Math 135: A Twelve Week Introduction to Mathematics.

1. From Meno to Pythagoras. The first math lesson on record: Socrates shows how to double a square. Gradually generalizing the method, we finally arrive at the theorem of Pythagoras. No numbers yet: lengths and areas need not always be quantified.

2. Diagonals and Heights. If you can find square roots, Pythagoras facilitates indirect measurement. Concentrating on lengths related to regular polygons, but also trying some three dimensional objects. Making use of scales and proportion.

3. Areas and Volumes. Stacking strips or slabs, you can use heights to determine areas and volumes. Prime examples: triangles and pyramids. Thence (regular) polygons and polyhedra.

4. Cavalieri's Principle. Reinterpreting slices of pyramids, we find the area under a parabola. Comparing a hemisphere (slice-by-slice) with a cratered cylinder, we find the volume of a sphere.

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5. Rational vs. Irrational Numbers. We compute square roots by repeatedly averaging the sides of rectangles (Newton), and approximate π by repeatedly doubling the number of vertices of a polygon (Archimedes). We show that periodic decimals represent rational numbers (cf. Achilles and tortoise), and prove that many square roots (e.g. $\sqrt{2}$) are irrational.

6. The Lore of Large Numbers. We find spatio-temporal representations for several very large quantities, like the national debt. Scientific notation and orders of magnitude. The technique of "counting factors" gives easy estimates for products and quotients.

7. Fractional Powers: Logarithms. Using square roots, we make a table of 10^r, where $r = k/32$ with $0 < k < 32$. The resulting graph gives a way of writing any positive number as 10^a, with obvious benefits for multiplicative calculations.

8. Growth and Decay. Many quantities evolve according to a scheme of the form $Q(t) = Q_0 a^t$, where $a > 1$ (growth) or $a < 1$ (decay). We look at a typical crop of such problems. Pitches in the well-tempered chromatic scale follow such a pattern with $a^{12} = 2$; irrational versus rational ("Pythagorean") musical intervals.

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9. Multiplicative Counting. Multiplying the number of choices available at each stage of a composite task. Counting with or without replacement. Permutations. Probabilties as relative counts.

10. Binomial Coefficients. If k elements become interchangeable, the total count is divided by $k!$. Reshuffling "words" with repeated letters. Binary words. More probabilities.

11. From Pascal to Gauss. Histograms derived from different lines of Pascal's triangle have a strong family resemblence. Extrapolating the limiting case. Probabilistic interpretation.

12. Testing and Sampling. The normal curve shows how the values of many (but certainly not all) random variables tend to cluster. Of course, it is also useful in computing binomial probabilities, especially when the number of "trials" is large. Standard deviation versus sample size.