**Fractional Powers.**

1. In a nutrient solution, a bacterial culture grows merrily (i.e., exponentially) from 2 to 20 grams in 16 hours. What was its mass after (a) 8 hours, (b) 4 hours, (c) 12 hours ?

2. (a) Make a table of the biomass described above for every hour from 1 to 16. (b) Graph this table in a coordinate system on millimeterpaper (1 cm horizontally = 1 hour, 1 cm vertically = 1 gram) and connect the 16 points by a smooth curve. (c) By rescaling the coordinates interpret this curve as an approximate graph of the function $y = 10^x$ $(0 \le x \le 1)$.?

3. My means of this reinterpreted graph, find numbers $u$ and $v$ such that $10^u \approx 3$ and $10^v \approx 5$. Using $u$ and $v$ compute the exponents $x$ for which $10^x$ is approximately equal to 15, 27, 2, 4, and compare these values with the $x$-coordinates of the appropriate points on the curve (for instance, at $y = 1.5 = 15/10$ and $y = 2 = 10/5$).

*These rough approximations are good enough for certain practical purposes illustrated in the following two questions. For the first one, it would have been better to work with fracional powers of 2, but we shall stick with 10 for the sake of uniformity. Conversion is easy:* $2 = 10^w$ *implies* $2^x = 10^{wx}$ *and vice versa.*

4. Living matter contains proportionally as much radioactive carbon $C^{14}$ as its environment. This isotope decays with a half-life of about 5700 years, and thus diminishes in dead bones, wood, etc.
   (a) About how old is a bone which has only 20% of the normal $C^{14}$-content ?
   (b) Trees crushed by the advancing glaciers of the Fourth Ice Age contain only 27% of normal amount of $C^{14}$. Approximately when did the Ice Age begin ?

5. Salt dissolves in water more or less exponentially. If 10 pounds of salt is placed in a tub of water, and 4 pounds dissolves in 20 minutes, how long will it take 2 more pounds to dissolve ?

*More delicate calculations require more accurate values for powers and exponents (logarithms). We shall now see how these can be obtained in elementary fashion.*

6. For every number $0 < a < 10$ the following BASIC program prints out a sequence $b(a)$ of zeroes and ones. What do these represent ? (Check this first for $a = 10^{1/2}, 10^{1/4}, 10^{3/4}$ ).

```
1 m=0:  z=10:  input a
2 m=m+1:  z=sqr(z):  b=0
3 if a<z goto 5
4 a=a/z:  b=1
5 print b;:  goto 2
```

(or: `if m < 30 goto 2` in order to stop after 30 places).

7. This program can also be worked with a calculator. Store the number $a$ in its memory, and successively reduce it by division as in line 4. If a second memory is available, it can be used to store the value $z$; otherwise $z$ can be recreated in line 2 by starting with $z = 10$ and hitting the square root button $m$ times.
   (a) In this manner find the missing 6 places for each of the sequences

$$b(3) = ??????1000100100100111100\ldots \quad \text{and} \quad b(5) = ??????1011101111101100101\ldots$$

   (b) Convert $b(3)$ and $b(5)$ into more accurate values for the numbers $u$ and $v$ in Exercise 3. For convenience, you may wish to change them first into hexadecimal notation.

8. Experts estimate that this planet can feed at most 40 billion people. If the human population grows at 2.4% per year and counted five billion in 1980, when will it reach the maximum ?

9. Carlos gets 1.25% interest per quarter in his term deposit. How long does it take him to triple his money? (Hint: $10125 = 81 \times 125$).

10. Explore *linear interpolation* as a way of approximating intermediate values of powers or exponents.