose a well-substantiated solution, read the article and then try again or check out the solutions at the end of this article.

Problem 1: A blue whale and a fin whale have almost the same lengths, 30 meters and 25 meters, respectively, for the largest representatives of the species. How would you explain that a fin whale weighs just half of the weight of a blue whale?



Problem 2: If you look in a supermarket at the price per pound of small and large grapefruits of the same kind, you will see that the large ones cost more per pound. Why do you think this is the case?

Problem 3: A camel can carry approximately the same weight as a mule and move with the same speed. Which pack animal is able to travel further without water?



Problem 4: How does the height of jump of an animal depend on its size?

II. Similarity in a one-dimensional world.

The first question is: who are these creatures that inhabit the line? These are segments, finite parts of straight lines.

One can say that any two segments are similar to each other. Really, they differ only in their lengths, and their shape is the same. That corresponds to our common sense notion of similarity.

Thus, our first experience with similarity can be summarized as follows:

> *Two objects are similar if one of them can be transformed into another by means of an expansion, contraction, or reflection.*

For segments of lengths S_1 and S_2 one can find such a coefficient κ that

$$S_1 = \kappa \cdot S_2.$$

For instance:

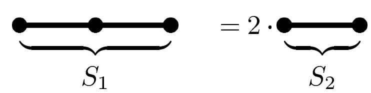


Figure 1

This coefficient, κ, is called the *magnification* or *similarity ratio*.

III. Similarity in a two-dimensional world.

There are infinitely many different kinds of objects lying in the plane.

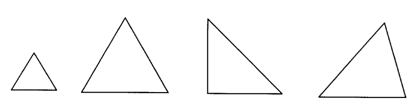


Figure 2(a)

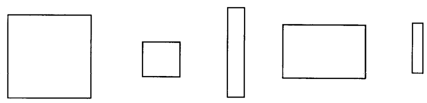


Figure 2(b)

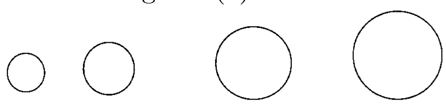


Figure 2 (c)

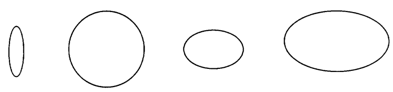


Figure 2(d)

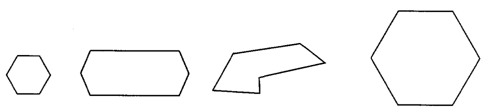


Figure 2(e)

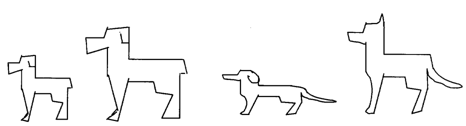


Figure 2(f)

Let us examine possible criteria for similarity based on the above examples and using our common sense.

(a) Not all triangles are similar; yet some of them are (for example two equilateral triangles).

(b) Two rectangles are seldom similar, but two squares are similar and two rectangles with the same ratio between their sides are similar.

(c) All circles look similar.

(d) Two ovals (ellipses) are not similar unless the ratios of their dimensions (length divided by width) are the same.

(e) Polygons with the same number of sides are not usually similar. Yet, regular polygons with the same number of sides are similar.

(f) Dogs are not similar to each other. Yet, two terriers, small and big, look similar.

It is clear that symmetric objects of the same kind, such as equilateral triangles, or circles, or regular hexagons, are similar to each other. What about non-symmetric objects?

Two figures of the same shape (e.g., two terriers) but of different sizes look similar. Another example is that the same face on various size photo portraits looks similar (Figure 3).

Figure 3

It would be probably a good idea to describe similar two-dimensional figures obtained from one another by a transformation similar to photo enlargement. Figure 4 illustrates in a simplified manner, how a photo enlarger works. Each elementary component of the picture grows proportionally, evenly in all directions.

What about the most elementary components of any figure? Each segment of the larger picture is obtained from the corresponding segment of the smaller picture by means of an enlargement, and the magnitude of the enlargement is the same for all segments lying in the picture.

We can formalize this "definition" to make it more rigorous:

We say the figure F is *similar* to figure F' with *ratio* κ if F can be transformed onto F' in such a way that for any pair of points P and M of figure F and their images P' and M' we have $P'M' = \kappa \cdot PM$.

