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Menelaos – Vot Iss?

I have yet to discover what this theorem is good for, but here goes:

Let the (extended) sides of a triangle ABC be cut by a line in points P, Q, R. Then

$$\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RA} = 1\,.$$

Without loss of generality, assume $A = (0,0)$, $B = (1,0)$, $C = (0,1)$. Moreover, let $P = (a,0)$, $Q = (u,v)$, $R = (0,b)$. Then we must show:

$$\frac{a}{a-1} \cdot \frac{u}{v} \cdot \frac{1-b}{b} = 1\,.$$

Since the line BC is given by $x + y = 1$, we have $u + v = 1$. Rescaling the lines AB and AC, so as to make AP and AR into unit vectors, we see that $u/a + v/b = 1$ as well. Solving these equations, we get $u = a(1-b)/(a-b)$ and $v = b(1-a)/(b-a)$, whence the result.