The case Fermat did for sure: n=4.

Recall the parametrization of Pythagorean Triples: if x , y, and z are relatively prime integers, then (after maybe switching x and y)

$$x^2 + y^2 = z^2 \iff \quad x = a^2 - b^2, \quad y = 2ab, \qquad z = a^2 + b^2 \tag{1}$$

for suitable integers a and b, which are again relatively prime. Note that x and z are odd while y is even. We shall prove:

If x, y, z is a Pythagorean Triple, no number of the form $2^\mu xy$ can be a square.

Clearly we may take x, y relatively prime. Then $2^\mu xy$ not being a square is equivalent to x and $2^\mu y$ not being squares. Of course, μ counts only modulo 2. From $\mu = 0$ we get FLT4: in a Pythorean Triple, x and y cannot both be squares. From $\mu = -1$, we see that there is no right triangle with integer sides and a square area.

Proof. If $x = w^2$ is itself a square, we can rewrite $x = a^2 - b^2$ as $w^2 + b^2 = a^2$, and repeat the same magic — with w odd and hence b even — to get

$$a = u^2 + v^2 , \quad \text{and} \quad b = 2uv . \tag{2}$$

If now $2^\mu y = 2^{\mu+1}ab$ were also a square, there would be integers c and d such that

$$a = c^2 \quad \text{and} \quad 2^{\mu+1}b = d^2 . \tag{3}$$

Now, $d^2 = 2^{\mu+2}uv$ shows that $2^\mu uv$ is a square as well, and we have

$$c^2 = u^2 + v^2$$

with c smaller than z. By induction (here called “descent”), the proof is finished.

Another interpretation of the case $\mu = -1$ is that the elliptic curve given by $\eta^2 = \xi^3 - \xi$ has no rational points. Indeed, with fractions in lowest terms, a rational square equal to $a^3/b^3 - a/b = (a^3 - ab^2)/b^3$ is necessarily of the form n^2/m^6 with $a^3 - ab^2 = n^2$ and $b = m^2$. Therefore $(a^2 - b^2) \cdot ab = n^2m^2$, in other words: $xy/2$ is a square.