MAED 320, March 1, 1995.

NUMBER SEQUENCES.

1. Repeated multiplication. Play with the sequence $1, 10, 100, \ldots, 10^n, \ldots$ just to learn the common names of its members. Now try 2^n as far as you can. How does this compare with 4^n? Note: $4^5 = 2^{10} = 1024$ is just over 1000. If Alice races along the sequence 2^n taking 10 steps at a time, and Bob skips along 10^n taking 3 steps at a time, who wins?

For a really fast sequence, look at $1, 1\times 2, 1\times 2\times 3, 1\times 2\times 3\times 4, \ldots, n!, \ldots$. Try the first 20 terms of this one. How does it stack up to the others?

Game. Students hand IOU's to one another: 1 cent, 2 cents, and up, doubling every time. The last one goes to the teacher. How would you organize this, so as to keep the whole class interested all along?

Experiment. Teacher brings a jar of uncooked rice. Gives out 1 grain, 2 grains, etc. to the students in turn. At one point, you will have to use larger units (teaspoons, cups, etc.) How long will the jar last?

2. Repeated division. To wean itself of its sweet tooth, the cookie monster puts a glass of sugar water by its bedside. Every night it drinks exactly half the glass and fills it up again with pure water. How much of the original sugar is left after a week, after a month? Can this go on forever? What if the glass initially contained a trillion trillion sugar molecules?

Experiments. Start with a graduated beaker filled with a strong food colouring solution. In turn, each student pours out half, tops it up again with water, and then hands it to her neighbour. Do you ever get clear water?

Hot water (say, $100°$) placed in an environment of constant temperature (say, $20°$) loses half its temperature difference (here, $80°$ initially) every so many minutes. Check this out. Does it ever reach the ambient temperature?

Carbon dating. The biosphere contains lots of carbon, a certain percentage of which is the unstable "heavy carbon". As a piece of wood (or bone) is taken out of the circulation of life, this heavy carbon begins to deplete, losing half its substance every 5700 years. How many years does it take before there is only one eighth of the original amount?

3. Growth and decay. A mass of bacteria in nutrient solution grows 9-fold every 24 hours. How much does it grow in 12 hours? (Try some guesses). Now repeat the question for another batch that grows 10-fold every 24 hours. Apply this to the Richter Scale used to rate earthquakes: if a quake rated 7 is ten times stronger than one rated 6, how much stronger than the latter is a tremor rated 6.5?

At an interest rate of 7.2% (i.e. multiplied by 1.072 each year), your money doubles every 10 years. Use your calculator to check this. What happens after 5 years, 15 years? If your account has \$10 000 today, how much was in it 5 years ago, 1 year ago? Back to carbon dating: what fraction of the heavy carbon is gone after 28850 years, after 570 years?

4. Achilles and periodic decimals. Achilles runs 400 meters a minute, ten times as fast as Alfred (a toddler), who has a head-start of 400 meters. After Achilles gets to where Alfred started, the latter has advanced 40 meters, and so on. To catch up, Achilles has to run $1 + 1/10 + 1/100 + 1/1000 + \cdots$ minutes. Will he ever do it? What if Alfred's head-start was 1200 meters?

The next day, Achilles races a Betsy (a tortoise), who runs only 4 meters a minute. How long will Achilles need to catch up if Betsy had a head-start of 800 meters? (Write the time in decimal notation). What if the head-start was 20,000 meters?

At 12 noon, the minute hand is right on top of the hour hand. When exactly does this happen again?