Since $\kappa = \frac{P'M'}{PM}$ is the same for any choice of points P and M, where P' and M' are their corresponding images, in order to determine the ration κ for two similar figures it is sufficient to consider just two such points, which can be chosen according to a certain rule. For example, in the case of a triangle we could consider two of its vertices or the endpoints of the height, and in the case of a circle just the endpoints of its diameter. We can think about the length L of the segment PM as a *characteristic linear size* of F that can be compared with the corresponding linear size L' of an other similar figure F'.

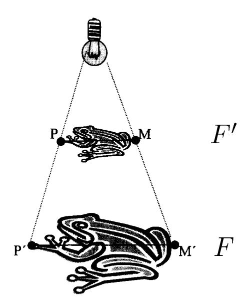


Figure 4

Note that after the enlarged photo is obtained, one can move the two pictures and the pictures will still be similar.

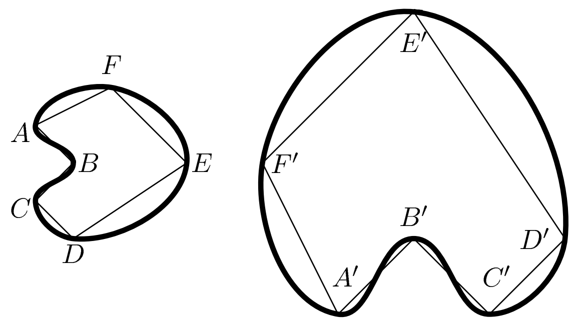


Figure 5

So, a similarity with magnification factor κ has the following properties (see Figure 5):

(1) transforms segments of length L in segments of length $\kappa \cdot L$;

(2) preserves angles;

(3) transforms triangle with sides a, b, c and angles A, B, C onto a triangle with sides $\kappa \cdot a$, $\kappa \cdot b$, $\kappa \cdot c$ and angles A, B, C;

(4) transforms a figure with area S onto a figure with area $\kappa^2 \cdot S$.

Let us prove the last property (4). We start with triangles:

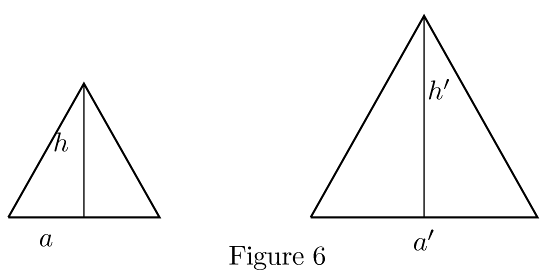


Figure 6

$$S = \frac{1}{2}ah, \qquad S' = \frac{1}{2}a'h' = \frac{1}{2}a\kappa \cdot h\kappa = \kappa^2 S.$$

For polygons: we decompose them into similar triangles:

$$\begin{aligned} S' &= S_{\Delta'_1} + S_{\Delta'_2} + \ldots + S_{\Delta'_n} \\ &= \kappa^2 S_{\Delta_1} + \kappa^2 S_{\Delta_2} + \ldots + \kappa^2 S_{\Delta_n} = \kappa^2 S. \end{aligned}$$

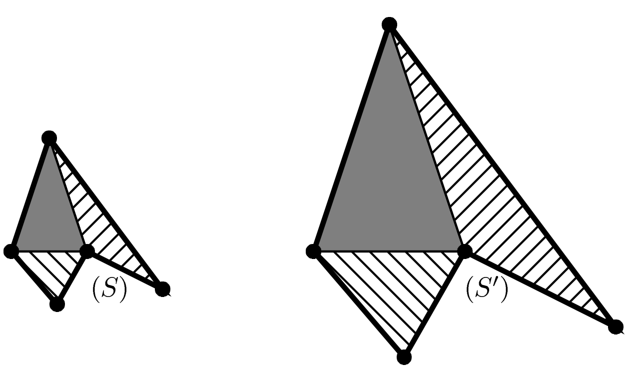


Figure 7

In simple words, Property (4) says that for similar figures:

The area of a figure is proportional to square of its linear size.

In particular, this statement means that for all similar figures the ratio S/L^2, where S is the area of a figure and L its characteristic linear size, is always the same. We can also say that for similar figures the area is growing proportionally to their linear size.

