Nov 8, 1999.

MULTIPLYING MAGIC SQUARES

On a problem by Jackson Ng.

Every 3-by-3 magic square is of the form $aM + bN + cE$, where

$$M = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \qquad N = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Every magic square is determined by the three parameters a, b, c, which are easily reconstructed if a row or column of the square is given.

The matrix multiplication of magic squares is governed by the obvious rules $E^2 = 3E$ and $EM = ME = EN = NE = 0$, together with easily verifiable

$$M^2 = -N^2 = 3I - E\,, \quad \text{and} \quad MN = -NM = 3J - E\,,$$

where J is the matrix with 1's on the "anti-diagonal" and 0's elsewhere. In particular, any product of two magic squares is a linear combination of I, J, E — which is never magic (unless it is constant).