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On a full quantization of the torus. (English. English summary)

Quantization, coherent states, and complex structures (Białowieża, 1994), 55–62, Plenum, New York, 1995.

The author shows that the "no-go" theorem of H. J. Groenewold [Physica 12 (1946), 405–460; MR 8, 301a] and L. Van Hove [Acad. Roy. Belgique. Bull. Cl. Sci. (5) 37 (1951), 610–620; MR 13, 519a; Acad. Roy. Belgique. Cl. Sci. Mem. Coll. in 8° 26 (1951), no. 6, 102 pp.; MR 15, 198d (pp. 61–102)] about the impossibility of a full quantization of \mathbf{R}^{2n} need not hold for all symplectic manifolds. More precisely he shows that there is a full quantization of the Poisson algebra on the torus T^2 . A full quantization of a symplectic manifold M here consists of a prequantization map Ω from $\mathfrak{C}^{\infty}(M)$ to the space of (essentially) self-adjoint operators on a Hilbert space \mathcal{H} and a complete set $\mathcal{F} \subset \mathfrak{C}^{\infty}(M)$ of observables such that the corresponding operators $\{\Omega(f): f \in \mathcal{F}\}$ form an irreducible set of operators.

First the author constructs for every $N \in \mathbf{Z}$ a prequantization \mathcal{Q}_N of the Poisson algebra of the torus T^2 on the (prequantum) line bundle L_N with Chern class N. By showing that the prequantization \mathcal{Q}_1 is unitarily equivalent to the Schrodinger quantization of the Heisenberg algebra the author succeeds in proving that \mathcal{Q}_1 gives rise to a full quantization of the torus T^2 .

At the end of the article some remarks about the interpretation of the proven result are made and further questions are posed.

{For the entire collection see MR 97c:81004}.

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