This fact can be explained more precisely. *The area of a figure is proportional to L^2, where L is a (characteristic) linear size of the figure. That is, $S' = \kappa^2 S$, where $\kappa = L'/L$. This means that*

$$S' = (L')^2 \cdot \frac{S}{L^2}.$$

As the ratio S/L^2 is the same for all similar figures, we observe that the area S' is proportional to $(L')^2$. We will write $S' \propto (L')^2$ to say that S' is proportional to $(L')^2$, i.e. the factor $S'/(L')^2$ is the same for all similar figures F'.

For example, if there are two similar rectangles, one with side of length L, the other with side of length L', then the ration of their areas S to S' is

$$\frac{S}{S'} = \left(\frac{L}{L'}\right)^2.$$

Question for discussion: All circles are similar to each other. Then, can we assert, basing on the above stated corollary, the existence of the universal[§] number π such that

$$S_{\text{circle}} = \pi R^2?$$

IV. Similarity in a three-dimensional world

One can define similar solids by means of similarity transformations.

Let us point out a property analogous to the property (4) of the previous section:

The volumes of similar figures are in the same proportion as the cubes of their corresponding linear sizes.

In other words:

$$V \propto L^3.$$

Now you are ready to attempt solving the applied problems in the beginning of the article.

Problem 1. The ratio of the linear sizes of the two whales is approximately 0.8, therefore the ratio of their volumes and, hence, their weights is $(0.8)^3 \approx 0.5$.

[§] *universal* means the same for all circles.

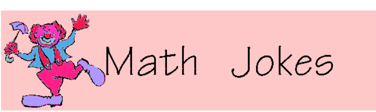
Problem 2. The mass of a grapefruit is proportional to its radius cubed, whereas the amount of waste, the volume of the skin, is proportional to its radius squared (if we assume that the thickness of the skin is constant). Therefore, the ratio of useful volume to total volume of the fruit is proportional to its radius.

Problem 3. Let L be characteristic linear size of the animal. The amount of water that the animal can store is proportional to L^3. Evaporation of water is proportional to the surface area of the animal, i.e. to L^2. Hence, the maximum time an animal keeps water is proportional to L, i.e. the bigger animal (the camel) can move longer without drinking.

Problem 4. Let L denote a characteristic linear size (e.g. height or length of a leg) of an animal. The energy E required to jump to height H is proportional to the height H and to the mass of the animal, i.e. $E \propto L^3 H$. The physical work done by the animal's muscles is equal to FL, where F, the strength of the muscles, is proportional to L^2 (the greater the area of the cross-section of a muscle, the more fibers it contains). Therefore, the physical work is proportional to $L^2 \cdot L = L^3$. Energy balance then requires that

$$L^3 H \propto L^3.$$

Thus H does not depend on L. (Observations shows that a jerboa and a kangaroo can jump to the same height.)



A somewhat advanced society has figured how to package basic knowledge in pill form.

A student, needing some learning, goes to the pharmacy and asks what kind of knowledge pills are available. The pharmacist says *"Here's a pill for English literature."* The student takes the pill and swallows it and has new knowledge about English literature!

"What else do you have?" - asks the student.

"Well, I have pills for art history, biology, and world history," replies the pharmacist.

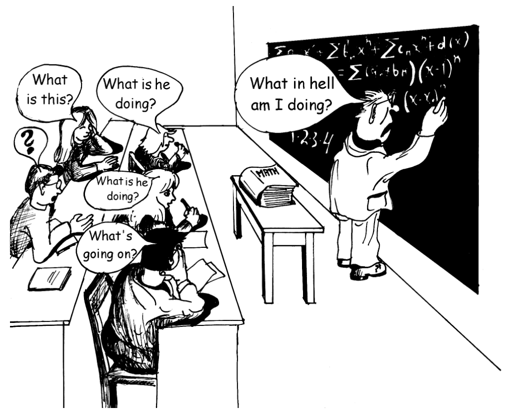
The student asks for these, and swallows them and has new knowledge about those subjects.

Then the student asks, *"Do you have a pill for math?"*

The pharmacist says *"Wait just a moment"*, and goes back into the storeroom and brings back a whopper of a pill and plunks it on the counter.

"I have to take that huge pill for math?" - inquires the student.

