The Chancellor of the Exchequer.

To understand why the British call their Finance Minister the “Chancellor of the Exchequer”, one has to keep in mind that, from very early times (before 500 BC) until quite recently, large financial computations were done on tables marked with a kind of checker-board pattern. To calculate on such an “exchequer”, tokens were moved around like beads on an abacus, but with more freedom and effiency. In fact, the abacus was a kind of laptop version of these computers. Of course, clerks found all sorts of games to play on the desk-top model when the boss was not looking.

One these — the game of chess — was so fascinating and time-consuming that Persia’s Chancellor of the Exchequer got wind of it. Instead of risking a scandal by sacking his entire staff, he showed the game to the Shah — presenting it as his own invention. His Majesty was delighted and asked the chancellor to name his reward. According to legend, the wily bureaucrat asked for “just some grains of wheat” (for details cf. Exercise 3) — but his request was designed to humiliate the Minister of Agriculture, whose daughter was receiving more favours at Court than his own.

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1. Everyone knows that a million is a thousand thousand, but there is disagreement about a billion: on this side of the Atlantic it means a thousand million $(1,000,000,000)$, but in Europe it means a million million $(1,000,000,000,000)$. Let us stay on this side and ask: how far is a billion millimetres, how big is a billion millilitres, how long is a billion seconds? Try to find striking ways of visualizing the Canadian federal debt of about 600 billion dollars.
2. John and Mary start pestering their parents for a Boxing Day Bonus on October 3, exactly 12 weeks before the event. Ann and Bill decide to give their kids an extra weekly allowance (payable every Friday): John starts with one dollar (on October 10) and gets a raise of $1 every week; Mary starts with one penny and has her allowance doubled every week. How much do they have, respectively, on Boxing Day?
3. The wizard of the exchequer (cf. above) asked for 1 grain of wheat on the first square, 2 on the second, 4 on the third, 8 on the fourth, 16 on the fifth — and so on through all 64 squares. Reasoning as in Execise 2, you find that he would get very nearly 2^{64} grains. Is that a lot? Noting that $2^{10} = 1024$ is just over a thousand (this is the famous “K” of computers), estimate $2^{20} = 2^{10} \times 2^{10}$ (the famous “Meg”) and $2^{20} = 2^{10} \times 2^{20}$ (the famous “Giga”). Roughly how many billion is he asking for? Try to visualize the amount by the strategies of Exercise 1.
4. To correct the approximation of 2^{64} obtained in the last exercise, you would have to multiply it by 1.024 six times (see?) — i.e., apply compound interest at 2.4% for 6 periods. To improve your grain estimate, pretend that this was simple interest. What do you get?
5. The last digit given by a calculator is often uncertain. My ten-digit scientific calulator, for instance, says that 2^{32} is an odd number, namely $4,294,967,295$. What is the correct last digit? What are the three last digits of 2^{64} ?
6. Writing $2^{32} = a \times 10^5 + b$ with a and b less than 10^5, compute the exact value of 2^{64} using your pocket calulator. (You will have to add the various pieces by hand). If your display has only 8 digits, you can still do this by means of a finer break-up. Try to find the most efficient one in that case.
7. Continue the factorization $2^{64} - 1 = (2^{32} + 1)(2^{32} - 1) = (2^{32} + 1)(2^{16} + 1)(2^{16} - 1) = \cdots$ as far as it will go. Fermat (ca. 1650) thought that all the factors were primes. What is *your* guess? Euler (ca. 1750) found $2^{32} + 1 = \big(2^{28} - 639\,(640^2 + 1)\big) \times 641$. He probably noticed the usefulness of $641 = 2^4 + 5^4$ and $640 = 2^7 \times 5$. How did he go from there?