Ten geese and ten hunters.

Let $n > 1$ be an integer, and consider n distinct symbols $s_1, \dots, s_n$, which will be referred to as *letters*. By a *word*, we mean any string of n letters, repeats allowed. Thus, a typical word W will have a certain number $m(W)$ of *missing* letters, i.e., symbols which do not occur in W.

Problem: find $\bar{m}$, the *average* number of missing letters.

Solution: Altogether there are $N = n^n$ words. Make a table of N rows and n columns, as follows. Each row corresponds to a word W, each column to letter. The i, j-th entry of your table will be 1 if the j-th symbol is missing from the i-th word; otherwise it will be 0. Thus, $m(W)$ is just the row-sum of row W, and $\bar{m}$ is the sum over all row-sums divided by N.

But the sum over all row-sums equals the sum over all column-sums. In the j-th column, every 1 indicates a word not containing s_j. There are $(n-1)^n$ such words — see? Hence, the sum over the whole matrix is $n \cdot (n-1)^n$ and $\bar{m}$ is that number divided by N. All told, we have

$$\bar{m} = n \cdot (1 - 1/n)^n \, .$$