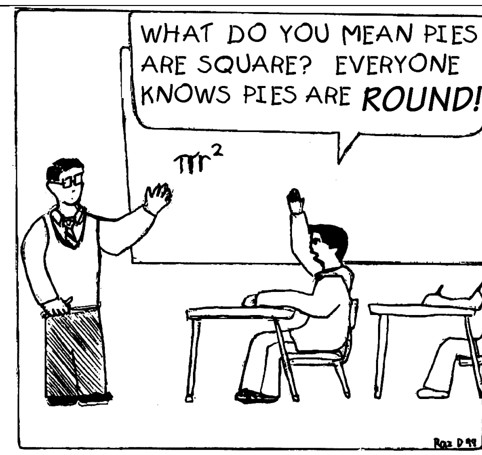
The pharmacist replied *"Well, you know math always was a little hard to swallow."*



A mathematician is showing a new proof he came up with to a large group of peers. After he's gone through most of it, one of the mathematicians says, "Wait! That's not true. I have a counter-example!"

He replies, "That's okay. I have two proofs."

Stefan Chakerian



"Science is built upon facts, as a house is built of stones; but an accumulation of facts is no more a science that a heap of stones is a house." Henri Poincare' in Science and Hypothesis

Albert Einstein, who fancied himself as a violinist, was rehearsing a Haydn string quartet. When he failed for the fourth time to get his entry in the second movement, the cellist looked up and said, "The problem with you, Albert, is that you simply can't count."

A guy decided to go to the brain transplant clinic to refreshen his supply of brains. The secretary informed him that they had three kinds of brains available at that time. Doctors' brains were going for \$20 per ounce and lawyers' brains were getting \$30 per ounce. And then there were mathematicians' brains, which were currently fetching \$1000 per ounce.

"1000 dollars an ounce!" he cried. "Why are they so expensive?"

"It takes more mathematicians to get an ounce of brains," she explained.

Math Strategies

The Box Principle

Dragos Hrimiuc

There are different versions of the *Box Principle* (or *Pigeonhole Principle*). Essentially it says:

It $n+1$ balls are distributed in n boxes then at least one box has more than one ball.

We can reformulate this principle into a slightly more general form:

If $mn+1$ balls are distributed in n boxes, then at least one box has more than m balls.

This elementary principle, used first by Dirichlet (1805-1859) in number theory, has a lot of unexpected and nice applications. It is easy to recognize if the Box Principle has to be applied when we solve a problem. The hard point is to identify the boxes and the balls.

Now, let's give some examples to illustrate how this principle can be used. First we begin with a list of simple questions without solutions:

1. A bag contains beads of five colors. What is the smallest number of beads that must be drawn from the bag without looking, to get two of the same color?

2. Show that among 37 persons, there are at least four born in one month.

3. Show that among 78 persons, there are at least twelve born on the same day of the week.

4. How many persons are needed to be sure that we have ten with the same birthday?

Let us now solve some typical questions by using the Box Principle. In this way you will be familiarized with the method and this will be useful for solving future problems.

Problem 1

Prove that in any group of five people there are two who have an identical number of friends within the group:

Solution:

Take 5 boxes labeled $0, 1, 2, 3, 4$. If a person has 0 friends put that person in box 0, if he has 1 friend put that person in box 1 and so on. Remark that the box 0 and 4 cannot be simultaneously occupied. Thus we have 5 persons and at most 4 occupied boxes. Therefore in at least one box there are two persons.

Problem 2

Given 2000 integers show that two of them can be chosen such that their difference is divisible by 1999.

Solution:

When we divide a number by 1999 then the remainder can be $0, 1, 2, \ldots, 1998$. Take 1999 boxes numbered $0, 1, \ldots, 1998$. Pick up a number from those 2000 and divide it by 1999. If the remainder is i put it in the box i. Since we have 2000 numbers and only 1999 boxes at least one box contains two numbers. Thus we get at least two numbers that provide the same remainder if we divide them by 1999. Then the difference of these two numbers is divisible by 1999.

Problem 3

Let $a_1, a_2, \ldots, a_n$ be n integers. Prove that we can choose a subset of these numbers such that their sum is divisible by n.

Solution:

We consider the following integers:

$$s_1 = a_1, \quad s_2 = a_1 + a_2, \ldots, \quad s_n = a_1 + a_2 + \ldots + a_n.$$

If any of these integers is divisible by n, then the proof is done. Otherwise, if we divide these integers by n the remainders can be $1, 2, \ldots, n-1$.

