MR1919840 (2003i:53130) 53D50 (17B37 81S10)
Gotay, Mark J. (1-HI)
On quantizing semisimple basic algebras. I. $\mathrm{sl}(2,\mathbf{R})$**. (English summary)**
Geometry, mechanics, and dynamics, 523–536, *Springer, New York*, 2002.

In the paper under review the author continues his study of Groenewold-Van Hove obstructions to quantization. Let M be a symplectic manifold and $\mathfrak{b}$ a finite-dimensional "basic algebra" of observables on M. Given a Lie subalgebra $\mathcal{O}$ of the Poisson algebra $C^\infty(M)$ containing $\mathfrak{b}$, the problem is to determine whether $\mathcal{O}$ can be "quantized". In this paper the problem of quantizing $(P(M),\mathfrak{b})$ when the basic algebra is semisimple is considered. Here $P(M)$ is the Poisson algebra of polynomials on M generated by $\mathfrak{b}$. In particular, the following theorem is proved: Let $\mathfrak{b}$ be a finite-dimensional semisimple Lie algebra, and M a basic nilpotent coadjoint orbit in $\mathfrak{b}^*$. Then there exists a polynomial quantization of $(P(M),\mathfrak{b})$. Furthermore, it is shown that any polynomial quantization of a nilpotent orbit in $\mathrm{sl}(2,\mathbf{R})^*$ must be essentially trivial. Thus, while polynomial quantizations of basic nilpotent orbits do exist, this example indicates that they are likely to be uninteresting. Finally, in the last section it is proved that polynomial quantizations are forced to be trivial for nilpotent orbits in $\mathrm{sl}(2,\mathbf{R})^*$, and are genuinely obstructed for all other basic orbits.

{For the entire collection see MR1919824 (2003c:00027)}

Reviewed by *Giovanni Giachetta*