Feb. 12, 1994

Suggestions for Math in Grades 8 to 12.

8. Dissection Puzzles. Pythagoras. Angles in a triangle. Theorem of Thales. Euclid's Theorem; quadrature (by dissection) of triangulable planar regions. Elementary constructions with ruler and compass. Emphasis on understanding and explaining phenomena.

Moving into mensuration. The circumference of the earth à la Eratosthenes. Heights, areas, volumes. NOT FORMULAS but processes. Square (and cube) roots à la Newton, geometrically motivated. Indirect measurement using Pythagoras. Approximating the area of the unit disc. Making a table of "rises and runs". Easy applications of similarity, e.g. Thales and the pyramids. Emphasis on "homemade" methods and results.

9. Changes of SCALE. Using the rise-run table (supplemented by graphs and linear interpolation) for a far-reaching extension of indirect measurement, with right-angle triangles as stepping stones. Trigonometry with a human face. NOT LAWS but common sense and ingenuity. Scaling areas and volumes. Cavalieri's Principle; volumes of cones and spheres.

Scales in other settings. Conversion of units (e.g. currencies) back and forth. NOT RULES but explanations that would stand up in court. Repeated multiplication: exponential growth (e.g. compound interest) and decay. Musical scales. Large numbers, fractional exponents (via square roots), and logarithms. Again start with "homemade" tools, later use calculators.

10. Multiplicative counting: multiplying the number of choices available at each stage of a composite task. Tree diagrams. Counting with or without replacement. Permutations. Reshuffling "words" with repeated letters. Binary words. Pascal's triangle.

Probabilities as relative counts. Conditional probabilities. Frequency distributions. The strong resemblence between histograms derived from different lines of Pascal's triangle. Extrapolating and interpreting the limiting case. The normal curve showing how the values of many (but not all!) random variables tend to cluster. Standard deviation versus sample size. Introduction to descriptive statistics.

11. Euclidean constructions with rigorous proofs, say, up to that of the pentagram. Some fallacies, e.g. Dürer's approximately regular pentagon. The mathematics of uncompromising precision.

Analytic plane geometry, especially lines and conic sections, but also some cubic curves. Solving linear and quadratic equations in one or two unknowns. Review of Grade 8, using modern mathematical notation. Some three-dimensional analytic geometry.

12. Functions and graphs. Review of Grade 9, especially trignometry and exponentials, in precalculus mode. Rational functions, partial fractions (by way of practicing symbolic manipulation).

Rate of growth of a function, slope of a graph. Newton quotients. Derivatives of polynomials. Optimization problems. Newton's method. Curve sketching for rational functions.

Mar. 6, 1994

Second Thoughts.

The “curriculum” on the previous page was deliberately written before I had any knowledge of the one in effect now. Surprisingly, the two do not disagree dramatically in either level or contents, but I got it all wrong as far as structure and strategy are concerned. The present B. C. mathematics curriculum is organized in five “strands”:

(i) number and operation
(ii) data analysis
(iii) geometry
(iv) measurement
(v) algebra

all enveloped by a kind of meta-strand called “problem solving”, which, however, is absent from Grades 9 to 11. These strands are revisited every year in spiral fashion, which is “the way people learn”, according to the experts. There is probably some truth in this — provided that learning has indeed taken place, and that the jumping from strand to strand does not appear too capricious to the student.

Since there is no point in railing against a system so ingrained, I will limit myself to some suggestions for its improvement.

Problem solving. This should be the loom on which the five strands are woven together. At present it appears (in Grades K-8 and 12) as a separate entity, raising the spectre of “rote problem solving” — which would be far worse than rote computation. To guard against this danger, every chapter should culminate in one or more problems which can be attacked and clarified, but not entirely solved, by the available techniques. This would also be a good place for collaborative work.

Algebraic overload. Inundated by a flood of disconnected and pointless algebraic manipulations, students reach a point of confusion which makes identities like $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ appear obvious and transparent. To correct this, algebra should be practiced almost exclusively in the two domains it inhabited before 1800: polynomial equations and coordinate geometry.

Strengthening geometry. Geometry, as opposed to measurement, does figure as a separate strand, but a very weak one. Some old-fashioned Euclidean constructions with ruler and compass, supplemented by a set of interesting dissection puzzles, would liven up the scene. Pythagoras, for instance, should be encountered first in the context of adding two squares to make a third — prior to quantification.

Integrating strands. The five strands are a bureaucratic creation. In the class-room, they should be visible (if at all) only as perspectives or modes of operation.

Inservice videos. One of the less pleasant realities of B. C. school math is the fact that 80% of its teachers have no post-secondary mathematical training — which means that most of them have serious difficulties beyond Grade 6. For the sake of scientific honesty, these people must endeavour to *learn math for real* as they teach it. Instead of letting them fumble their way by flipping pages among overviews, curriculum guides, planning guides, and textbooks — how about a set of videos showing the exploration, discussion, and solution of typical “integrative” problems and their spin-offs, for every major item in the curriculum ?