Take n boxes labeled from 1 to $n-1$. Now divide s_k by n for each $k = 1, 2, \ldots, n$, find the remainder, say r, and then put s_k in the box r. Since there are only $n-1$ such boxes and n numbers, at least two of the sums, say s_p and s_q with $p < q$, will be in the same box.

In this case $s_q - s_p = a_{p+1} + a_{p+2} + \ldots + a_q$ is divisible by n.

Problem 4

Prove that there is an integer whose decimal representation consists entirely of $1'$s and that is divisible by 1999.

Solution:

Consider the integers

$$1, 11, \ldots, \underbrace{11\ldots1}_{1999 \text{ digits}}.$$

If we divide these integers by 1999, we get the remainders $0, 1, \ldots, 1998$. If 0 occurs, the proof is finished. If not, at least two numbers have the same remainder (Box Principle) when they are divided by 1999. Therefore, their difference $111\ldots10\ldots00$ will be divisible by 1999. Canceling all zeros from the end, we get a number consisting of ones and divisible by n.

Problem 5

201 points are selected inside a square of side 1. Prove

that there exists a disk of radius $1/14$ that covers at least three points.

Solution:

If we subdivide the square into 100 small squares of side $1/10$, by the Box Principle there will be a square that contains at least 3 points. The smallest circle containing this square has radius $\sqrt{2}/20$. Since $\sqrt{2}/20 < 1/14$, the circle with radius $1/14$ centered at the center of the square will cover the entire square.

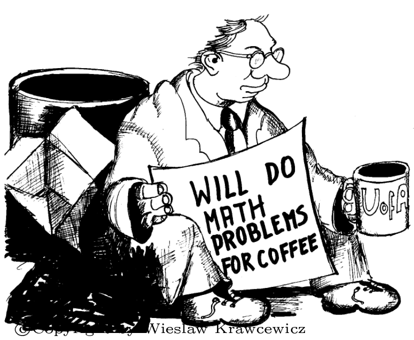
Now, try yourself to solve the following problems by using the Box Principle:

1. The Empire State Building has 102 floors. Suppose that an elevator stops 52 times as it descends from the top floor. Show that it stops at two floors whose sum is 102.

2. Twenty-eight points are selected inside a cube with edge 1. Show that there are at least two points that are separated by a distance not greater than $\sqrt{3}/3$.

Math Jokes

The guy gets on a bus and starts threatening everybody: *"I'll integrate you! I'll differentiate you!!!"* So everybody gets scared and runs away. Only one person stays. The guy comes up to him and says: *"Aren't you scared, I'll integrate you, I'll differentiate you!!!"* And the other guy says; *"No, I'm not scared, I'm e^x."*



A shoeseller meets a mathematician and complains that he does not know what size shoes to buy. *"No problem,"* says the mathematician, *"there is a simple equation for that,"* and he shows him the Gaussian normal distribution. The shoeseller stares some time at the equation and asks, *"What is that symbol?"*

"That is the Greek letter pi."

"What is pi?" "That is the ratio between the circumference and the diameter of a circle."

Upon this the shoeseller cries out: *"What does a circle have to do with shoes?!"*

Diploma Exams Around the World

Wieslaw Krawcewicz

It is interesting to examine the standards of high school math education in countries other than Canada and the U.S. See if you can solve these mathematics exam problems from other parts of the world. All the problems discussed in this section come from actual Diploma Exams that were given in some European or Asian countries in recent years.

Problem 1: Solve the following equation

$$x^3 + 4x^2 + 8x + \frac{1}{x^3} + \frac{4}{x^2} + \frac{8}{x} = 70. \tag{1}$$

Solution: Before attempting to solve this problem it is appropriate to rearrange the expression (1) in the following form

$$\left(x^3 + \frac{1}{x^3}\right) + 4\left(x^2 + \frac{1}{x^2}\right) + 8\left(x + \frac{1}{x}\right) = 70. \tag{2}$$

Notice that the terms containing the second and third powers can be written using the standard procedure of completing the square and cube. More precisely, it is possible to use the well-known formulae

$$(a+b)^2 = x^2 + 2ab + b^2$$

and

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

to rewrite Eq. (2) as

$$\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + 4\left[\left(x + \frac{1}{x}\right)^2 - 2\right] + 8\left(x + \frac{1}{x}\right) = 70,$$

so Eq. (1) is equivalent to

$$\left(x + \frac{1}{x}\right)^3 + 4\left(x + \frac{1}{x}\right)^2 + 5\left(x + \frac{1}{x}\right) = 78. \tag{3}$$

