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The Equation of Time.

For civil purposes, we pretend that the sun runs around the celestial equator at a constant angular speed — something like $Q(t) = (\cos t, \sin t)$. This gives us *mean time*, which our cuckoo clocks and quartz watches can easily handle. Folks who go by sundials are dealing with *solar time*. They have to keep in mind that the sun actually does an ellipse on a plane inclined at an angle $\varepsilon \approx 23.5°$ to the xy-plane suggested above. The difference between mean and solar time is called the *Equation of Time*, even though it is not an equation but a difference (go figure).

To get an idea of how it arises, let us first consider a second point $P(t) = M\,P(t)$ in xyz-space, where

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{bmatrix},$$

in other words, doing uniform circular motion at the same angular speed as $Q(t)$, but on a slanted plane. In the (equatorial) xy-plane, this projects to a point $R(t) = (\cos t, \cos\varepsilon \sin t)$ whose angle $\alpha(t)$ bears the distinguished name of *right ascension*. The famous Equation of Time is simply the angular difference between $Q(t)$ and $R(t)$, namely $t - \alpha(t)$.

Peering out at $Q(t)$ and $R(t)$, it is obvious that they agree whenever t is an integer multiple of $\pi/2$. This yields the famous *analemma* (a kind of lemniscate) used to correct sundials for mean time. The easiest way to compute $\alpha(t)$ is by

$$\tan\alpha(t) = \cos\varepsilon\ \tan t\,.$$

Of course, in real life, $P(t)$ will have to describe an ellipse instead of a circle, and the analemma will no longer be symmetric with respect to the y-axis.