Temper Versus Purity in Money and Music.

To celebrate the Year of the Piggy Bank, some financial institutions in Diaton allow children to double their money in just one year — by offering 5.95% interest per month on accounts up to 25 dollars.

1. How does this double their money? Explain.

Since the beginning of the year, Jack has 24 dollars deposited with the Tempered Trust Co. He goes to check the state of his account on the first days of March, May, June, August, October, and December.

2. Make a list of the numbers (dollars and cents) he obtains, starting with 24.00 and ending with 48.00.

His sister Jill has a similar account with the Caisse Populaire d'Orémy, which she checks on the same days as her brother. However, *her* statements are always rounded off to the nearest dollar.

3. List Jill's numbers $(24, 27, \ldots, 48)$.

Multiplied by eleven, Jack's numbers represent the pitches (in cycles per second) of the notes C D E F G A B C' found on a piano, while Jill's correspond to the "same" notes sung or played on a violin. Jack's is the so-called tempered scale, invented in the seventeenth century. It has only two sizes of steps: tone or semi-tone. The latter multiplies the pitch by $\alpha = 1.05946\ldots$, the former by α^2.

4. List the ratios of all pitches with respect to that of C.

5. Do the same for Jill's scale.

6. List the ratios between pitches of successive notes in her scale.

You should find that there are three sizes of steps: a long and a short tone, as well as a semi-tone. How do these compare to Jack's? Jill's is sometimes called the "Pythagorean" scale. Guess why.

Suppose you tune one of the strings of a guitar to C. If it is 660 mm long, the fret for c is at 330 mm, the one for G at 440 mm. In other words, the pitch is *inversely* proportional to the length.

7. Where are the frets for D, E, F, etc. according to to Jack?

8. Where should they be according to Jill?

Note: The length of a guitar string "decays" by 5.6% *from one fret to the next. In twelve steps it is down to one half — check this.*