Next, we apply the substitution

$$X = x + \frac{1}{x},$$

to obtain the equation

$$X^3 + 4X^2 + 5X - 78 = 0. \tag{4}$$

Since $78 = 1{\cdot}2{\cdot}3{\cdot}13$, we look for a solution of this equation among the integers ±1, ±2, ±3 and ±13. It is not hard to find out that one root of Eq. (4) is the number $X = 3$. The polynomial $P(X) = X^3 + 4X^2 + 5X - 78$ then has the factor $X - 3$, so by dividing the polynomial $P(X)$ by $X - 3$ we obtain

$$X^3 + 4X^2 + 5X - 78 = (X - 3)(X^2 + 7X + 26).$$

Notice that the discriminant $\triangle = 7^2 - 4{\cdot}26$ of the quadratic equation $X^2 + 7X + 26 = 0$ is negative. Therefore there exists only one solution to Eq. (4), namely $X = 3$. The roots of Eq. (1) are thus given by the roots of the equation

$$x + \frac{1}{x} = 3.$$

The last equation can be written as the quadratic equation

$$x^2 - 3x + 1 = 0,$$

from which we obtain the solutions to Eq. (1):

$$x_1 = \frac{3 - \sqrt{5}}{2}, \quad x_2 = \frac{3 + \sqrt{5}}{2}.$$

Problem 2: For what values of the parameter m does the quadratic equation

$$(m + 1)x^2 - 2x + m - 1 = 0 \tag{5}$$

have two different roots belonging to the interval $(0, 2)$?

Solution: Notice that the quadratic equation

$$ax^2 + bx + c = 0$$

has two different roots if and only if its discriminant $\triangle = b^2 - 4ac$ is positive and $a \neq 0$. Consequently, Eq. (5) has two different roots if and only if $m \neq 1$ and

$$\begin{aligned}\triangle &= 2^2 - 4(m + 1)(m - 1) = 4 - 4(m^2 - 1) = 4(2 - m^2)\\ &= 4(\sqrt{2} - m)(\sqrt{2} + m) > 0.\end{aligned}$$

The last inequality is satisfied if and only if $m \in (-\sqrt{2}, \sqrt{2})$. Consequently, Eq. (5) has two different solutions if and only if $m \in (-\sqrt{2}, \sqrt{2})$ and $m \neq -1$. (If $m = -1$, Eq. (5) is just a linear equation.) The two different roots are

$$x_1 = \frac{1 - \sqrt{2 - m^2}}{m + 1} \quad \text{and} \quad x_2 = \frac{1 + \sqrt{2 - m^2}}{m + 1}.$$

It is clear that the value $1 + \sqrt{2 - m^2}$ always positive, so x_2 will be positive if $m + 1$ is also positive, i.e. if $m > -1$. On the other hand, for x_1 to be positive, it is necessary that $1 - \sqrt{2 - m^2} > 0$. That means the following inequality must be satisfied

$$2 - m^2 < 1,$$

so $m < -1$ or $m > 1$. That means, that Eq. (5) will have two different positive roots x_1 and x_2 if and only if $m \in (1, \sqrt{2})$. Since $x_1 < x_2$ it is sufficient to solve the inequality

$$x_2 = \frac{1 + \sqrt{2 - m^2}}{m + 1} < 2. \tag{6}$$

The inequality (6) is equivalent to

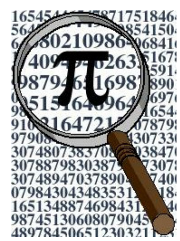
$$\sqrt{2 - m^2} < 2m + 1.$$

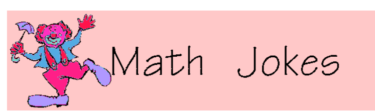
By taking square of the last inequality we obtain

$$5m^2 + 4m - 1 = (5m - 1)(m + 1) > 0,$$

which implies that $m > 1/5$ or $m < -1$. Therefore for $m \in (1, \sqrt{2})$ Eq. (5) will have two different roots belonging to the interval $(0, 2)$.

Notice that both of the above problems involve solving quadratic equations. This will be the topic of a future article in an upcoming issue of the π *in the Sky*.





"But we just don't have the technology to carry it out."

There was this statistics student who, when driving his car, would always accelerate hard before coming to an intersection, whiz straight over it, then slow down again once he was beyond it. One day, he took along a passenger, who was understandably unnerved by his driving style and asked him why he went so fast over intersections. The statistics student replied, *"Well, statistically speaking, you are far more likely to have an accident at an intersection, so I just make sure that I spend less time there."*

There are three kinds of people in the world: those who can count and those who can't. (Richard Harter)

In modern mathematics, algebra has become so important that numbers will soon only have symbolic meaning. (Peter Bengtsson)

If God is perfect, why did He create discontinuous functions?

And God said "Let there be numbers", and there were numbers. Odd and even created he them, and he said unto them be fruitful and multiply; and he commanded them to keep the laws of induction. (Bill Taylor)

Math Teacher: Now suppose the number of sheep is x....

Student: Yes sir, but what happens if the number of sheep is not x? (Dave McQuillan)

Theorem: \$1 = 1c.
Proof: To give you a sense of money disappearing...

$$\begin{aligned} \$1 &= 100\text{c} \\ &= (10\text{c})^2 \\ &= (\$0.1)^2 \\ &= \$0.01 \\ &= 1\text{c}. \end{aligned}$$

(Benjamin J. Tilly)

Theorem: \$1= 10 cents.
Proof:
We know that

$$\$1 = 100 \text{ cents}.$$

Divide both sides by 100

$$\$\frac{1}{100} = \frac{100}{100} \text{ cents}.$$

Thus

$$\$\frac{1}{100} = 1 \text{ cent}.$$

Take the square root of both sides

$$\sqrt{\$\frac{1}{100}} = \sqrt{1} \text{ cent}.$$

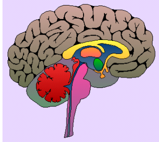
Therefore, we get

$$\$\frac{1}{10} = 1 \text{ cent}.$$

Multiply both sides by 10

$$\$1 = 10 \text{ cents}.$$

(Brijesh)



Math Challenges

We have selected several problems of various types and would like to invite everybody to try to solve them. Some of these problems are quite easy, others are more difficult. Please send your solutions to:

π *in the Sky*– Math Challenges
Pacific Institute for the Mathematical Sciences
501 Central Academic Building
University of Alberta
Edmonton, Alberta
T6G 2G1

We will publish the most interesting solutions of these problems and for some selected problems there will be book prizes.

Problem 1: Solve the equation

$$\sqrt{16x+1} - 2\left(\sqrt[4]{16x+1}\right) = 3.$$

Problem 2: A man has 15878 equilateral triangular pieces of mosaic, all of side length one cm. He constructs the largest possible mosaic in the shape of an equilateral triangle.

(a) What is the side length of the mosaic?

(b) How many pieces will he have left over?

Problem 3: From 12 cm $\times$ 18 cm sheet of tin, we wish to make a box by cutting a square from each corner and turning up the sides. Determine the size of the square that yields the largest box.

Problem 4: From a point P on the circumference of a circle, a distance PT of 10 meters is laid out along the tangent. The shortest distance from T to the circle is 5 meters. A straight line is drawn through T cutting the circle at X and Y. The length of TX is 15/2 meters.

(a) Determine the radius of the circle,

(b) Determine the length of XY.

Problem 5: Some playing cards from an ordinary deck are arranged in a row. To the right of some King is at least one Queen. To the left of some Queen is at least one other Queen. To the left of some Heart is at least one Spade. To the right of some Spade is at least one other Spade. Find the minimum number of cards in this row.

Problem 6: Suppose that your height this year is 10% more than it was last year, and last year your height was 20% more than it was the year before. By what percentage has your height increased during he last two years?

Problem 7: A gambler starts with \$1. The probability that he will win \$1 on each bet is 1/2 and the probability that he loses \$1 is 1/2.

(a) If the gambler can draw on an infinite credit, what is the probability $P_n(x)$ that he will have x dollars after n bets?

(b) Suppose now that the bank won't give the gambler any credit, so that he must stop playing when his balance reaches zero. What is the probability that he is able to continue playing after n bets?

Problem 8: A 10×10 chessboard is divided into eighteen rectangles along the grid lines. The area of each rectangle is written in one of the squares inside it. The diagram below shows the areas but not the rectangles, with 0 standing for 10. Reconstruct the rectangles.

					0				
6									
			4				6		6
	2	4						2	
						0			
					2				
		8		4				9	
						4			
			6						6
5				6					

Problem 9: Each square of 9×9 chessboard is painted in one of six colours: R=red, O=orange, Y=yellow, G=green, B=blue and V=violet. All squares of the same colour are connected edge to edge. The diagram below shows the colours of some of the squares. Reconstruct the colours of the other squares.

								R
	O							O
						R		Y
								G
				B				
		G						B
								V
	V							Y

Problem 10: An 8×8 chessboard is the map of twenty-one islands on a lake. Each island is the size of one square. There are bridges connecting pairs of islands. All bridges are parallel to the grid line, and no two bridges intersect. There may be multiple bridges between the same pair of islands. The number of bridges to each island is recorded on the square representing it. The diagram below shows these numbers but not the bridges. Reconstruct the bridges.

	4			5			4
2			2			4	
		3			2		
	1		3			5	
4		2		1			
	2		4				
				3		3	
2			3				3

Our Forum

A number of my grade 9 students and a few grade 8's looked at the magazine. The higher achieving students definitely were more interested in it. They liked the puzzles especially and thought that more puzzles and math games should be included. Few were impressed with the articles - perhaps a little beyond them? I thought they were great. A suggestion for future articles - feature someone whose career involves the use of math, and have them explain how the ability to do and understand math has benefitted them in their career. If they could draw parallels to what they are doing and what is being taught in our curriculum, that would be a bonus.

Wendy Richards

In future issues of π in the Sky we plan to publish more material related to the present high school curriculum,including problem solving techniques. There will be also some articles about careers for students specializing in mathematics.

— *Editors*

AHSMC

Alberta High School Mathematics Competition

ANNOUNCEMENT

High School Students in Alberta and the North West Territories are invited to try the Alberta High School Mathematics Competition.

This is a two part in school competition that takes place on the following dates:

Part I: Thursday, November 21, 2000
Part II: Wednesday, February 7, 2001.

Substantial prizes are awarded. For more details visit:
http://www.math.ualberta.ca/~ahsmc
or send e-mail to tlewis@math.ualberta.ca.

Math Links

Encyclopedia of Mathematics

We highly recommend the **Concise Eric Weisstein's Encyclopedia of Mathematics** that can be found at the address
http://mathworld.wolfram.com/
It is a wonderful resource of information, including interactive 3D graphics.

Torus Games

We have found an extraordinary collection of games (Tic-Tac-Toe, Maze, Crossword, Word search, Jigsaw, Chess) on the torus and the Klein bottle. You can also find an explanation of what exactly is a torus and the Klein's bottle. These games are really cool, so you must try them!
http://www.northnet.org/weeks/TorusGames/TorusGames.html

Images of Famous Mathematical Works

This is a collection of images of title pages or pages from some famous mathematical works. Go to the address
http://www.ma.iup.edu/~gstoudt/history/images/images.html

History of Mathematics

The web archive devoted to the History of Mathematics at the School of Mathematics, Trinity College, Dublin. Check out this address
http://www.maths.tcd.ie/pub/HistMath/

Mathematics Archives

This web site, containing a MSDOS Software Collection for K–12 (Freeware, Shareware, Commercial Software and Demo of Commercial Software), is located at
http://archives.math.utk.edu/software/msdos/k-12/.html

Earliest Use of Mathematical Terms

These pages attempt to show the first uses of various words used in mathematics. This web site located at
http://member.aol.com/jeff570/mathword.html is maintained by Jeff Miller, a teacher at Gulf High School in New Port Richey, Florida.

Mathematically Correct

This web site is devoted to the concerns raised by parents and scientists about the invasion of US schools by the New-New Math and the need to restore basic skills to math education. It may be of great interest for both teacher and students. Look at
http://ourworld.compuserve.com/homepages/mathman/

CyberMath

Play with various geometrical figures at
http://www.maplesoft.com/cybermath/samples.html

Alberta High School Mathematics Competition

A two-part math competition takes place in November and February of each school year, with book prizes for the first part and cash prizes and scholarships for the second part. Find more about this competition at
http://www.math.ualberta.ca/~ahsmc/

New Chronology of the World History

The accepted traditional chronology of ancient and medieval world is incorrect. The new mathematical and statistical methods help to determine the real sequence of events in the ancient and medieval history. To learn more about this new sensational research checkout
http://univ2.omsk.su/foreign/fom/fom.htm